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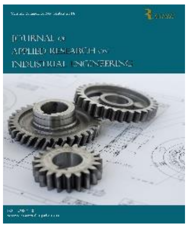

Modifying the Interconnecting Activities through an Adjusted Dynamic DEA Model: A Slacks-Based Measure Approach

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ABSTRACT

A new approach to the dynamic Data Envelopment Analysis (DEA) referred to as the adjusted dynamic DEA, is proposed in this study. Adjusted dynamic DEA optimizes the production activity of DMUs by introducing adjustment variables to modify the interconnecting activities between consecutive terms, solving conflicts that arise between terms and between management and shareholders. The non-oriented Slack Based Model (SBM) is used as a base model and is extended to the adjusted dynamic framework, where adjustment variables are introduced. And also, in this paper, an attempt has been made to consider ratio data and extend traditional ratio DEA models to dynamic DEA-R model. In order to examine the applicability of the proposed method, the model is applied to evaluate the efficiency of ten branches of an Iranian bank during three consecutive terms. The adjusted dynamic SBM model under Variable Return to Scale (VRS) is solved and reference units for each inefficient DMU are identified. In addition, the slacks and adjustment variables are analyzed and further suggestions about the efficient conditions to the management are provided.

1. Introduction

Performance evaluation of bank branches is a major concern for both, the managers and the shareholders, since it has a strong effect on the performance of economy. As strong banking system will result in a developed economy and society. One of the most important issues in bank performance evaluation is measuring the operational and technical efficiency. Measuring the efficiency of bank branches, using new mathematical Data Envelopment Analysis (DEA) techniques, has received a great deal of attention in recent years. Emrouznejad et al. [1] evaluated the researches in efficiency and productivity and provided a comprehensive bibliography of the first 30 years of scholarly literature in DEA.

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DEA is a non-parametric method for evaluating the efficiency of multiple Decision Making Units (DMUs), considering output data vs. input data. The efficiency value is calculated by a relative comparison among similar DMUs which are under competition, and enables analysis to know the strengths or weaknesses of DMUs that operate in the same industry, under the same circumstances. At first, Charnes et al. [2] introduced CCR model and then Banker et al. [3] proposed BCC model. Tone proposed a slack based DEA model to evaluate DMUs and compute input excesses and outputs shortfalls directly. Also he shown that results of Slacks-Based Measure (SBM) model has close connection with CCR and BCC results.

After introducing the initial models in data envelopment analysis, these models were quickly mastered in various fields and researchers studied many in this field, including studies conducted in this field can be cited [5, 6, 7]. Although standard DEA models are powerful tools for evaluation of DMUs, in traditional DEA models, efficiency is measured in single period using cross-sectional data. Such measuring could not be a comprehensive approach for performance evaluation from a long-term point of view. Note that effective management of an organization requires knowing the performance of the overall terms rather than just simply the performance of the individual term. So, DEA researchers extended the normal DEA model to the dynamic structure of organizations during the multiple terms, where some outputs at period t are inputs in the next period, $t + 1$ [8]. In dynamic models actions taken in one term can affect the efficiency of the firms in future terms. Bogetoft et al. [9], Fare and Grosskopf [10], Nemoto and Goto [11], Sueyoshi and Sekitani [12], Park and Park [13], Chang et al. [14], and many others contributed in this field. Sengupta presented a dynamic DEA model by introducing the shadow values of quasi-fixed inputs and their optimal paths into an analytic linear programming problem [15]. Färe and Grosskopf formulated several kinds of intertemporal substitution among inputs, outputs and intermediate outputs using a network theory by which more realistic production processes across periods can be described [8]. Nemoto and Goto extended DEA to a dynamic framework [11]. Their dynamic DEA not only provided a measure of efficiency, but also had the ability to be used as a non-parametric alternative to the economic modeling of the intertemporal behavior of a firm. They incorporated two different types of inputs (variable inputs and quasi-fixed inputs) into a framework of dynamic DEA. A unique feature of quasi-fixed inputs is that they are considered as outputs in the current period, while being treated as inputs in the next period. Sueyoshi and Sekitani developed a method of how to incorporate the concept of Return To Scale (RTS) into the dynamic DEA [12]. Classical DEA models did not account for the effect of carry-over activities (links) between two consecutive terms. For each term, these models have inputs and outputs, but the connecting activities between terms are neglected [16]. The dynamic DEA model proposed by Färe and Grosskopf was the first action for dealing with these interconnecting activities [8]. Then Tone and Tsutsui developed Färe and Grosskopf's model into a SBM framework [16]. They classified the carry-overs into four categories, i.e. desirable or good link like net profit, undesirable or bad link like bad debt, free link that is under control of management and fixed link that is beyond the control of management. According to the characteristics of carry-overs or links, they treated the desirable link as output and undesirable link as input.

