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A Study on Investment Problem in Chaos Environment

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ABSTRACT

Investment problem is one of the most important and interesting optimization problems. This problem becomes more difficult when we deal with it in an uncertain and vague environment with chaos data. This paper attempts to study the investment problem with uncertain return data. These data are represented as chaos numbers. Dynamic programming is applied to obtain the optimal policy and the corresponding best return. Finally, a numerical example is given to illustrate the utility, effectiveness, and applicability of the approach to the problem.

1. Introduction

Investment plays an important social role in helping severs to meet their financial needs over time and in doing so, the investment process contributes to growth through the efficient allocation of capital. In active management, the investment processes depend on research to identify opportunities risk-adjusted return over client's chosen time horizons. The research used in investment process can take many different forms and can be sourced from multiple locations. Meza and Webb [14] examined the effects of symmetric information on aggregate investment and the financial structure of firms. Hargitay [7] identified the property of portfolio problem in order to have the way for the applications of recent developments in investment and portfolio theory. Yahaya [26] provides the optimal solution to the Markowitz's mean- variance portfolio selection problem based on the analytical solution developed in previous research that lead to the convergence of an important model. Yahaya et al. [27] compared investment opportunities and combined them with portfolios.

In many scientific areas, such as systems analysis and operations research, a model has to be set this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [5] extended the use of algebraic operations on real numbers to fuzzy numbers by the use of a fuzzification principle. Jana et al. [9] added an entropy objective function to the multiobjective portfolio selection model that is to generate a well-diversified asset portfolio based on the possibilities mean value and variance of constraints distribution. Ammar and Khalifa [3]

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presented portfolio selection problem as quadratic programming problem with inexact rough interval in the objective function and constraints. Kheirfam [28] used a fuzzy ranking and arithmetic operations to transform the quadratic programming problem with fuzzy numbers in the coefficients and variables into the corresponding deterministic one and solved it to obtain a fuzzy optimal solution. Khalifa and ZeinEldein [12] presented portfolio selection problem as multi- objective quadratic- linear programming problem with fuzzy objective functions coefficients and applied fuzzy programming approach for solution. By addressing randomness in the project costs and making individual project budgets decision variables, Hu and Szmerekovsky [8] introduced the problem of balancing associated with budgetary slack and cost overruns in the portfolio selection problem. In the presence of transaction costs, Suh [23] proposed a new portfolio selection problem rule formed by combining an extant portfolio rule with the no-rebalancing portfolio rule. Abel and Eberly [1] unified an investment model under uncertainty in a dynamic programming problem. Ammar and Khalifa [2] characterized the optimal solution on uncertainty investment problem with trapezoidal fuzzy numbers. Kahraman et al. [10] applied dynamic programming to the situation where each investment in the set has several possible values and the rate of return varies with the amount invested. Modarres et al. [15] used a dynamic programming approach to obtain the optimal policies for an investor who faces as stochastic number of investing chances (with Poisson Distribution) and a stochastic profit for every chance accruing (with Uniform Distribution). Xu [25] developed new two-stage fuzzy optimization methods for production and financial investment planning problem, in which the exchange rate is uncertain and characterized by possibility distribution. Sirbiladze et al. [21] introduced a new methodology of making a decision on an optimal investment in several projects. Ammar and Khalifa [3] studied investment problem with rough interval data and applied the dynamic programming for obtaining the optimal policy and the corresponding best return. Tahar et al. [24] presented an extension of the Merton optimal investment problem to the case where the risk asset is subjected to transaction costs and capital gains taxes. Roshan and Afsharinezhad [18] suggested and demonstrated the novel approach dividing a broad target market into subsets of customers who have common needs. Shin et al. [19] applied a dynamic programming method for analyzing the optimal consumption and investment problem of an agent having quadratic-type utility function and faced a substitute consumption constraints. Lee and Shin [13] studied an optimal consumption and portfolio selection problems with subsistence consumption constraints. Shin and Roh [20] analyzed the optimal consumption and investment problem of agent by incorporating stochastic hyperbolic preferences with constant relative risk aversion utility.

Nature forces and processes are infinite. They interact with other in a chaotic manner. They follow a spontaneous order that is chaos. Chaos is the ordered disorder of nature. The conventional definition and perception of order is short-term oriented since it does not allow change or evaluation. However, nature is in continuous change, progress, and evaluation because it is governed by infinity of dimensions including time, space, and state [11]. Nature knowledge requires the development of new tools to understand better its variables and process and their diverse interaction [6, 16, 17, 22].

