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# A Multi-Objective Optimization Approach for a Nurse **Scheduling Problem Considering the Fatigue Factor (Case Study: Labbafinejad Hospital)**

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PAPER INFO	ABSTRACT
Chronicle: Received: 02 July 2020 Reviewed: 08 August 2020 Revised: 21 October 2020 Accepted: 11 November 2020	The current study, according to ergonomic factors, aims to model the nurses' work shift scheduling problem. Considering the urgent needs of the hospitals in providing better services to patients, it seems significant to take the preferences of nurses in scheduling shifts into account. Therefore, in this paper, a multi-objective model of nurses' scheduling with emphasis on reducing their fatigue during the career shift is presented. To evaluate the outputs of the model, two numerical instances in small and large sizes
Keywords: Combined Optimization. Nurses' Scheduling. Ergonomic. Fatigue. Meta-Heuristic Algorithm.	with real data of Labbafinejad Hospital were designed in 18-person and 90-person wards. To solve a small size problem, a comprehensive standard decision method is employed, the results of which showed that nurses take their most rest during the night shift and in the middle of their working hours to reduce fatigue. Furthermore, due to the NP-Hard nature of the nurses' scheduling problem, in the problem of the 90-person ward, MOPSO and NSGA II algorithms are applied based on the design of a new chromosome. Using the TOPSIS method and entropy weighting method shows that the designed NSGA II algorithm can solve the nurses' scheduling problem of Labbafinejad Hospital faster and better.

#### 1. Introduction

In recent years, presenting the scientific approaches to make appropriate decisions in various fields of scheduling, where time is an important factor, has attracted the attention of many researchers. The issue of nurses' scheduling, as one of the important issues related to decision-making in hospitals, has received a lot of attention due to health care and the difficulty of nurses' work. In hospitals, the head nurse of each ward usually develops a monthly time schedule and tries to make sure that this schedule is

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compatible with all the restrictions and demands of the nurses and other staff of the [25]. The work schedule should meet the needs of the required number of skilled people in each shift. On the other hand, the program has other restrictions such as people's preferences for days off, the presence of people with special situations such as breastfeeding mothers, or people who are on leave due to illness or other accidents [26]. Mostly, for the nurses working in turning shifts, night shifts can have unpleasant consequences on their normal lives, many of which are out of control. Night shift has negative physical, psychological, and social effects on nurses' personal lives and consequently, these effects can also affect their families and long working hours endanger their health and safety [27]. Nurses are always prone to health threats in various dimensions due to long work shifts and the consequent resulting fatigue. Cardiovascular disease and heart attacks are more common among shift-work nurses than day-work ones. These different work shifts such as working night shifts, on weekdays and at different hours of the day and overtime can be a serious threat to the physical and mental health of nurses. These different work shifts such as working night shifts, on all weekdays and at different hours of a day as well as overtime work can lead to a serious threat to the physical and mental health of nurses [8]. Those nurses, who are not in good general health, will not be able to provide good care such as physical and psychological support to patients. As a result, the risk of mistakes and accidents at work will be increased, and ultimately the consequences of which affect the patient and the nurse [29].

In most developing countries, 5 to 10% of government costs are allocated to the health sector, and among the various components of the health system, hospital services are the major driver of costsgrowth. Hospitals, as the largest center for providing health services, occupy the major part of the resources and credits of the health sector of a country, for instance, in Iran, about 40% of government health expenditures are related to hospital care. However, among the operating costs of the hospital, the costs related to human resources account for the largest share of total hospital costs, and in Iran, on average, the cost of humanpower is estimated about 55-60% of the total costs of the hospital operations. According to research, lack of nursing staff or its inappropriate distribution is usually one of the main problems of hospitals. Therefore, standardizing the number and distribution of nursing staff in clinical wards is necessary to improve the efficiency and quality of services provided to patients, to make the best use of available facilities and improve productivity in hospitals. Nowadays, hospital managers decide to increase their job satisfaction by providing appropriate scheduling by assigning optimal work shifts to nurses leading to improving the quality of services provided to patients [30]. Nowadays, hospital managers decide to increase their job satisfaction by providing appropriate scheduling by assigning optimal work shifts to nurses leading to improving the quality of services provided to patients [30]. In fact, the more the schedule assigned to each nurse is consistent with that nurse's preferences for his/her preferred work shifts, the more patients the nurse will treat with a higher spirit. As a result, the services provided to patients will be of better quality. From this point of view, the issue of scheduling nurses in hospitals has been considered by many researchers in recent years. Considering nurses' scheduling, the number of nurses required to meet the demand for location-shifts is available during the scheduling period, and the purpose of solving the problem is to assign nurses to shifts. So that the demand for shifts is met [31]. Finally, a table presenting the time shifts assigned to nurses is provided and nurses are required to serve them in the assigned work shifts. Meanwhile, even by considering the optimal allocation of work shifts to nurses and taking their preferences into account, nurses can't provide services continuously in each work shift because with each passing hour, nurses' fatigue increases. Therefore, short-term rest in each work shift can lead to rehabilitation in the workforce of nurses so that they can provide services with a higher spirit. As a result, in addition to assigning work shifts to nurses, the fatigue caused by the continuous activity of nurses on each working day should also be considered. Accordingly, in this paper, a mathematical model of nurses' work shift scheduling is presented by considering ergonomic factors. One of the main goals of this paper is presenting the optimal allocation



of work shifts according to nurses' preferences and government laws besides the hospital policies, as well as reducing nurses' daily fatigue by taking short breaks to retrieve nurses. Due to the NP-Hard nature of the nurses' scheduling problem, the Multi-Objective Particle Swarm Optimization (MOPSO) and the Nondominated Sorting Genetic Algorithm II (NSGA-II) as metaheuristic algorithms have been used to solve the model in a case study.

#### 2. Literature Review

The importance of nurses' shift scheduling issues has led to many researchers modeling such issues and offering different methods to solve the problem in recent years. Various features in nursing shift scheduling issues such as cost reduction, social, ergonomics, etc. have been considered by researchers, the most important of which are discussed in this section. A mathematical model for cyclic and non-cyclic planning of 12-hour shift nurses was introduced, and a concept called stint was introduced, a pattern that is characterized by start date, length, cost, and work shifts. Using stints as nodes in a network, a rotation diagram was created on which nurse programs could be defined. The models are shown on data samples from a local hospital [3]. A branch and price algorithm is employed to solve the problem of nurses' shift scheduling. The objectives of the model in this study include the optimal allocation of nurses to work shifts in a way that minimizes the cost of deviations from nurses' work in their non-specialized field [22].

Considering recent studies, the Cplex method has been used to maximize nurses' flexibility in a single-objective problem in which the model shows the optimal allocation of nurses in each work shift [16]. Genetic algorithms have been used to improve nurses' scheduling problem solving time with the aim of minimizing the cost of assigning nurses to lower skill levels and the results show the high efficiency of this algorithm [17]. Refrigeration simulation algorithm has been used to solve the nurses' scheduling model by considering work rules and regulations, hospital policy and with the aim of maximizing nurses' preferences for work shifts and weekends. The results showed that the refrigeration simulation algorithm offers far better solutions than the programs provided by the head nurses [18].

The mathematical model of nurses' work shift scheduling is presented with the aim of maximizing nurses' preferences for work shifts and weekends. In this model, the last days of the previous planning horizon are considered to determine the shift of nurses in the early days of the current planning horizon. Additionally, leave days requested by nurses are not fully considered [19]. A two-step innovative algorithm is proposed to achieve the goals of fairness and flexibility in determining the nurse shift. The first step identifies a combination of shift types, and the second step creates nurse lists that allocate weekend shifts as evenly as possible. The proposed model is based on a case study using data from a US hospital to demonstrate the applicability of the method. It was found that the cost savings and fairness can be achieved through proper shift design [14]. A study has examined the factors that lead to or prevent nurses' fatigue. Interviews were conducted using a qualitative content analysis method by the Patient Safety System Engineering Initiative (SEIPS) model and analyzed. The findings show what nurses perceive as a cause of fatigue and the factors that are beneficial and harmful to coping with fatigue in their work system [12].

A nurse-balanced scheduling model was also developed where the case study was used at a local hospital in Ratchaburi Province to test the model. The aim of this study is to balance the load of each shift for all nurses. Objective constraints were applied to determine positive and negative deviations from the mean load of each shift. The proposed model was solved by Premium Solver in Microsoft Excel and it was found that the balanced load was improved [8]. In the study of a new strategy, a hybrid



approach based on the experimental model of fuzzy regular weight average has been used to identify the best solution to the problem of nurses' scheduling regeneration. This strategy has been used on the nurses' scheduling model at the Vojvodina Oncology Institute in Serbia [21].

In one study, a traditional algorithm was proposed to solve the nurse scheduling problem in which the results of the return algorithm and other innovative algorithms including genetic algorithm and refrigeration simulation were compared. Experimental results showed that the recursive algorithm produces an optimal solution with small applications compared to traditional innovative algorithms [11]. In one study, a proposed mathematical model for the nurses' scheduling problem was proposed, which is based on the idea of a multi-product network flow model. The proposed model was validated by hypothetical cases as well as standard cases, and then it was applied to a real case study in an Egyptian hospital. The results showed the advantage of using the proposed model in producing the program needed to solve the problem [7].