Although there are some studies about dynamic DEA situation, adjusted variable were ignored in these studies. On the other hand, in previous studies, interconnecting activities between consecutive terms are not analyzed. In order to solve this problem, this study proposes a method for optimizing the production activity of DMUs by introducing adjustment variables to modify the interconnecting activities between consecutive terms which are separated into output from term t and input to term $t+1$. This model views the production activity of a DMU as a transformation process between terms by focusing on input and output data. The proposed model addresses the conflicts between consecutive terms arising from intermediate or carry-over activities, when they are output of term *t* and input to term *t+1*. In this situation optimizing the output of term *t* or input to term *t+1* will result in inefficiency of the other term. Adjustment variables have the ability to modify these conflicts between terms. In this paper we considered net profit as an intermediate activity (carry-over) between consecutive terms, which is output from term *t* and input to term *t+1*. The unique feature of this study is that a combination of slacks and adjustment variables enables us to determine the amount of net profit that should be distributed among shareholders as earning per share and amount of net profit that should be carried to the next term as loanable funds or reinvestment. In other words, the proposed model is able to address the potential conflicts that may arise between shareholders and management at the end of each term when shareholders aim to maximize their revenues through the dividends and managers aim toward more loans to customers or reinvestment by incorporating non-distributed amount of net profits. To resolve this conflict, the adjusted variable optimizes the shareholders' revenues and manager decisions on reinvestment over the evaluation period. In general, adjustment variables can modify different conflicts between terms in different cases resulting from the nature of intermediate input and output, a feature which gives adjusted dynamic DEA model an edge over the previous studies on dynamic DEA.

As you know, traditional DEA model consider a set of DMUs with multiple inputs and outputs and each DMU consume a set of inputs to produce outputs. But this assumption is not true in some literature and ratio variable considered as ratio input or ratio output. Thanassoulis [17] and Hollingsworth and Smith [18] applied a method where ratio data are substituted by absolute measures. Despic et al. [19] showed standard DEA models cannot be applied for ratio data and consider ratio data in DEA model and it name DEA-R. DEA-R models were applied by Wei et al. [20, 21] to evaluate efficiency in health care. Also Emrouznejad and Amin modified convexity assumption in DEA models in presence of ratio data [22]. Khoshnevis and Teirlinck [23] applied ratio measure based on defined notifications in Emruznejad and Amin [22] and evaluated R&D firms. Olesen et al. [24] discussed about ratio measure in production possibility set with Constant Return to Scale (R-CRS) and Variable Return to Scale (R-VRS). Gidion et al. [25] by considering of intermediate production, developed DEA-R model for performance evaluation of network units.

As can be seen, in previous studies about ratio data, all measure considered in a term of time and do not consider conflicts between consecutive terms so in this paper we developed dynamic DEA models to R-dynamic DEA models.

Briefly in this paper, Section 2 deals problem statement, here we have described that why we need to consider adjusted variable. Section 3 deals preliminaries; in this section some useful DEA models are reviewed. Our proposed DEA models are described in Section 4. In this section at first, adjusted dynamic DEA model are proposed, then dynamic R-DEA models are introduced. Also in order to examine the applicability of the proposed method, in Section 4, the proposed models are applied to evaluate the efficiency of ten branches of an Iranian bank during three consecutive terms. Final, in Section 5, some results are concluded.