Mathematics is the study of relationships using numbers, shapes, and quantities. It employs signs, symbols, and proofs. It includes arithmetic, algebra, calculus, geometry, and trigonometry. In classical mathematics, a number is static and meaningless. However, in chaos mathematics, a number is dynamic, evolutionary, and value-added.

In this paper, the investment problem with chaos numbers return is studied. A dynamic programming provides an optimal policy and the corresponding best return are applied. The outline of the paper is proceeded as follows: In Section 2, some preliminaries are presented. In Section 3, investment problem with chaos numbers is introduced. Section 4 introduces a dynamic approach to obtain chaos optimal policy and hence best return. In Section 5, a numerical example is given for illustration. Finally some concluding remarks are reported in Section6.

2. Preliminaries

In order to discuss our problem conveniently, the basic concepts and results related to chaos numbers are recalled [11].

Definition 1. (Chaos number). A chaos number $\mathrm{x_a}$ is neither static nor rigid. It changes over time and space.

Definition 2. Let $\mathrm{x_a}$ and $\mathrm{y_b}$ be two chaos numbers. The arithmetic operations on $\mathrm{x_a}$ and $\mathrm{y_b}$ are:

$$\mathrm{x_a + y_b = (x+y)_{a+b}},$$
$$\mathrm{x_a - y_b = (x-y)_{a-b}},$$
$$\mathrm{x_a \times y_b = (x\times y)_{bx+ay+ab}},$$
$$\mathrm{x_a \div y_b} = \left(\frac{\mathrm{x}}{\mathrm{y}}\right)_{\mathrm{u}}, \text{ where } u = \frac{(x+a)\times y - x\times(y+b)}{(y+b)\times y},$$
$$\mathrm{0_a \times x_b = 0_{ax+ab}},$$
$$\mathrm{0_a \div x_b = 0_{\left(\frac{a}{x+b}\right)}},$$
$$\mathrm{x_a \div 0_b = (b\times x)_{ab}}.$$

3. Problem Statement

3.1. Notation

In this subsection, some notation needed in the problem formulation is introduced:

$z_1(x)$: Profit function with investment in policy 1,
$z_2(x)$ Profit function with investment in policy 2,
.
.
.
$z_N(x)$: Profit function with investment in policy N

l: Amount of investment

$\beta_{1,2}(l)$: Return on combined investment in policy 1 and 2,
$\beta_{1,2,3}(l)$: Return on combined investment in policy 1, 2 and 3,
.
.
.
$\beta_{1,2,3\dots,N}(l)$: Return on combined investment in policy 1, 2, 3… and N.

Assume that we have disposal \$ M for investment in N possible production programs $V_1, V_2, \dots, V_N$. The expected profits for period h years are not known, but they will be estimated and given in the chaos numbers form.

Here, a dynamic programming approach is developed to obtain the best return.

4. Numerical Example

Consider one investor has at his disposal $10 millions for investment in four possible production programs V_1,V_2, V_3, V_4. The expected profit for 3 years given in the chaos numbers are shown in the following table.

Table1. *Return on an investment for a period of three years.*

Investment $*10^6$	**Profit investing in V_1**	**Profit investing in V_2**	**Profit investing in V_3**	**Profit investing in V_4**
0	*0*	*0*	*0*	*0*
1	$.25_{.03}$	$.20_{.05}$	$.13_{.02}$	$.18_{.02}$
2	$.40_{.05}$	$.33_{.08}$	$.19_{.06}$	$.25_{.08}$
3	$.58_{.07}$	$.48_{.07}$	$.25_{.03}$	$.25_{.03}$
4	$.70_{.08}$	$.62_{.03}$	$.45_{.05}$	$.40_{.01}$
5	$.81_{.09}$	$.68_{.07}$	$.53_{.09}$	$.51_{.02}$
6	$.95_{.07}$	$.70_{.10}$	$.70_{.03}$	$.55_{.01}$
7	$1.04_{.09}$	$.83_{.02}$	$.76_{.06}$	$.56_{.02}$
8	$1.20_{.03}$	$.85_{.03}$	$.89_{.01}$	$.59_{.01}$
9	$1.31_{.01}$	$.88_{.01}$	$.95_{.01}$	$.59_{.01}$
10	$1.35_{.03}$	$.89_{.01}$	$.89_{.02}$	$.59_{.01}$

We will now define:

$z_1(x)$: The profit function for investing in V_1,

$z_2(x)$: The profit function for investing in V_2,

$z_3(x)$: The profit function for investing in V_3,

$z_4(x)$: The profit function for investing in V_4,

$\beta_{1,2}(l)$: The optimal profit where l is invested in V_1 and V_2 together,

$\beta_{1,2,3}(l)$: The optimal profit where l is invested in V_1, V_2 and V_3 together,