An innovative solution to the nurses' scheduling problem has been proposed. This solution is based on the practice of changing shifts performed by nurses who receive an unfavorable program. Constraints are arranged in order of importance. First, a program is created that meets all the strict constraints and ensures fairness. The second level is trying to meet as many soft constraints as possible while maintaining hard constraints [20]. Using the Gurobi optimizer in Python 3.7, a comparative analysis was presented to examine nurses' scheduling models in terms of target performance and their time complexity. A case study was also presented to analyze the performance of the model. Techniques based on bee colony optimization, simulated annealing and memetic algorithm were studied. Finally, the results were confirmed using statistical analysis [31]. A dual objective model was presented with the aims of minimizing the cost of allocating staff to the skill level and balancing the burden of each shift for all nurses in a nurses' scheduling problem. To form the Pareto Front, the constraint planning technique in Python has been used to solve the problem [13].

A mathematical programming model was presented to maximize nurses' preferences for shift work, and then the Werner fuzzy operator-based fuzzy modeling approach was used, and several randomized test problems were generated and solved using the fuzzy model. In addition, a sensitivity analysis was performed to investigate the effects of parameter changes on the results [9]. To improve productivity, a two-step approach to planning treatment for new patients, planning the nurse needs, and assigning the patient's daily composition to existing nurses was proposed, using a mathematical formula to use the waiting list to use last minute cancellations. In the first step, at the end of each day, an appointment with new patients is made, the nurses' daily needs are estimated, and a waiting list is generated. The second stage assigns patients to nurses while minimizing the number of nurses required [10].

A genetic algorithm was used to solve the nursing scheduling problem at the Bringkoning Community Health Center. It is found that the value of the genetic algorithm parameter affects the optimization results. The small parameter makes the search area in the genetic algorithm more limited, while if the parameter size is too large, it requires more computational time and does not guarantee that for some of those variables it will lead to a desirable value. Therefore, achieving the optimal result of the nursing care program in the emergency room depends on the number of admission days [6].

A two-step strategy is proposed to solve the nurses' scheduling problem. In the first stage, three innovative algorithms - HRA1 (allocation of human resources based on hospital size), HRA2 (allocation of human resources based on average allocation) and HRA3 (allocation of human resources based on the severity of fines) - were proposed for allocating mailboxes. In the second stage, the improved



Particle Swarm Optimization (PSO) algorithm was used to schedule the nursing staff in a reasonable time. The findings of this study help hospital managers make decisions about the allocation and planning of nursing staff [4]. The issue of integrated nursing planning and rescheduling was considered under demand uncertainty. This problem was set and solved as a correct two-step random program. The value of the random solution was estimated using real problem cases on a monthly planning horizon based on data provided by a private health care provider in Ankara [5].

Table 1 examines the most important papers and the differences between the solution methods and the defined goals of nurses' shift work scheduling models. According to the literature and the research background, there is no comprehensive multi-objective model including ergonomic factors such as staff fatigue in a nurses' scheduling problem. Therefore, in the following, a multi-objective problem of nurses' work shift scheduling is modeled by considering ergonomic factors.

# 3. Problem Definition and Modeling

In this case, there is nurse staff with the skills of nurse, practical nurse, and assistant nurse who must be present in the morning, evening, and night shifts according to the predefined schedule in the hospital. Due to the limited number of nurses and the urgent need for any skills in each shift, there is a need for proper staff scheduling. There are various limitations and assumptions regarding nurses' shift work scheduling, imposing of which has led to the complexity of the problem and its proximity to the real world. On the other hand, ergonomic factors are considered in this issue and the designed model seeks to reduce nurses' fatigue and in fact balance their working hours to reduce fatigue during work shifts. Dawson and Fletcher [1, 2] hypothesized that employee fatigue follows a specific rhythm during their working hours, which can be prevented by excessive rest during work. Therefore, along with other aspects of hospital staff work shift scheduling, the goal is to reduce their fatigue

The limitations and assumptions of the nurses' shift scheduling model studied in this article are as follows:

- The number of nurses (nurse, practical nurse, and assistant nurse) in each work shift (morning, evening, and night) is determined in advance daily.
- Each nurse can work at a lower skill level than his real skill.
- Each nurse in each shift of each day can not work at more than one skill level.
- The minimum and maximum working hours of nurses are known every day and every month and this
  range of working hours must be observed for each nurse.
- Each nurse can not work more than 12 hours in a day or 12 hours in a row (morning and night shifts or
  evening and night shifts in one day and also night shifts in one day and morning shifts in the next day
  are not allowed).
- Morning and evening shifts are 6 working hours and night shifts are 12 working hours.
- If a nurse works in the morning and evening shifts of one day or in the night shift, she/he should rest the
  next day and there is no need for her/him to be in the hospital.
- Nurses' fatigue in each work shift is considered as a sinusoidal function.
- The number of night shifts that a nurse can work during the monthly planning period is limited.
- Each nurse can not rest for more than 4 sequent days and not present in the hospital.
- There should be at least one nurse with the highest level of skills in each shift of each day.
- Every nurse tends not to be assigned to work on pre-arranged days and shifts.
- The lower and upper limit on the total number of hours worked by each nurse during a week is clear that should be observed as much as possible.



*Table 1.* Literature review and the comparison of the studies done in the field of nurses' shift work schedule.

Reference	Multi- Objective	The Objective Function	Ergonomic Factors	Model Type	Solution Method
Tsai and Li [23]	-	Maximizing nurses' preferences for work shifts and weekends.	-	MILP	Genetic Algorithm
Maenhout and Vanhoucke [22]	-	Minimizing the cost of allocating staff to the skill level.	-	MILP	Branch & Price
Maenhout and Vanhoucke [24]	*	<ul><li>1- Balancing the load of each shift for all nurses.</li><li>2- Maximizing nurses' preferences for work shifts.</li></ul>	-	MILP	Epsilon Constraint
Kumar et al. [16]	-	Maximizing staff work flexibility. Minimizing the cost of	*	MILP	Cplex
Kim et al. [17]	-	allocating staff to low skill levels.	-	MILP	Genetic Algorithm
Jafari and Salmasi [18]	-	Maximizing nurses' preferences for work shifts and weekends.	-	MILP	Simulated Annealing
Thongsanit et al. [8]	-	Balancing the load of each shift for all nurses.	-	MILP	Premium Solver
Jafari et al. [19]	-	Maximizing nurses' preferences for work shifts and weekends.	-	MILP	Fuzzy Mathematical Model
Ko et al. [11]	-	Maximizing nurses' preferences for work shifts and weekends.	-	MILP	BackTrack Algorithm
Youssef and Senbel [20]	-	Minimizing the cost of allocating staff to lower skill levels.	-	MILP	Simulated Annealing
Alade and Amusat [13]	*	<ul><li>1- Minimizing the cost of allocating personnel to a lower skill level.</li><li>2- Balancing the load of each shift for all nurses.</li></ul>	-	MILP	Constraint Programming Technique
Jafari [9]	-	Maximizing nurses' preferences.	-	MILP	Fuzzy Mathematical Model
Mala Sari Rochman et al. [6]	-	Maximizing nurses' preferences.	-	MILP	Genetic Algorithm
Chen et al. [4]	-	Maximizing nurses' preferences.  1- Minimizing the cost of allocating personnel to the skill level.  2- Minimizing the total deviations from the shifts that personnel tend not to be assigned to work.	-	MILP	Particle Swarm Optimization
The Present Study	*	3- Minimizing the amount of morning and evening shifts that personnel are assigned to work continuously.  4- Minimizing the total deviations from the lower and upper limits on the total number of hours worked.  5- Minimizing the total fatigue of nurses.	*	MINLP	LP-Metrics - MOPSO – NSGA II



The aims of this issue are divided into two types, the objectives of employers and the preferences of nurses. If nurses of any skill level are assigned to work at a lower skill level than their actual skills, employers will have to pay a fine. Therefore, employers tend to pay the minimum penalty for assigning nurses at lower skill levels than their actual skills. Nurses also tend to work as many hours as they choose during the planning period and not be assigned to work on the days or shifts that they themselves have announced at the beginning of the planning period. Therefore, 5 different objective functions have been considered for the problem, which include minimizing (1) the cost of assigning a nurse to a skill level lower than their actual skill level, (2) the total deviations from the days and shifts that nurses tend not to assign to work, (3) the amount of morning and evening shifts that nurses are assigned to work continuously, (4) the sum of deviations from the lower and upper limits on the total number of hours worked by nurses during a week, and (5) the sum of maximum total fatigue of nurses every day and in every work shift.

According to the assumptions, limitations, and explanations of the problem, the nurses' work shift scheduling model will be as follows:

#### 3.1. Sets

```
I Set of nurses (i = 1,2,...,I);

J Shift set (j = 1,2,...,J);

K Set of working days (k = 1,2,...,K);

LK Set of working weeks (lk = 1,2,...,LK);

S Set of skill levels (s = 1,2,...,S).
```

As mention previously in this section, three work shifts and three different skill levels are considered. The work shifts are presented as morning shift (j = 1), evening shift (j = 2) and night shift (j = 3). Besides, the level of nursing skills is indicated by s = 1, the level of practical nursing skills is shown by s = 2, and the level of assistant nursing skills is represented by s = 3.