2. Problem Statement

Previous dynamic studies applied standard DEA approach to each term and do not consider conflicts between consecutive terms arising from the nature of intermediate activities or links. For example, the second term may have to decrease its input (intermediate activity) in order to achieve an efficient status. Such an action would, however, imply a reduction in the first term output (intermediate activity), thereby reducing the efficiency of that term. In this paper we assume that every DMU at each term, employs two different inputs; direct input (X_t^D) and intermediate input (Z_{t-1}^{Ii}) in order to produce two different outputs, normal output (Y_t) and intermediate output (Z_t^{Io}). We consider intermediate outputs from term *t* and intermediate inputs to term *t*+1 as interconnecting activities or carry-overs that make connection between consecutive terms as depicted in *Fig. 1*. The clearest example for carry-over is net profit at the end of term *t* that is divided into two parts. One part which is distributed among shareholders and the other part which is carried to the next term as loanable funds or reinvestment. Introducing adjustment variables in dynamic SBM model in DEA enables us to optimize these two parts in a way that the efficiency of whole terms is optimized. The two arrows that connect consecutive terms are the possible adjustment parts, so management can adjust the amount of intermediate output of term *t* and intermediate input to term *t+1*. For example the intermediate output (Z_t^{Io}) and the intermediate input (Z_t^{Ii}) can be adjusted so that input Z_t^{Ii} decreases if output Z_t^{Io} is restricted.

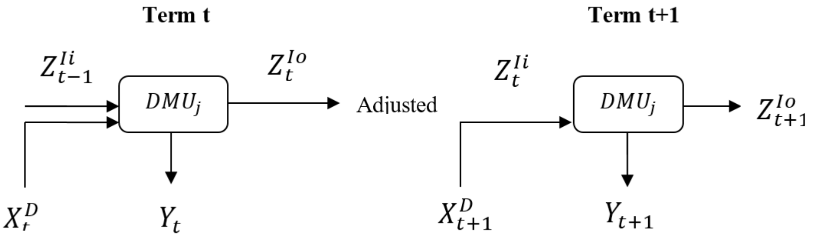

Fig. 1. *Adjusted dynamic DEA model.*

3. Preliminaries

In this section, we briefly provide some useful models such as SBM and R-DEA Models.

3.1. SBM Model

There are different kinds of DEA models such as CCR, BCC, additive models and many others, but this study employs the non-oriented SBM model proposed by Tone in order to treat the surplus of input and lack of output at the same time and show the improvement with an ideal management state [4]. Also, SBM model is non-radial and can deal with the inputs and outputs individually and non-proportional changes can occur. Therefore, the SBM model is suitable for our study since it can consider adjustment between consecutive terms through carry-over activities. The non-oriented SBM model under VRS is defined by *Model (1).*

$$\min \rho = \left(1-\frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}\right).\left(1+\frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}\right)^{-1}, \qquad (1)$$

$$s.t.$$

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}, \qquad (i = 1,2,\dots,m)$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, \qquad (r = 1,2,\dots,s)$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$s_i^-, s_r^+, \lambda_j \geq 0.$$

There are n evaluated DMUs having m input and s output elements. The minimization to an objective function is carried out. This means that we want to reduce the proportion of input excesses and output shortfalls to the observed values of inputs and outputs. A DMU is efficient if all slacks (output shortfalls and input excesses) in the SBM model are zero. On the other hand, a DMU is inefficient if one or more areas of slacks have a value.

3.2. DEA-R

A number of researchers have drawn attention to DEA-R model. Despic et al. [19] were one of the first to consider to ratio DEA model. DEA-R models were applied by Wei et al. [20, 21] to evaluate efficiency in health care. Mozaffari et al. [26] implied this idea to compute cost efficiency of DMUs. In another study, Mozaffari et al. [27] focused on relationship between ratio DEA models without explicit data and classic DEA.

The input-oriented DEA-R model for efficiency evaluation of DMU_o is formulated below:

$$\min \gamma_R, \tag{2}$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j \frac{x_{ij}}{y_{rj}} \leq \gamma_R \frac{x_{io}}{y_{ro}}, \quad i = 1,\dots,m, r = 1,\dots,s$$

$$\lambda_j \geq 0; \quad j = 1,\dots,n.$$

Definition 1. DMU_o is input-oriented CCR-R efficient, if and only if, the optimal objective function value of *Model (2)* equals 1 (See [26]).

4. Proposed Model

Classic DEA models deal with efficiency of DMUs at given time so they are suitable for interconnecting activities between the two periods. Although there are some studies about dynamic, adjusted variable were ignored in these studies. Therefore, in this section based on idea in Tone and Tsutsui [16, 28], we proposed a new Dynamic DEA model.

4.1. Non-Oriented Adjusted Dynamic SBM Model

Suppose that $DMU_j, j = 1, \dots, n,$ consume a set of inputs to produce some outputs in each terms. We can apply SBM model to evaluate each unit in each period of time. But sometimes, we need to assess DMU_O, under evaluation of DMU, in whole of time, $t=1, \dots, T$.