$\beta_{1,2,3,4}(l)$: The optimal profit where l is invested in V_1, V_2, V_3 and V_4 together.
Let us now introduce the computation of the optimal profits in investments in V_1 and V_2 for various values of l as:

$$\beta_{1,2}(l) = \max_{x+y=l} (z_1(x) + z_2(x)). \tag{1}$$

Table 2. *Optimal policy investments V_1 and V_2.*

Investment *10^6	$\beta_{1,2}(l)$	**Optimal Policy with V_1 and V_2**
0	*0*	*(0, 0)*
1	$.25_{.03}$	*(1, 0)*
2	$.45_{.08}$	*(1, 1)*
3	$.60_{.10}$	*(2, 1)*
4	$.78_{.12}$	*(3, 1)*
5	$.91_{.15}$	*(3, 2)*
6	$1.06_{.14}$	*(3, 3)*
7	$1.18_{.15}$	*(4, 3)*
8	$1.29_{.16}$	*(5, 3)*
9	$1.48_{.11}$	*(6, 3)*
10	$1.57_{.10}$	*(6, 4)*

Let us now compute $\beta_{1,2,3}(l)$, the optimal return on the investments in V_1, V_2 and V_3 for various values of l as:

$$\beta_{1,2,3}(l) = \max_{x+y=l} \left(\beta_{1,2}(l) + z_3(x)\right) \quad (2)$$

Table 3. *Optimal policy investments V_1, V_2, and V_3.*

Investment *10^6	$\beta_{1,2,3}(l)$	**Optimal Policy with V_1, V_2 and V_3**
0	*0*	*(0, 0, 0)*
1	$.25_{.03}$	*(1, 0, 0)*
2	$.45_{.08}$	*(1, 1,0)*
3	$.60_{.10}$	*(2,1,0)*
4	$.78_{.12}$	*(3, 1, 0)*
5	$.91_{.14}$	*(3, 1, 1)*
6	$1.06_{.14}$	*(3, 3, 0)*
7	$1.19_{.16}$	*(3, 3, 1)*
8	$1.31_{.17}$	*(4, 3,1)*
9	$1.42_{.18}$	*(5, 3, 1)*
10	$1.56_{.16}$	*(6, 3, 1)*

Now, let us compute $\beta_{1,2,3,4}(l)$, the optimal return on the investments in with V_1, V_2, V_3 and V_4 for various values of l as:

$$\beta_{1,2,3,4}(l) = \max_{x+y=l} \left(\beta_{1,2,3}(l) + z_4(x)\right) \quad (3)$$

The results of these computations are given in the table below:

Table 4. *Optimal policy investments V_1, V_2, V_3, and V_4.*

Investment *10^6	$\beta_{1,2,3,4}(l)$	**Optimal Policy with V_1, V_2 , V_3 and V_4**
0	*0*	*(0, 0, 0, 0)*
1	$.25_{.03}$	*(1, 0, 0, 0)*
2	$.45_{.08}$	*(1, 1,0, 0)*
3	$.63_{.10}$	*(1,1,0, 1)*
4	$.78_{.12}$	*(3, 1, 0, 0)*
5	$.96_{.14}$	*(3, 1, 0, 1)*
6	$1.09_{.16}$	*(3, 1 1, 1)*
7	$1.24_{.16}$	*(3, 3, 0, 1)*
8	$1.37_{.18}$	*(3, 3,1,1)*
9	$1.49_{.19}$	*(4, 3, 1,1)*
10	$1.60_{.2}$	*(5, 3, 1,1)*

Hence, the best investment for $10 million as computed in Table 4 is:

\$5.0 million in V_1 with an optimal interval-valued fuzzy return: $\$.81_{.09}$ million,

\$3.0 million in V_2 with an optimal interval-valued fuzzy return: $\$.48_{.07}$ million,

\$1.0 million in V_3 with an optimal interval-valued fuzzy return: $\$.13_{.02}$ million, and

\$1.0 million in V_4 with an optimal interval-valued fuzzy return: $\$.18_{.02}$ million.

Thus, the total optimal return on a \$10 million investment is $\$\ 1.60_{.2}$ million.

5. Conclusion

In this paper, the investment problem involving data represented in the chaos numbers was introduced. A dynamic programming approach was applied to obtain chaos return. The significant benefit of using such approach where the decision maker facing a problem is including ambiguity in the data. It is clear that from this study, the investment methodology provided the framework in which the planned investment is fully investigated and all options are explored so as to ensure that it is aligned with the organizations business objectives and strategies directions.

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Conflicts and Interest

The author declares no conflict of interest.

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