# 3.2. Parameters

```
h_{kj}
           Shift length j per day k;
 Dh_{max}
           Maximum working hours of nurses per day;
 Wh_{min}
           Minimum working hours of nurses per week;
           Maximum working hours of nurses per week;
Wh_{max}
 Mh_{min}
           Minimum working hours of nurses per month;
Mh_{max}
           Maximum working hours of nurses per month;
 RN_{kis}
           Total number of nurses required at skill level s in shift j from day k;
           The maximum number of night shifts that a nurse can work during a one-month
Max_{night}
           planning period;
           The amount of penalty for assigning nurse i to work at a lower skill level in shift j from
   \alpha_{ki}^{l}
           day k;
   k_i
           The set of days that the nurse i tends not to be assigned to work in some or all of the
           shifts;
           The set of shifts from day k_i that nurse i tends not to be assigned to work;
  j_{k_i}
           The set of half-hour intervals in each shift j;
 T_{j-half}
           Parameter (0 and 1) that takes 1 if nurse i can be assigned to work at her actual skill
 RSL_{kjs}^{\iota}
           level or any lower skill level s in shift j from day k; otherwise it takes the value 0.
```



#### 3.3. Decision variables

- $d_{ki}^{1i}$ Deviation from days or shifts that the nurse i does not want to be assigned to work;
- $d_{k_1}^{2i}$ Deviation from the lower limit on the total number of hours worked by nurse i during a week;
- Deviation from the upper limit on the total number of hours worked by nurse i during a week;
- $d_{k_1}^{3i}$   $d_{kjs}^{4i}$ The rest rate of the nurse i with skill level s while working in shift j from day k;
- $r_{kits}^i$ The fatigue score of the nurse i with skill level s while working in shift j from day k at the hour t;
- The fatigue score of the nurse i with skill level s while working in shift j from day k at the  $f_{kits}^{i}$ end of hour t:
- $x_{kis}^i$ Takes 1 if nurse i is assigned to work at skill level s in shift j from day k, otherwise it takes the value 0;
- $0_k^i$ Takes 1 if nurse i is assigned to work continuously in the morning and evening shifts from day k, otherwise it takes the value 0;
- $F_k^i$ Takes 1 if nurse i is assigned to work in night shift from day k, otherwise it takes the value 0;
- Takes 1 if nurse i is assigned to work at skill level s in shift i from day k in half an hour t,  $\omega_{kits}^{i}$ Otherwise it takes the value 0.

## 3.4. Modeling of the Multi-Objective Problem of Nurses' Work Shift Scheduling

$$Z_{1} = Min \sum_{i} \sum_{k} \sum_{i} \sum_{s} \left[ \alpha_{kj}^{i} \cdot (s - RSL_{kjs}^{i}) \cdot X_{kjs}^{i} \right], \tag{1}$$

$$\mathbf{Z}_2 = \operatorname{Min} \sum_{\mathbf{i}} \sum_{\mathbf{k} \in \mathbf{k}_i} \sum_{\mathbf{j} \in \mathbf{i}_{\mathbf{k}}} \mathbf{d}_{\mathbf{k}\mathbf{j}}^{1\mathbf{i}}, \tag{2}$$

$$Z_3 = Min \sum_{i} \sum_{k} O_k^i, \tag{3}$$

$$Z_4 = Min \sum_{i} \sum_{k_1=1}^{|LK|} (d_{k_1}^{2i} + d_{k_1}^{3i}), \tag{4}$$

$$Z_5 = Min(\sum_{i} max_{k,j,t,s} f_{kjts}^i), \tag{5}$$

$$\sum_{i} \sum_{s} h_{kj} x_{kjs}^{i} \le Dh_{max}, \quad \forall i, k$$
 (6)

$$\sum_{k} \sum_{i} \sum_{s} h_{kj} x_{kjs}^{i} \ge M h_{min}, \quad \forall i$$
 (7)

$$\sum_{k} \sum_{i} \sum_{s} h_{kj} x_{kjs}^{i} \le Mh_{max}, \quad \forall i$$
 (8)

$$\sum_{i} x_{kjs}^{i} = RN_{kjs}, \quad \forall k, j, s$$
 (9)

$$\sum_{s} x_{kjs}^{i} \le 1, \quad \forall i, j, k$$
 (10)

$$\mathbf{x}_{kjs}^{i} \le RSL_{kjs}^{i}, \quad \forall i, k, j, s$$
 (11)

$$\sum_{k=k}^{k_1+1} \sum_{s} x_{k(j \in 3)s}^{i} \le 1, \quad \forall i, k_1 < K$$
 (12)



$$\sum_{k} \sum_{s} x_{k(j\in3)s}^{i} \le \max_{\text{night}}, \quad \forall i$$
 (13)

$$\sum_{s} x_{k(j \in 1)s}^{i} + \sum_{s} x_{k(j \in 3)s}^{i} \le 1, \quad \forall i, k$$
 (14)

$$\sum_{s} x_{k(j\in2)s}^{i} + \sum_{s} x_{k(j\in3)s}^{i} \le 1, \quad \forall i, k$$
 (15)

$$\sum_{s}^{i} x_{k(j \in 3)s}^{i} + \sum_{s}^{i} x_{(k+1)(j \in 1)s}^{i} \le 1, \quad \forall i, k < K$$
 (16)

$$\sum_{j} \sum_{s} x_{kjs}^{i} \le 2, \quad \forall i, k$$
 (17)

$$0_k^i - \sum x_{k(j \in 1)s}^i \le 0, \quad \forall i, k$$
 (18)

$$0_k^i - \sum_{s} x_{k(j \in 2)s}^i \le 0, \quad \forall i, k$$
 (19)

$$0_k^i - \sum_{s} x_{k(j \in 1)s}^i - \sum_{s} x_{k(j \in 2)s}^i \ge -1, \quad \forall i, k$$
 (20)

$$\sum_{s} x_{k(j \in 1)s}^{i} + \sum_{s} x_{k(j \in 2)s}^{i} + \sum_{j} \sum_{s} x_{(k+1)js}^{i} + 0_{k}^{i} \le 3, \quad \forall i, k < K$$
 (21)

$$F_k^i - \sum_s x_{k(j \in 3)s}^i = 0, \quad \forall i, k$$
 (22)

$$\sum_{s} x_{k(j \in 3)s}^{i} + \sum_{j} \sum_{s} x_{(k+1)js}^{i} + F_{k}^{i} \le 2, \quad \forall i, k < K$$
 (23)

$$\sum_{k=1}^{1+4} \sum_{i} \sum_{s} x_{ljs}^{i} \ge 1, \quad \forall i, k \le K - 4$$
 (24)

$$\sum_{i} x_{kj(s\in 1)}^{i} \ge 1, \quad \forall j, k$$
 (25)

$$\sum_{i} x_{kj(s\in 1)}^{i} \ge 1, \quad \forall j,k$$

$$\sum_{i} x_{kjs}^{i} - d_{kj}^{1i} = 0, \quad \forall i,k \in k_{i}, j \in j_{k_{i}}$$
(25)

$$\sum_{k=7k_{1}-6}^{7k_{1}} \sum_{j} \sum_{s} h_{kj} x_{kjs}^{i} + d_{k_{1}}^{2i} \ge Wh_{min}, \quad \forall i, k_{1} \in LK$$
 (27)

$$\sum_{k=7k_1-6}^{7k_1} \sum_{j} \sum_{s} h_{kj} x_{kjs}^i - d_{k_1}^{3i} \le Wh_{max}, \quad \forall i, k_1 \in LK$$
 (28)

$$r_{kjts}^{i} = \frac{1}{2} sin \left( \frac{2\pi}{24} (t+1) \right) + 1.5, \quad \forall i, j, k, t \le T_{j-half}, s \tag{29}$$

$$f_{kjts}^{i} = f_{kj(t-1)s}^{i} + \omega_{kjts}^{i}.r_{kjts}^{i} - 2(1 - \omega_{kjts}^{i}).r_{kjts}^{i}, \quad \forall i, j, k, t \le T_{j-half}, s$$

$$T_{i-half}$$
(30)

$$\sum_{k=1}^{1} 0.5(\omega_{kjts}^{i}) + d_{kjs}^{4i} = h_{kj}x_{kjs}^{i}, \quad \forall i, j, k, s$$
 (31)

$$d_{kjs}^{4i} \le \frac{T_{j-half}}{12}, \quad \forall i, j, k, t, s \tag{32}$$

$$d_{kj}^{1i}, d_{k_1}^{2i}, d_{k_1}^{3i}, d_{kjs}^{4i}, r_{kjts}^i, f_{kjts}^i \ge 0, \tag{33}$$

$$\mathbf{x}_{kjs}^{i}, \mathbf{O}_{k}^{i}, \mathbf{F}_{k}^{i}, \boldsymbol{\omega}_{kjts}^{i} \in \{0,1\}.$$
 (34)



In this modeling, Eq. (1) minimizes the cost of assigning nurses to the skill level below their actual skill level. Eq. (2) shows the minimization of the total deviations from the days and shifts that nurses tend not to assign to work. Eq. (3) minimizes the number of morning and evening shifts that nurses are assigned to work continuously. Eq. (4) describes the fourth objective function of the problem and involves minimizing the sum of deviations from the lower and upper bounds on the total number of hours worked by nurses during a week. Eq. (5) sums the total maximum fatigue score of all nurses in each skill level in each shift of each day. This objective function actually reduces nurses' fatigue during work shifts. Eq. (6) illustrates the maximum number of working hours that each nurse can work in a day. Relationships (7) and (8) show the minimum and maximum number of working hours that each nurse can work in a one-month planning period, respectively. Eq. (9) also shows the total number of case nurses at each skill level in each shift of each day. Eq. (10) ensures that each nurse in each shift of each day can not work at more than one skill level. Eq. (11) ensures that each nurse can work at their actual skill level or at a lower skill level. Eq. (12) indicates that night shifts are not permitted. Eq. (13) presents the maximum number of night shifts that each nurse can work in a one-month planning period. Relationships (14) - (17) show that morning and night shifts or evening and night shifts on one day as well as night shifts on one day and morning shifts on the next day are not allowed, meaning that each nurse cannot work more than 12 hours a day or 12 hours in a row. Relationships (18) - (23) express that if a nurse works one day in the morning and evening shift or night shift, the next day should not be working. Eq. (24) ensures that each nurse can not rest more than 4 days in a row and not be assigned to any shifts. Eq. (25) ensures that there should be at least one nurse with the highest level of skill in each shift of each day. Eq. (26) shows that every nurse tends not to be assigned to work on pre-determined days and shifts. Eqs. (27) and (28) show the upper and lower bounds on the total number of hours worked by each nurse during a week. Eq. (29) shows the nurse's fatigue score every half hour of her shift. Eq. (30) also shows the nurse's fatigue score at the end of each half hour of work, taking into account her rest. Eq. (31) indicates the total amount of nurse rest hours in each shift of each working day. Eq. (32) presents the maximum number of half-hour breaks for a nurse in each work shift of each day. Relationships (33) and (34) show the type and gender of decision variables.