So in this situation we need to consider interconnecting activities between consecutive terms which are separated into output from term t and input to term $t+1$.

Therefore we should apply a model for optimizing of the production activity of DMU_O by introducing adjustment variables to modify interconnecting activities. So this model should view the production activity of a DMU as a transformation process between terms by focusing on input and output data.

In this section, in order to introduce a new model, at first we define the elements as shown below in order to extend the normal SBM model to the adjusted dynamic SBM model.

- The Number of terms: T.
- The Number of input element: m.
- The Number of output elements: s.
- The Number of carry-overs (input to term t): l.
- The Number of carry-overs (output of term t): h.
- The Number of DMUs: n.
- The Observed value of inputs: x_{ijt}.
- The Observed value of outputs: y_{rjt}.
- The Observed value of carry-overs (intermediate input to term t): z^{Ii}_{pjt}.
- The Observed value of carry-overs (intermediate output of term t): z^{Io}_{pjt}.
- The Slack of input elements: s^-_{it}.
- The Slack of output elements: s^+_{rt}.
- The slacks of carry-overs (input to term t): s^{c-}_{pt}.
- The Slack of carry-overs (output of term t): s^{c+}_{pt}.
- The Adjustment variables of carry-overs (input to term t): $i^+_{z_{pt}}, i^-_{z_{pt}}$.
- The Adjustment variables of carry-overs (output of term t): $o^+_{z_{pt}}, o^-_{z_{pt}}$.

Non-oriented adjusted dynamic SBM model is illustrated below and calculates total efficiency of DMUs over T consecutive terms under VRS.

$$\mathrm{Min}\left[\sum_{t=1}^{T} w^t\left[\left(1-\frac{1}{m+l}\left(\sum_{i=1}^{m}\frac{s^-_{it}}{x_{iot}}+\sum_{p=1}^{l}\frac{s^{c-}_{pt}}{z^{Ii}_{pot}}\right)\right)\right]\right]\left[\sum_{t=1}^{T} w^t\left[\left(1+\frac{1}{s+h}\left(\sum_{r=1}^{s}\frac{s^+_{rt}}{y_{rot}}+\sum_{p=1}^{h}\frac{s^{c+}_{pt}}{z^{Io}_{pot}}\right)\right)\right]\right].^{-1} \quad (3)$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n}\lambda^t_j x_{ijt}+s^-_{it}=x_{iot}, \quad i=1,\dots,m;\ t=1,\dots,T$$

$$\sum_{j=1}^{n} \lambda_j^t z_{pjt}^{Io} - s_{pt}^{c+} + o_{z_{pt}}^{+} - o_{z_{pt}}^{-} = z_{pot}^{Io},\ \ p = 1, \dots, h;\ t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t z_{pjt}^{Ii} + s_{pt}^{c-} + i_{z_{pt}}^{+} - i_{z_{pt}}^{-} = z_{pot}^{Ii},\ \ p = 1, \dots, l;\ t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t y_{rjt} - s_{rt}^{+} = y_{rot},\ \ r = 1, \dots, s; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t = 1\ ;\ t = 1, \dots, T$$

$$o_{z_{pt}}^{+} - o_{z_{pt}}^{-} = i_{z_{pt}}^{+} - i_{z_{pt}}^{-};\ \ (p = 1, \dots, l; t = 1, \dots, T)$$

$$\lambda_j^t \geq 0;\ \ (j = 1, \dots, n; t = 1, \dots, T)$$

$$i_{z_{pt}}^{+}, i_{z_{pt}}^{-} \geq 0;\ \ (p = 1, \dots, l; t = 1, \dots, T)$$

$$o_{z_{pt}}^{+}, o_{z_{pt}}^{-} \geq 0;\ \ (p = 1, \dots, l; t = 1, \dots, T$$

$$s_{it}^{-} \geq 0;\ \ (i = 1, \dots, m; t = 1, \dots, T)$$

$$s_{rt}^{+} \geq 0;\ \ (r = 1, \dots, s; t = 1, \dots, T)$$

$$s_{ptl}^{-} \geq 0;\ \ (p = 1, \dots, l; t = 1, \dots, T)$$

$$s_{pth}^{+} \geq 0.\ (p = 1, \dots, l; t = 1, \dots, T)$$

The proposed model analyzes the improvement in efficiency of the whole terms by introducing adjustment variables between terms, however, it cannot obtain an optimum solution without restriction on adjustment variables. It is essential for the proposed model that adjustment variables be equal if output from term *t* is equal to input of term *t+1*. It is then possible to find the optimum input and output.