#### 4. Solution Methods and Comparison Indicators

After presenting the mathematical model of the multi-objective problem of nurses' work shift scheduling, in this section, the solution methods, as well as the comparison indicators of the solution methods used in the article are presented. Due to the multi-objective nature of the developed mathematical model, different methods should be used to find the Pareto front and efficient solutions. Therefore, to solve the developed multi-objective problem, comprehensive standard decision methods, as well as NSGA II and MOPSO meta-heuristic algorithms, have been used. In the following, each of the solution methods as well as the primary chromosomes of the meta-heuristic algorithms are described.

#### 4.1. LP-Metrics Method

In LP-Metrics, it is necessary to obtain the best value of each objective function by individual optimization method. That is, the value of each objective function must first be obtained without considering the other objective function to be used in the calculations. *Eq.* (35) shows the multi-objective decision-making method named LP-Metrics.



$$L_{p} = \left\{ \sum_{i=1}^{n} w_{i} \left[ \frac{(f_{i} - f_{i}^{*})}{(f_{i}^{*})} \right]^{p} \right\}^{\frac{1}{p}} , p \ge 1$$
 (35)

In the above relation  $w_i$  is the weight assigned to each objective function,  $f_i$  is the objective function of the problem and  $f_i^*$  is the optimal value of each objective function obtained from the individual optimal method. Also, the soft p (norm) of the problem indicates that in this paper the linear norm, ie (p = 1)has been used to solve the multi-objective problem.

# 4.2. MOPSO Algorithm

Eberhart and Kennedy [15] first proposed a method called particle motion by modeling the movement of birds in the air and discovering the logical relationship between the change of direction and speed of birds and using the knowledge of physics. The scientists later discovered in their research that these movements were interdependent and found that a bird's movement was due to information it received from birds around it, so they completed the proposed method and named it a swarm movement. In general, the cumulative particle motion algorithm has many similarities with algorithms such as ants or genetics, but there are serious differences with them that make this algorithm different and simple. For example, this algorithm does not use operators such as crossover and mutation, so the algorithm does not require the use of number strings and the decryption step, so it is much simpler than algorithms such as genetics. This algorithm divides the solution space into several paths using a quasi-probabilistic function, which are formed by the movement of individual particles in space. The motion of a group of particles consists of two main components (definite and probable). Each particle is interested in moving in the direction of the best current answer  $x^*$  or the best answer obtained so far  $g^*$ . Eqs. (36) and (37) show the speed and motion functions of birds in each iteration of the MOPSO algorithm.

$$\begin{aligned} V_i^{t+1} &= wV_i^t + C_1 rand(pbest_i - X_i^t) + C_2 rand(gbest_i - X_i^t), \\ X_i^{t+1} &= X_i^t + V_i^{t+1}. \end{aligned} \tag{36}$$

$$X_{i}^{t+1} = X_{i}^{t} + V_{i}^{t+1}. (37)$$

In the above relations,  $V_i^{t+1}$  is the velocity of the particle i in the new iteration t.  $V_i^t$  is the velocity of the particle i in the current iteration t.  $X_i^{t+1}$  is the current position of the particle t+1.  $X_i^t$  is the position of the particle in the new iteration, phest; is the best position has ever taken by the particle i. rand is the best position of the best particle (the best position that all particles have ever taken). The rand is a random number between zero and one that is used to maintain group diversity.  $C_1$  and  $C_2$  are cognitive and social parameters, respectively. Selecting the appropriate value for these parameters leads to accelerating the convergence of the algorithm and preventing premature convergence in local optimizations. Recent research shows that choosing a larger value for the cognitive parameter  $C_1$  is more appropriate than the social parameter  $C_2$ . The parameter w is called weight inertia, which is used to ensure convergence in the particle group. Gravitational inertia is used to control the effect of previous velocity records on current velocities.

### 4.3. NSGA II Algorithm

Genetic algorithms start by randomly generating an initial population of chromosomes, while satisfying the boundaries or limitations of the problem. In other words, chromosomes are strings of proposed values for problem-solving variables, and each represents a possible answer to the problem. Chromosomes are inferred from consecutive repeats called generations. During each generation, these



chromosomes are evaluated according to the goal of optimization, and the chromosomes that are the better answer to the problem are more likely to reproduce the answers to the problem. It is important to formulate the chromosome evaluation function to help speed up the convergence of calculations toward the general optimal answer. Because in the genetic algorithm, the value of the evaluation function must be calculated for each chromosome, and because we usually have a significant number of chromosomes in many problems, the time consuming calculation of the evaluation function can make it practically impossible to use the genetic algorithm in some problems. Therefore, based on the obtained values of the objective function in the population of strings, a fit number is assigned to each string. This fitness number will determine the probability of selection for each discipline. Based on this selection probability, a set of strings is selected first. To produce the next generation, new chromosomes called offspring are created by linking two chromosomes from the current generation using a combination operator or by modifying the chromosome using a mutation operator. Therefore, new strings replace strings from the original population so that the number of strings in the various computational iterations is constant. The random mechanisms that act on the selection and removal of strings are such that strings that are more fitting are more likely to combine and produce new strings and are more resilient than other strings in the replacement phase. Thus, the population of sequences in a competition based on the objective function is completed over different generations and increases by the value of the objective function in the population of the strings, so that after several years, the algorithm converges to the best chromosome, which hopefully indicates optimal or suboptimal solution. In general, in this algorithm, while in each computational iteration, new points of the response space are searched by genetic operators, by the selection mechanism, it explores the space in which the statistical average of the objective function is higher. Usually the new population that replaces the previous population is more appropriate. This means that the population is improving from generation to generation. The search will be fruitful when we have reached the maximum possible generation, or if convergence has been achieved, or the cessation criteria have been met, and as a result, the best chromosome obtained from the last generation will be selected as the optimal solution or optimal solution to the problem.

#### 4.3.1. Initial chromosome design

The most important part in designing and using algorithms in problem solving is how to define problem variables called problem chromosomes. In this section, the primary chromosome design of the nursing shift work schedule in the hospital is discussed. According to the model assumptions, in this article, three types of personnel (nurse A, practical nurse B and assistant nurse C) are considered who should be present in the work shifts according to the predefined schedule. Therefore, to define the primary chromosome as well as its decoding, a schedule with four type A personnel, three type B personnel and two type C personnel is considered for 3 working days. *Fig. 1* shows the basic information and the initial chromosome of the nurses' shift work schedule.

Day	Personnel allocation	A1	A2	A3	A4	B1	B2	В3	C1	C2
first day	Personnel	2	4	3	1	-	-	-	-	-
second	type A	4	1	3	2	-	-	-	-	-
day										
third day		1	3	2	4	-	-	-	-	-
first day	Personnel	3	1	4	5	2	6	7	-	-
second	type A and B	5	2	7	3	6	4	1	-	-
day										
third day		3	2	1	4	5	4	2	-	-
first day	Personnel	2	3	1	8	9	4	6	7	5
second	type A and B	4	3	5	7	9	1	8	6	2
day	and C									
third day		5	7	6	2	9	4	3	8	1
	The first	shift		The	second sl	nift		The thir	d shift	
first day	1 C	1 B	-	1 C	-	i	IA	-	2 B	1 A
second day	1 C	-	1 A	-	1 B	-	-	1 C	-	IA
third day	-	-	1 A	1 C	1 B	i	l A	1 C	1 B	-

Fig. 1. Initial chromosome for the nurses' work shift scheduling problem.

According to Fig. 1, the primary chromosome is the replacement for number of personnel of type A, B and C per day. Fig. 1 also shows the number of personnel required per day and per shift. The following are the steps for decoding the primary chromosome.

<u>Step 1.</u> To decipher the problem, first the assignment of type A personnel and from the third shift is done. Therefore, according to the assumptions of the problem, the highest priority of the primary chromosome is selected on the first day and the personnel is assigned to the third shift. According to the random priority created in *Fig. 1*, personnel A2 are assigned as the first personnel to the first day and the third shift. In this case, due to the non-assignment of personnel A2 on the second day (according to the assumptions of the problem), the priorities related to personnel A2 on the second day will be reduced to zero. After assigning personnel in the third shift, type A personnel will be assigned in the second and first shifts. *Fig. 2* shows the allocation of type A personnel according to the assumptions of the problem based on chromosome in *Fig. 1*.

Personnel allocation	A1	A2	<b>A3</b>	<b>A4</b>	The third shift
first day	2	4	3	1	A2
		₩			
second day	4	0	3	2	AI
third day	0	3	2	4	-
			$\prod$		
Personnel	I				The second
Personnel allocation	A1	A2	A3	<b>A4</b>	The second shift
	<b>A1</b>	<b>A2</b>	A3 3	<b>A4</b>	
allocation					shift
allocation					shift
allocation first day	2	0	3	1	shift
allocation first day	2	0	3	1	shift



Personnel allocation	A1	A2	A3	A4	The first shift
first day	2	0	0	1	-
second day	0	0	3	2	A3
third day	0	3	2	0	A2
			<b></b>		

	The first shift	The second shift The	third shift
first day	-	A3	A2
second day	A3	-	AI
third day	A2	A4	-

Fig. 2. Assignment of type A personnel based on initial chromosome decoding.

<u>Step 2.</u> After assigning type A personnel in each shift and every day, type B personnel will be assigned from the third shift. Given that type A personnel have the ability to perform the activities of type B personnel, so the chromosome of the second part is a combination of type A and B personnel. For example, personnel A1 and A4 along with personnel B1, B2 and B3 can be assigned as Type B personnel. *Fig. 3* shows the allocation of Type B personnel according to the assumptions of the problem based on the chromosome of *Fig. 1* and the modified *Fig. 2*.

Personnel allocation	A1	A2	A3	<b>A4</b>	B1	B2	В3	The third shift
first day	3	0	0	5	2	6	7	B2-B3
second day	0	0	0	3	6	0	0	-
third day	0	0	1	0	5	4	2	B1
					ļ			
				_				
Personnel allocation	A1	A2	A3	A4	B1	B2	В3	The second shift
	<b>A1</b>	<b>A2</b>	<b>A3</b>	<b>A4</b>	<b>B1</b>	<b>B2</b>	<b>B3</b>	second
allocation								second shift



Personnel allocation	A1	<b>A2</b>	<b>A3</b>	<b>A4</b>	B1	B2	В3	The first shift
first day	3	0	0	5	2	0	0	A4
second day	0	0	0	3	0	0	0	-
third day	0	0	1	0	0	4	2	-

	The first shift		The second shift			
first day	A4	-	-	A3	B2-B3	A2
second day	-	A3	BI	-	-	AI
third day	-	A2	-	A4	<i>B1</i>	-

Fig. 3. Assignment of type A and B personnel based on initial chromosome decoding.

<u>Step 3.</u> After assigning type A and B personnel in each shift and every day, type C personnel will be assigned from the third shift. Given that type A personnel have the ability to perform the activities of type B and C personnel and also type B personnel have the ability to perform the activities of type C personnel, so the chromosome of the third part is a combination of type A, B and C personnel. *Fig. 4* shows the allocation of Type C personnel according to the problem assumptions based on chromosomes in *Fig. 1* and the modified *Fig. 2* and *Fig. 3*.

Personnel allocation	A1	A2	A3	<b>A4</b>	B1	B2	В3	<b>C</b> 1	C2	The third shift
first day	0	0	0	0	9	0	0	7	5	-
second day	0	0	0	0	0	0	0	6	2	CI
third day	0	0	6	0	0	4	3	0	1	<i>A3</i>
						7				
Personnel allocation	A1	A2	<b>A3</b>	<b>A4</b>	B1	B2	В3	C1	C2	The second shift
	<b>A1</b>	A2	<b>A3</b>	<b>A4</b>	<b>B1</b>	<b>B2</b>	<b>B3</b>	<b>C1</b>	<b>C2</b>	second
allocation										second shift
first day	2	0	0	0	9	0	0	7	5	second shift



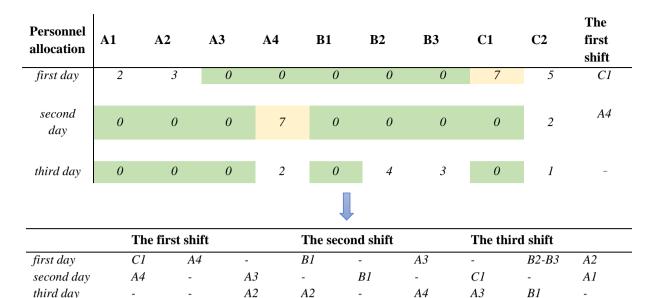


Fig. 4. Assignment of type A, B and C of personnel based on the initial chromosome decoding.

The above decoding should be updated and modified based on the following assumptions at each stage:

- If the staff is working in the third shift, they should not be working the next day.
- There is a limit to the number of third shifts for staff.
- Should not be assigned to any other shift if each staff member's working hours are exceeded.
- Every day, each staff member can be employed in only one skill.
- If the staff is working in the first and second shifts, they are exempted from working in the third shift and the next day.

# 4.3.2. Combining operators

The Combining operators is one of the operators of the NSGA II algorithm, which is modeled by changing the genes of the parents to create new children and improve fitness. In this paper, a single point combination is used to combine chromosomes. *Fig.* 5 shows the combination on one of the problem chromosomes.

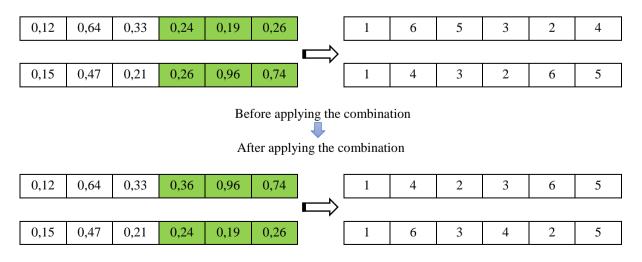


Fig. 5. Performing a single point combination.



#### 4.3.3. Mutation operator

Mutation is another operator that produces other possible answers. In a genetic algorithm, after a member is created in a new population, each of its genes mutates with a certain probability of mutation. In mutation, a gene may be removed from a population of genes or a gene that did not previously exist in the population may be added. In this paper, the transfer mutation is used for the mutation operator between the problem chromosomes. *Fig.* 6 shows how this chromosome is applied and converted into a valid answer.

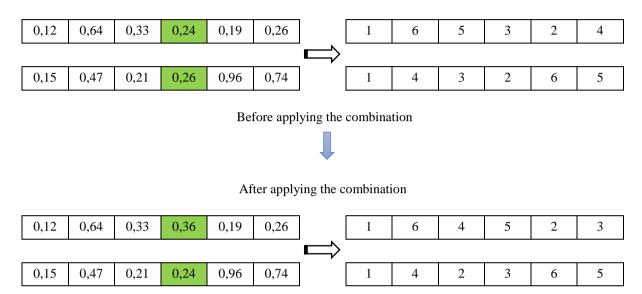


Fig. 6. How the mutation operator works.

Due to the continuous space of NSGA II and MOPSO algorithms, as well as the discrete space of the primary chromosome response space, it is possible to create an impossible answer at any stage of the algorithm repetition. Therefore, using the following mechanism, the continuous answers generated in each iteration of the algorithm are converted to discrete answers. *Fig.* 7 shows how to convert continuous space to discrete space. In this mechanism, the largest continuous number is identified and the corresponding house number becomes the highest priority of discrete space. Then the next largest number in continuous space is selected and the highest priority of discrete space is assigned to that house number. This operation continues until the last continuous number is converted to discrete.

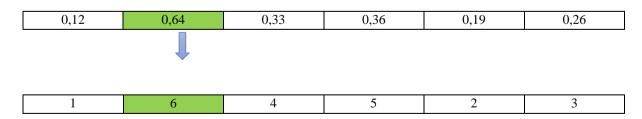


Fig. 7. Mechanism of conversion for continuous to discrete space.



## 4.4. Comparative Indicators of Metaheuristic Algorithms

One of the problems in solving multi-objective problems is how to evaluate the quality of the final solutions, which can sometimes be complicated due to the conflicting used goals. For this purpose, in the early 1990s, visual (observational) methods were used to compare Pareto collections. Convergence to Pareto optimal solutions and providing density and variability among the obtained set of solutions are the two main goals of any multi-objective evolutionary algorithm. But since these two goals are somewhat at odds with each other, there is no standard by which to make an absolute decision about the performance of algorithms. Therefore, to evaluate the performance of the proposed algorithms, the following criteria are used:

- Computational time: An algorithm that has less computational time will be more desirable.
- Number of answers in Pareto: Shows the number of unsuccessful answers in the Pareto set obtained for each problem.
- Maximum expansion: This criterion shows how many of the answers of a Pareto set are distributed in the answer space, which is calculated from Fig. 38. The larger the value of this criterion, the better the variety of Pareto set answers.

$$MSI = \sqrt{\sum_{m=1}^{M} \left( \max_{i=1:|Q|} f_m^i - \max_{i=1:|Q|} f_m^i \right)^2}.$$
 (38)

– Distance: Indicates the uniformity of the answers, which is calculated using Eq. (39).

$$SM = \sqrt{\frac{1}{|Q|} \sum_{i=1}^{|Q|} (d_i - \bar{d})^2}.$$
 (39)

In the above relation, /Q / represents the size of the Pareto archive, and the values  $d_i$  and  $\bar{d}$  can be calculated from Eqs. (40) and (41), respectively. An algorithm with less of this criterion would be more desirable.