$$o_{z_{pt}}^{+} - o_{z_{pt}}^{-} = i_{z_{pt}}^{+} - i_{z_{pt}}^{-}.\ \ (p = 1, \dots, l; j = 1, \dots, h; t = 1, \dots, T).$$

Adjustment variables, like other slacks, can show the direction for higher efficiency, for example, to optimize the whole efficiency of terms, the input to term *t* should be decreased when $i_{z_{pt}}^{+} \geq 0$ and the output of term *t* should be increased whe $o_{z_{pt}}^{-} \geq 0$n. And also the relative importance of a unit in term *t* is illustrated by w^t which $\sum_{t=1}^{T} w^t = 1$.

4.1.1 Solving non-oriented adjusted dynamic SBM model

According to Cooper et al. [29], the fractional SBM model can be transformed into the linear programming by introducing a positive scalar variable k as follows:

$$\tau = \min \sum_{t=1}^{T} w^t \left[k - \frac{1}{m+l} \left(\sum_{i=1}^{m} \frac{ks_{it}^{-}}{x_{iot}} + \sum_{i=1}^{l} \frac{ks_{ptl}^{-}}{z_{pot}^{Ii}} \right) \right], \tag{4}$$

$$\text{s.t.}$$

$$\sum_{t=1}^{T} w^{t}\left[k+\frac{1}{s+h}\left(\sum_{r=1}^{s}\frac{ks_{rt}^{+}}{y_{rot}}+\sum_{q=1}^{h}\frac{ks_{pth}^{+}}{z_{pot}^{Io}}\right)\right]=1\,.$$

$$\sum_{j=1}^{n} k\lambda_{j}^{t}x_{ijt}+ks_{it}^{-}=kx_{iot},\;\; i=1,\dots,m;\, t=1,\dots,T$$

$$\sum_{j=1}^{n} k\lambda_{j}^{t}z_{pt}^{Io}-ks_{pth}^{+}+ko_{z_{pt}}^{+}-ko_{z_{pt}}^{-}=kz_{pot}^{Io},\;\; p=1,\dots,l;\; t=1,\dots,T$$

$$\sum_{j=1}^{n} k\lambda_{j}^{t}z_{pt}^{Ii}+ks_{ptl}^{-}+ki_{z_{pt}}^{+}-ki_{z_{pt}}^{-}=kz_{pot}^{Ii},\;\; p=1,\dots,l;\; t=1,\dots,T$$

$$\sum_{j=1}^{n} k\lambda_{j}^{t}y_{rjt}-ks_{rt}^{+}=ky_{rot},\;\; r=1,\dots,s;\, t=1,\dots,T$$

$$\sum_{j=1}^{n} k\lambda_{j}^{t}=k\,;\; t=1,\dots,T$$

$$ko_{z_{pt}}^{+}-ko_{z_{pt}}^{-}=ki_{z_{pt}}^{+}-ki_{z_{pt}}^{-};\quad (p=1,\dots,l;\, t=1,\dots,T)$$

$$k\lambda_{j}^{t}\;\geq 0;\quad (j=1,\dots,n;\, t=1,\dots,T)$$

$$ki_{z_{pt}}^{+}, ki_{z_{pt}}^{-}\geq 0;\;\; (p=1,\dots,l;\, t=1,\dots,T)$$

$$ko_{z_{pt}}^{+}, ko_{z_{pt}}^{-}\geq 0;\;\; (p=1,\dots,l;\, t=1,\dots,T)$$

$$ks_{it}^{-}\geq 0;\;\; (i=1,\dots,m;\, t=1,\dots,T)$$

$$ks_{rt}^{+}\geq 0;\quad (i=1,\dots,s;\, t=1,\dots,T)$$

$$ks_{ptl}^{-}\geq 0;\quad (p=1,\dots,l;\, t=1,\dots,T)$$

$$ks_{pth}^{+}\geq 0.\;\; (p=1,\dots,l;\, t=1,\dots,T)$$

Now let us define:

$S_{it}^{-}=ks_{it}^{-}$, $S_{rt}^{+}=ks_{rt}^{+}$, $S_{ptl}^{+}=ks_{ptl}^{+}$, S_{pth}^{-}= ks_{pth}^{-} ,$O_{z_{pt}}^{+}=ko_{z_{pt}}^{+}$, $O_{z_{pt}}^{-}=ko_{z_{pt}}^{-}$, $I_{z_{pt}}^{+}=ki_{z_{pt}}^{+}$, $I_{z_{pt}}^{-}=ki_{z_{pt}}^{-}$, $\Lambda_{j}^{t}=k\lambda_{j}^{t}$.