$$d_i = \min_{k \in Q \cap k \neq i} \sum_{m=1}^{M} \left| f_m^i - f_m^k \right|. \tag{40}$$

$$\bar{\mathbf{d}} = \sum_{i=1}^{|\mathbf{Q}|} \frac{\mathbf{d_i}}{|\mathbf{Q}|}.\tag{41}$$

 Distance from the ideal point: This criterion is used to measure the degree of proximity to the optimal level of the real Pareto, which is calculated from Eq. (42):

$$MID = \frac{\sum_{i=1}^{n} c_i}{n}.$$
 (42)



In this relation n is the number of answers in the Pareto optimal set and  $c_i$  is the Euclidean distance of each member of the Pareto set from the ideal point which is calculated from Eq. (43):

$$c_{i} = \sqrt{(f_{1i} - f_{1}^{*})^{2} + (f_{2i} - f_{2}^{*})^{2} + \dots + (f_{mi} - f_{m}^{*})^{2}}.$$
(43)

# 4.5. Parameter Setting of Metaheuristic Algorithms

In this section, the basic parameters of MOPSO and NSGA II meta-heuristic algorithms are set by Taguchi method. In Taguchi method, first the appropriate factors should be identified and then the levels of each factor should be selected and then the appropriate test design for these control factors should be determined. Once the test design is determined, the experiments are performed and the experiments are analyzed in order to find the best combination of parameters. In this study, for each factor, 3 levels are considered according to Table 2. For each algorithm, according to the number of factors and the number of their levels, the design of the experiment and their execution are determined. Given the multiobjective nature of the proposed model, the value of each experiment must first be calculated from Eq. (44). This relation is used in case of subtraction of the indicators used in comparison of meta-heuristic algorithms including indicators (number of answers in Pareto, maximum expansion, distance, distance from the ideal point and computational time). After determining the value of each experiment, the scaled value of each experiment, RPD, is calculated from Eq. (45) to analyze the design of the Taguchi experiment.

$$S_{i} = \left| \frac{NPF + MSI + SM + MID + CPU\_time}{5} \right|. \tag{44}$$

$$S_{i} = \left| \frac{\text{NPF} + \text{MSI} + \text{SM} + \text{MID} + \text{CPU\_time}}{5} \right|. \tag{44}$$

$$RPD = \frac{S_{i} - S_{i}^{*}}{S_{i}^{*}}. \tag{45}$$

In Relation (45),  $S_i$  is the index value obtained from each Taguchi experiment and  $S_i^*$  is the best index value among all Taguchi experiments.

Table 2. Levels of proposed parameters for parameterization of meta-heuristic algorithms by Taguchi method.

Algorithm	Parameter	Symbol	Level 1	Level 2	Level 3
	Maximum number of repetitions	Max it	50	100	200
NSGA II	Number of population	Npop	50	100	200
NSGA II	Combination rate	Pc	0,3	0,5	0,7
	Mutation rate	Pm	0,3	0,5	0,7
	Maximum number of repetitions	Max it	50	100	200
	Number of particles	N particle	50	100	200
MOPSO	Individual learning coefficient	C1	1	1,5	2
	Collective learning coefficient	C2	1	1,5	2
	Gravity coefficient	w	0,7	0,8	1

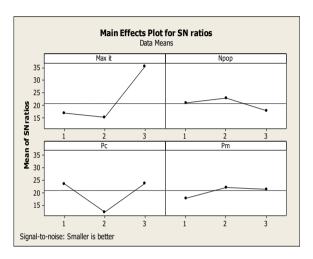
By comparing the difference between the maximum and minimum values obtained in the NSGA II algorithm for the SN index, the significant effect of the Max it parameter (maximum number of repetitions) in improving the solution process of the NSGA II algorithm is evident. The parameters Pm (Mutation rate), Npop (population size) and Pc (combination rate) are in the next influential ranks, respectively (Table 3).



Level	Maximum number of Repetitions	Number of Population	Combination Rate	Mutation Rate
1	0,14794	0,15478	0,09527	0,08686
2	0,19249	0,11135	0,16801	0,11512
3	0,01125	0,08556	0,08840	0,14970
Delta	0,18146	0,06922	0,07961	0,06284
Rank	1	3	2	4

**Table 3.** Results of parameter analysis of NSGA II algorithm.

Fig. 8 shows the mean S / N ratio and the mean for the NSGA II algorithm. As stated, the maximum value of the SN criterion is the criterion for selecting the values of the parameters.



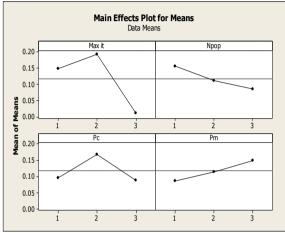


Fig. 8. Average graph of S / N ratio and average of averages in NSGA II algorithm.

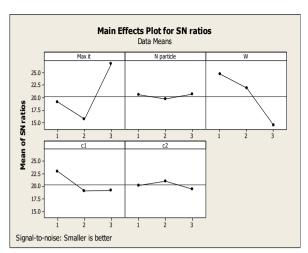
According to the results shown in *Fig.* 8, the NSGA II algorithm will be most effective if the maximum number of iterations is at level 3, the population at level 2, the combination rate at level 3, and the mutation rate at level 2. By comparing the difference between the maximum and minimum values obtained in the MOPSO algorithm for the SN index, the significant effect of the Max it parameter (maximum number of iterations) in improving the solution process of the MOPSO algorithm is evident. The parameters w (gravity coefficient), c1 (individual learning coefficient), Nparticle (number of particles) and c2 (collective learning coefficient) are in the next ranks of influence (*Table 4*).

Maximum Individual Collective Number of Gravity Level Number of Learning Learning **Particles** Coefficient Repetitions Coefficient Coefficient 1 0,13846 0,11466 0,08044 0,10832 0,13885 2 0,19160 0,14906 0,10510 0,12855 0,13836 3 0,06227 0,12861 0.20678 0,15546 0,11511 Delta 0,12933 0,12634 0,02374 0,03440 0,04713 Rank 1 2 3 5

**Table 4.** Results of parameter setting analysis of MOPSO algorithm.

Fig. 9 shows the mean S / N ratio and the mean for the MOPSO algorithm. As stated, the maximum value of the SN criterion is the criterion for selecting the values of the parameters.





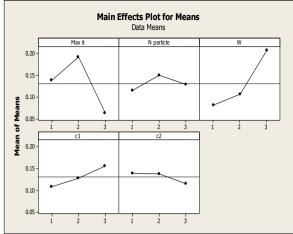


Fig. 9. Average graph of S/N ratio and average of averages in MOPSO algorithm.

According to the observable results of *Fig. 9*, if the maximum number of repetitions is at level 3, the number of particles at level 2, the gravity coefficient at level 3, the individual learning coefficient at level 3, and the collective learning coefficient at level 1, the MOPSO algorithm will be most efficient.

# 5. Results Analysis

In this section, in order to investigate the issue of multi-objective scheduling of nurses' work shifts by considering ergonomics, a small example is considered for Shahid Labbafinejad Hospital in Iran. Dr. Labbafinejad Hospital started operating in 1981 and gradually continued its growth trend with the provision of facilities and equipment and began its serious activities by attracting committed, and specialized medical personnel. Simultaneously with the establishment of the hospital, preparations were made for its incorporation into the educational system. At present, this center is one of the hospitals of the Social Security Organization, but it is under the supervision of Shahid Beheshti University of Medical Sciences. All wards of the hospital have been approved by the Secretariat of the Medical and Specialized Council, and specialized and sub-specialized assistants, postgraduate students, interns and interns, in other words, all-inclusive categories are engaged in training in this center. The total number of nurses (including nurses, practical nurses and assistant nurses) in the study ward of Shahid Labbafinejad Hospital is 18 people who work in three shifts during 24 hours. The length of the morning and evening shifts is 6 hours and the length of the night shifts is 12 hours. All nurses are divided into 3 skill levels: nurse, practical nurse and assistant nurse. The nurse skill level is the highest skill level and the nurse assistant skill level is the lowest skill level. In this study, all these 3 skill levels are called nurse (according to the hospital custom). When necessary, the skill level is precisely specified. The characteristics of the whole problem are presented in Table 5 and the characteristics of the staff and the required number of each nurse at different skill levels in an 18-person ward in Tables 6 and 7 for Shahid Labbafinejad Hospital.



Table 5. Required data for the small size problem.

Planning horizon	30 days		
The length of each shift.	(Morning and noon shift 6 hours and night shift 12		
The length of each ship.	hours)		
Maximum working hours per workforce per day.	12 hours		
Minimum working hours required for each workforce per week.	35 hours		
Maximum working hours per workforce per week.	42 hours		
Minimum working hours required for each workforce	100 hours		
during the month.	100 nours		
Maximum working hours allowed for each workforce	180 hours		
during the month.	100 nours		
Penalty coefficient for allocating each workforce at a	15 to 20 currencies		
lower skill level.	13 to 20 currencies		
Maximum allowed night shifts in a planning period.	15 shifts		

Table 6. Characteristics of employees in 18 people department.

Employees Number	Real Skill Level	Preferred Days for Leave	Employees Number	Real Skill Level	Preferred Days for Leave
1	Nurse	-	10	Nurse	-
2	Nurse	29	11	Nurse	7,14
3	Nurse	28,29	12	Practical Nurse	1,2,3,4
4	Nurse	-	13	Practical Nurse	22
5	Nurse	-	14	Practical Nurse	19,20,21,22
6	Nurse	24,25,26	15	Practical Nurse	-
7	Nurse	9,10,11,12	16	Practical Nurse	-
8	Nurse	26	17	Assistant Nurse	-
9	Nurse	16,17,18	18	Assistant Nurse	-

**Table 7.** The required number of each skill level of employees in each shift of each day in the planning period in 18 people department.

Day	Shift	Nurse	Practical Nurse	Assistant Nurse
	Morning	3	0	1
1,2,15,28	Noon	2	1	1
	Night	2	1	0
	Morning	2	1	0
6,10,13,20,22,27	Noon	2	1	0
	Night	2	1	0
3,4,5,7,8,9,11,12	Morning	4	0	1
14,16,17,18,19,21	Noon	2	1	1
23,24,25,26,29,30	Night	2	1	0

According to the Lp metrics method in *Relation* (35), to obtain an efficient answer, it is necessary to obtain a receiving table. Therefore, by solving the problem by individual optimization method, the value of the first objective function 1867, the second and third objective functions 0, the fourth objective function 181 and the fifth objective function 573.117 are obtained.

Based on the comprehensive criterion method, the obtained efficient solution optimizes all objective functions simultaneously. Therefore, the shift schedule of the problem under study can be shown in *Table 8*.



 Table 8. Scheduling problem for the small sample.

	Morning	Morning Shift Noon shift			Night S	Shift			
Day	Nurse	P- nurse <sup>1</sup>	A- nurse <sup>2</sup>	Nurse	P-nurse	A- nurse	Nurse	P-nurse	A-nurse
1	2-3-4		17	5-7	16	18	1-9	12	
2	4-5-11		17	6-7	15	18	2-10	13	
3	1-3-4-9		18	5-7	14	16	8-11	15	
4	1-5-7-10		18	2-3	1	17	4-6	12	
5	3-5-7-10		17	9-11	13	16	2-8	14	
6	9-10	16		1-6	13		3-11	15	
7	4-5-7-10		13	1-9	12	17	6-8	14	
8	1-2-3-4		18	5-10	13	17	7-9	15	
9	3-4-5-6		17	2-8	12	13	10-10	14	
10	6-9	15		1-2	16		5-7	12	
11	1-3-4-8		16	6-10	14	18	2-11	15	
12	1-4-5-6		18	7-8	13	17	3-9	16	
13	4-11	13		2-8	14		6-10	12	
14	3-7-9-11		17	4-5	13	14	1-8	16	
15	2-10-11		17	3-7	12	18	6-9	15	
16	1-2-4-10		12	5-11	16	17	3-8	14	
17	2-4-5-11		17	6-9	13	18	7-10	16	
18	3-4-8-11		18	2-6	12-13		1-5	15	
19	2-3-4-9		18	7-8	16	17	6-10	13	
20	1-8	16		4-5	15		7-9	12	
21	2-4-5-8		17	3-6	13	18	1-11	14	
22	2-9	12		5-10	13		3-8	15	
23	1-2-5-7		17	4-10	12	18	9-11	13	
24	3-4-5-7		16	1-8	14		6-10	15	12
25	2-3-4-11		18	1-7	16	17	8-9	14	
26	2-3-7-10		18	4-5	16	1	6-11	12	
27	1-7	14		3-10	13		2-5	15	
28	1-7-8		18	9-11	13	17	4-6	16	
29	1-5-9-11		18	2-3	12	7	8-10	15	
30	1-2-4-9		13	6-11	14	3	5-7	12	
	ical nurse int nurse								



Ergonomic factor in this study is considered in the form of short breaks in each work shift for staff. Personnel can take a short break in each shift to avoid excessive fatigue and rehabilitate their work force. *Table 9* shows the number of rest times of each staff in each work shift to answer the workload.

Table 9. Number of rest times of each staff in each shift for small sample problem.

D	Morning Shift			Noon Shift			Night Shift		
Day	Nurse	P- nurse <sup>1</sup>	A- nurse <sup>2</sup>	Nurse	P-nurse	A- nurse	Nurse	P-nurse	A-nurse
1	-	-	(2) 17	-	-	(2) 18	-	(1) 12	-
2	-	-	(2) 17	-	(1) 15	(2) 18	-	-	-
3	_	-	-	-	-	-	-	_	-
4	_	-	(2) 18	-	_	-	-	_	-
5	-	-	(2) 17	-	-	(1) 16	-	(1) 14	-
6	_	-	-	-	_	-	-	-	-
7	_	-	-	-	_	-	-	_	-
8	_	-	(2) 18	-	(1) 13	(2) 17	-	_	-
9	_	-	-	-	(1) 12	(1) 13	-	_	-
10	-	_	-	-	-	-	-	_	-
11	-	_	(2) 16	-	(1) 14	(2) 18	-	_	-
12	-	-	(2) 18	-	-	-	-	-	-
13	_	-	-	-	(2) 14	-	-	_	-
14	_	-	(2) 17	-	-	(1) 14	-	_	-
15	-	-	-	-	(1) 12	-	-	-	-
16	-	-	(2) 12	-	_	-	-		-
17	-	-	(2) 17	-	-	(2) 18	-	(3) 16	-
18	-	-	(2) 18	-	-	-	-	-	-
19	-	-	(2) 18	-	(2) 16	(2) 17	-	-	-
20	-	-	-	-	(1) 15	-	-	-	-
21	-	-	-	-	-	-	-	(4) 14	-
22	-	-	-	-	(2) 13	-	-	-	-
23	-	-	-	-	(1) 12	(2) 18	-	-	-
24	-	-	-	-	-	-	-	(1) 15	-
25	-	-	(2) 18	-	(2) 16	(2) 17	-	(1) 14	-
26	-	-	(2) 18	-	-	-	-	-	-
27	-	-		-	(2) 13	-	-	(4) 15	-
28	-	-	(2) 18	-	-	-	-	-	-
29	-	-	(2) 18	-	12	(2) 7	-	-	-
30	-	-	(23) 13	-	-	-	-	-	-
<sup>1</sup> Practical nurse									
<sup>2</sup> Asista	int nurse								

As can be seen from *Table 9*, the highest number of breaks occurred during one shift is in the night shift, which is due to the 12-hour night shift. Therefore, some personnel in the night shift during their service need 4 short breaks to rehabilitate their workforce. Based on the results, it is observed that most of the break time occurs in the middle of staff working hours, which continue to operate after a short break. It is also observed that a person's behavior every day is almost predictable and his rest period is also estimated. Therefore, by obtaining information before the schedule, a more detailed and accurate schedule regarding the shift schedule and also the working hours of nurses in the hospital can be provided.

In the following, a large sample size problem in Labbafinejad Hospital is solved by considering 90 staff and the information of *Table 5* with NSGA II and MOPSO algorithms. The number of personnel required in different shifts is as described in *Table 10*.



**Table 10.** The required number of each skill level of nurses in each shift of each day in the planning period in a 90-person department.

Day	Shift	Nurse	Practical nurse	Asistant nurse
	morning	15	0	5
1,2,15,28	noon	10	5	5
	night	10	5	0
	morning	10	5	0
3,10,13,20,22,27	noon	10	5	0
	night	10	5	0
3,4,5,7,8,9,11,12	morning	20	0	5
14,16,17,18,19,21	noon	10	5	5
23,24,25,26,29,30	night	10	5	0

Due to the inefficiency of the comprehensive standard method in solving the nurses' scheduling problem, the large size model has been solved by NSGA II and MOPSO algorithms and the efficient response comparison indices are obtained as described in *Table 11*. Furthermore, *Fig. 10* demonstrates the Pareto front resulting from problem solving by two algorithms.

Table 11. Comparison indicators of meta-heuristic algorithms.

Algorithm	NSGA II	MOPSO
The average of the first objective function.	3411,89	3407,17
The average of the second objective function.	46,02	50,70
The average of the third objective function.	0,45	0,62
The average of the forth objective function.	5379,43	5379,43
The average of the fifth objective function.	2710,77	2732,09
Number of answers in Pareto	64	24
The most spread.	466,94	325,98
Distance.	0,36	0,38
Distance from the ideal point.	225,89	158,49
Computational time.	2641,35	2248,63

According to the results of *Table 11*, it can be seen that NSGA II algorithm has obtained 64 efficient answers in 2641.35 seconds and MOPSO algorithm has obtained 24 efficient answers in 2248.63 seconds. Moreover, NSGA II algorithm has worked better than MOPSO algorithm in reducing nurses' shift scheduling.

#### 6. Conclusion

In this paper, the nurses' work shift scheduling problem is modeled by considering ergonomic factors. Given the urgent need of hospitals to provide better staff services to patients, it is necessary to consider the preferences of nurses in scheduling shifts. Therefore, a multi-objective model of scheduling nurses meeting different objectives as well as reducing nurses' fatigue during their day-to-day activities is presented in this article. The main objectives of the article include minimizing 1- the cost of allocating personnel to the skill level, 2- the total deviations from the shifts that personnel tend not to be assigned to work 3- the amount of morning and evening shifts that personnel allocate to work continuously 4- The sum of deviations from the lower and upper limits on the total number of hours worked and 5- fatigue is the sum of nurses. Moreover, all government regulations and hospital policies are included in the modeling. To solve the developed model, two algorithms, NSGA II and MOPSO, have been used by presenting a new chromosome. The designed chromosome first programs the hard limits of the problem and then tries to improve the soft limits of the problem. Therefore, to evaluate the outputs of



the model, two numerical examples in small and large size with real data of Labbafinejad Hospital were designed. In the first example, the schedule of an 18-person ward of Labbafinejad Hospital was analyzed and the results showed that nurses take the most breaks during the night shift and in the middle of their working hours to reduce fatigue. Then, due to the inability of the comprehensive standard method to solve large size problems, a problem with 90 staff was designed for Labbafinejad Hospital and the problem was solved with MOPSO and NSGA II algorithms. The output of the problem showed that the MOPSO algorithm was more efficient than the NSGA II algorithm in obtaining distance point indices from the ideal point as well as computational time. While the NSGA II algorithm performed well in obtaining Pareto response rate indices, the maximum expansion and metric distance. Therefore, the TOPSIS method and the use of entropy weighting method showed that the designed NSGA II algorithm has the ability to solve the problem of scheduling the nurses of Labbafinejad Hospital faster and better.