Then we have:

$$\tau=\min\sum_{t=1}^{T} w^{t}\left[k-\frac{1}{m+l}\left(\sum_{i=1}^{m}\frac{S_{it}^{-}}{x_{iot}}+\sum_{i=1}^{l}\frac{S_{ptl}^{-}}{z_{pot}^{Ii}}\right)\right], \tag{5}$$

$$\text{s.t.}$$

$$\sum_{t=1}^{T} w^t \left[k + \frac{1}{s+h} \left(\sum_{i=1}^{s} \frac{S_{rt}^{+}}{y_{rot}} + \sum_{i=1}^{h} \frac{S_{pth}^{+}}{z_{pot}^{Io}} \right) \right] = 1 ,$$

$$\sum_{j=1}^{n} \Lambda_j^t x_{ijt} + S_{it}^{-} = k x_{iot}, \quad i = 1, \dots, m; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \Lambda_j^t z_{pjt}^{Io} - S_{pth}^{+} + O_{z_{pt}}^{+} - O_{z_{pt}}^{-} = k z_{pot}^{Io}, \quad p = 1, \dots, l; \; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \Lambda_j^t z_{pjt}^{Ii} + S_{ptl}^{-} + I_{z_{pt}}^{+} - I_{z_{pt}}^{-} = k z_{pot}^{Ii}, \quad p = 1, \dots, l; \; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \Lambda_j^t y_{rjt} - S_{rt}^{+} = k y_{rot}, \quad r = 1, \dots, s; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \Lambda_j^t = k ; \; t = 1, \dots, T$$

$$O_{z_{pt}}^{+} - O_{z_{pt}}^{-} = I_{z_{pt}}^{+} - I_{z_{pt}}^{-}; \quad (p = 1, \dots, l; j = 1, \dots, n; t = 1, \dots, T)$$

$$I_{z_{pt}}^{+}, I_{z_{pt}}^{-} \geq 0; \quad (p = 1, \dots, l; j = 1, \dots, n; t = 1, \dots, T)$$

$$O_{z_{pt}}^{+}, O_{z_{pt}}^{-} \geq 0; \quad (p = 1, \dots, l; j = 1, \dots, n; t = 1, \dots, T)$$

$$\Lambda_j^t \geq 0, t = 1, \dots, T, j=1,\dots, n$$

$$S_{it}^{-} \geq 0; \quad (i = 1, \dots, m; t = 1, \dots, T)$$

$$S_{rt}^{+} \geq 0; \quad (r = 1, \dots, s; t = 1, \dots, T)$$

$$S_{pt}^{c+} \geq 0; \quad (p = 1, \dots, h; t = 1, \dots, T)$$

$$S_{pt}^{c-} \geq 0. \quad (p = 1, \dots, l; t = 1, \dots, T)$$

Let an optimal solution be ($\{\Lambda_j^{t*}\}$, $\{S_{it}^{-*}\}$, $\{S_{rt}^{+*}\}$, $\{S_{pt}^{c+*}\}$, $\{S_{pt}^{c-*}\}$, $\{k*\}$, $\{I_{z_{pt}}^{+*}\}$, $\{I_{z_{pt}}^{-*}\}$, $\{O_{z_{pt}}^{+*}\}$, $\{O_{z_{pt}}^{-*}\}$), we define optimal solution of adjusted dynamic SBM model as follows:

$$\lambda_j^{t*} = \Lambda_j^{t*}/k^* , s_{it}^{-*} = S_{it}^{-*}/k^*, \; s_{rt}^{+*} = S_{rt}^{+*}/k^* , \; s_{pth}^{+*} = S_{pth}^{+*}/k^*, \; s_{qtl}^{-*} = S_{qtl}^{-*}/k^*, i_{z_{qt}}^{+*} = I_{z_{pt}}^{+*}/k^* ,$$
$$i_{z_{pt}}^{-*} = I_{z_{pt}}^{-*}/k^*, o_{z_{pt}}^{+*} = O_{z_{pt}}^{+*}/k^*, \quad o_{z_{pt}}^{-*} = O_{z_{pt}}^{-*}/k^*.$$