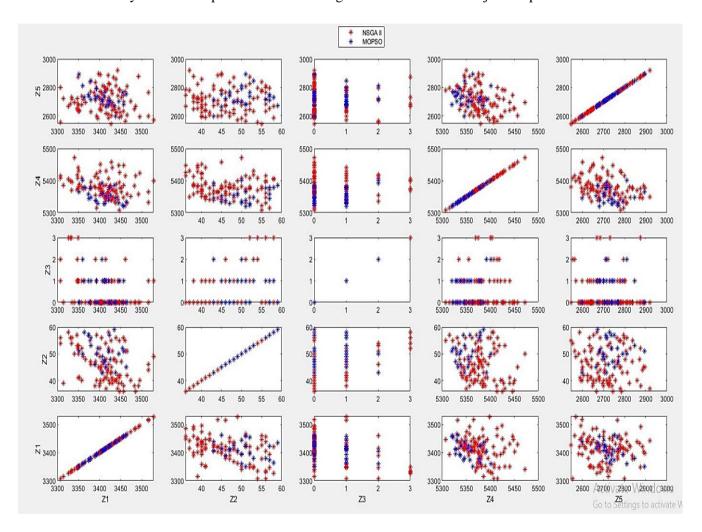


Fig. 10. Pareto front resulting from large-scale problem solving.



# References

- [1] Dawson, D., & Fletcher, A. (2001). A quantitative model of work-related fatigue: background and definition. *Ergonomics*, 44(2), 144-163.
- [2] Lin, R. C., Sir, M. Y., Sisikoglu, E., Pasupathy, K., & Steege, L. M. (2013). Optimal nurse scheduling based on quantitative models of work-related fatigue. *IIE transactions on healthcare systems engineering*, 3(1), 23-38.
- [3] Millar, H. H., & Kiragu, M. (1998). Cyclic and non-cyclic scheduling of 12 h shift nurses by network programming. *European journal of operational research*, 104(3), 582-592.
- [4] Chen, P. S., Huang, W. T., Chiang, T. H., & Chen, G. Y. H. (2020). Applying heuristic algorithms to solve inter-hospital hierarchical allocation and scheduling problems of medical staff. *International Journal of computational intelligence systems*, *13*(1), 318-331.
- [5] Batun, S., & Karpuz, E. (2020). Nurse scheduling and rescgeduling under uncertainty. *Hacettepe University Journal of Economics & Administrative Sciences/Hacettepe Üniversitesi Iktisadi ve Idari Bilimler Fakültesi Dergisi*, 38(1).
- [6] Mala Sari Rochman, E., Rachmad, A., & Santosa, I. (2020). The application of genetic algorithms as an optimization step in the case of nurse scheduling at the Bringkoning Community health center. *JPhCS*, 1477(2), 022026.
- [7] El Adoly, A. A., Gheith, M., & Fors, M. N. (2018). A new formulation and solution for the nurse scheduling problem: A case study in Egypt. *Alexandria engineering journal*, *57*(4), 2289-2298.
- [8] Thongsanit, K., Kantangkul, K., & Nithimethirot, T. (2016). Nurse's shift balancing in nurse scheduling problem. *Science, engineering and health studies (former name: Silpakorn University science and technology journal*), 43-48. https://doi.org/10.14456/sustj.2016.6
- [9] Jafari, H. (2020). Developing a fuzzy model for the nurse scheduling problem. *Journal of operational research in its applications (applied mathematics)-Lahijan Azad University*, 17(2), 93-107.
- [10] Benzaid, M., Lahrichi, N., & Rousseau, L. M. (2020). Chemotherapy appointment scheduling and daily outpatient–nurse assignment. *Health care management science*, 23(1), 34-50.
- [11] Ko, Y. W., Kim, D. H., Uhmn, S., & Kim, J. (2017). Nurse scheduling problem using backtracking. *Advanced science letters*, 23(4), 3792-3795.
- [12] Steege, L. M., & Dykstra, J. G. (2016). A macroergonomic perspective on fatigue and coping in the hospital nurse work system. *Applied ergonomics*, 54, 19-26.
- [13] Alade, O. M., & Amusat, A. O. (2019). Solving nurse scheduling problem using constraint programming technique. arXiv preprint arXiv:1902.01193.
- [14] Zhong, X., Zhang, J., & Zhang, X. (2017). A two-stage heuristic algorithm for the nurse scheduling problem with fairness objective on weekend workload under different shift designs. *IISE transactions on healthcare systems engineering*, 7(4), 224-235.
- [15] Eberhart, R., & Kennedy, J. (1995, October). A new optimizer using particle swarm theory. MHS'95 proceedings of the sixth international symposium on micro machine and human science (pp. 39-43). IEEE
- [16] Kumar, B. S., Nagalakshmi, G., & Kumaraguru, S. (2014). A shift sequence for nurse scheduling using linear programming problem. *Journal of nursing and health science*, *3*(6), 24-28.
- [17] Kim, S. J., Ko, Y. W., Uhmn, S., & Kim, J. (2014). A strategy to improve performance of genetic algorithm for nurse scheduling problem. *International journal of software engineering and its applications*, 8(1), 53-62.
- [18] Jafari, H., & Salmasi, N. (2015). Maximizing the nurses' preferences in nurse scheduling problem: mathematical modeling and a meta-heuristic algorithm. *Journal of industrial engineering international*, 11(3), 439-458.
- [19] Jafari, H., Bateni, S., Daneshvar, P., Bateni, S., & Mahdioun, H. (2016). Fuzzy mathematical modeling approach for the nurse scheduling problem: a case study. *International journal of fuzzy systems*, 18(2), 320-332.
- [20] Youssef, A., & Senbel, S. (2018, January). A bi-level heuristic solution for the nurse scheduling problem based on shift-swapping. 2018 IEEE 8th annual computing and communication workshop and conference (CCWC) (pp. 72-78). IEEE.
- [21] Simić, S., Simić, D., Milutinović, D., Đorđević, J., & Simić, S. D. (2017, September). A fuzzy ordered weighted averaging approach to rerostering in nurse scheduling problem. *International joint conference SOCO'17-CISIS'17-ICEUTE'17 León, Spain, September 6–8, 2017, proceeding* (pp. 79-88). Springer. Cham. https://doi.org/10.1007/978-3-319-67180-2\_8



- [22] Maenhout, B., & Vanhoucke, M. (2010). Branching strategies in a branch-and-price approach for a multiple objective nurse scheduling problem. *Journal of scheduling*, 13(1), 77-93.
- [23] Tsai, C. C., & Li, S. H. (2009). A two-stage modeling with genetic algorithms for the nurse scheduling problem. *Expert systems with applications*, *36*(5), 9506-9512.
- [24] Maenhout, B., & Vanhoucke, M. (2011). Reactive personnel scheduling: insights and policy decisions. 25th annual conference of the Belgian operations research society (ORBEL 25) (pp. 29-30). http://hdl.handle.net/1854/LU-1147910
- [25] Ikeda, K., Nakamura, Y., & Humble, T. S. (2019). Application of quantum annealing to nurse scheduling problem. *Scientific reports*, *9*(1), 1-10.
- [26] Hamid, M., Tavakkoli-Moghaddam, R., Golpaygani, F., & Vahedi-Nouri, B. (2020). A multi-objective model for a nurse scheduling problem by emphasizing human factors. *Proceedings of the institution of mechanical engineers, Part H: journal of engineering in medicine*, 234(2), 179-199.
- [27] Bagheri, M., Devin, A. G., & Izanloo, A. (2016). An application of stochastic programming method for nurse scheduling problem in real word hospital. *Computers & industrial engineering*, *96*, 192-200.
- [28] Thongsanit, K., Kantangkul, K., & Nithimethirot, T. (2016). Nurse's shift balancing in nurse scheduling problem. *Science, Engineering and Health Studies (Former name: Silpakorn University Science and Technology Journal*), 43-48.
- [29] Nasiri, M. M., & Rahvar, M. (2017). A two-step multi-objective mathematical model for nurse scheduling problem considering nurse preferences and consecutive shifts. *International journal of services and operations management*, 27(1), 83-101.
- [30] Sangai, J., & Bellabdaoui, A. (2017, April). Workload balancing in nurse scheduling problem models and discussion. 2017 international colloquium on logistics and supply chain management (LOGISTIQUA) (pp. 82-87). IEEE.
- [31] Svirsko, A. C., Norman, B. A., Rausch, D., & Woodring, J. (2019). Using mathematical modeling to improve the emergency department nurse-scheduling process. *Journal of emergency nursing*, 45(4), 425-432.



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