4.2. Dynamic DEA-R

Based on notations were defined in *Section 4.1*, dynamic ratio SBM model in input-oriented is illustrated below and calculates total efficiency of DMUs over *T* consecutive terms under VRS.

$$\gamma_{O-I} = \min \sum_{t=1}^{T} w^{t} \left[1 - \frac{1}{m.h + s.l} \left[\sum_{p=1}^{h} \sum_{i=1}^{m} \frac{s_{ipt}^{-}}{\frac{x_{iot}}{z_{pot}^{Io}}} + \sum_{p=1}^{l} \sum_{r=1}^{s} \frac{s_{rpt}^{+}}{\frac{z_{pot}^{Ii}}{y_{rot}}} \right] \right], \qquad (6)$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j^t \frac{x_{ijt}}{z_{pjt}^{Io}} + s_{ipt}^{-} = \frac{x_{iot}}{z_{pot}^{Io}}, \quad i = 1, \dots, m \qquad , p = 1, \dots, h, \; t{=}1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t \frac{z_{pjt}^{Ii}}{y_{rjt}} + s_{rpt}^{+} = \frac{z_{pot}^{Ii}}{y_{rot}}, \quad r = 1, \dots, s \qquad , \; p = 1, \dots, l, \; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t = 1 \;, \qquad t = 1, \dots., T,$$

$$\lambda_j^t \geq 0, \qquad j = 1, \dots, n \;,, \qquad t = 1, \dots., T,$$

$$s_{ipt}^{-} \geq 0, \quad i = 1, \dots, m, p = 1, \dots, l,, \qquad t = 1, \dots., T,$$

$$s_{rpt}^{+} \geq 0, \quad r = 1, \dots, s, p = 1, \dots, l \;, \qquad t = 1, \dots, T.$$

And also the relative importance of a unit in term t is illustrated by w^t which $\sum_{t=1}^{T} w^t = 1$.

Definition 2. DMU_o is output-oriented DEA-R efficient, if and only if, the optimal value of s_{iot}^{-} and s_{rpt}^{+} in *Model (6)* equals 0.

By using of similar manner, output-oriented of *Model (6)* can be gained as follows:

$$\gamma_{O-O} = \max \sum_{t=1}^{T} w^{t} \left[1 + \frac{1}{m.h + s.l} \left[\sum_{p=1}^{h} \sum_{i=1}^{m} \frac{s_{ipt}^{-}}{\frac{z_{pot}^{Io}}{x_{iot}}} + \sum_{p=1}^{l} \sum_{r=1}^{s} \frac{s_{rpt}^{+}}{\frac{y_{rot}}{z_{pot}^{Ii}}} \right] \right], \qquad (7)$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j^t \frac{s_{ipt}^{-}}{\frac{z_{pjt}^{Io}}{x_{ijt}}} - s_{ipt}^{-} = \frac{z_{pot}^{Io}}{x_{iot}}, \quad i = 1, \dots, m \qquad , p = 1, \dots, h, \; t{=}1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t \frac{y_{rjt}}{z_{pjt}^{Ii}} - s_{rpt}^{+} = \frac{y_{rot}}{z_{pot}^{Ii}}, \quad r = 1, \dots, s \qquad , \; p = 1, \dots, l, \; t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j^t = 1 \;, \qquad t = 1, \dots., T,$$

$$\lambda_j^t \geq 0, \qquad j = 1, \dots, n \;,, \qquad t = 1, \dots., T,$$

$$s_{ipt}^{-} \geq 0, \quad i = 1, \dots, m, p = 1, \dots, l,, \qquad t = 1, \dots., T,$$

$$s_{rpt}^{+} \geq 0, \quad r = 1, \dots, s, p = 1, \dots, l \;, \qquad t = 1, \dots, T.$$

And , γ_{O-O} in *Model (7)* illustrates the efficiency of DMU_o in output oriented.

Definition 3. DMU_o is output-oriented DEA-R efficient, if and only if, the optimal value of s_{iot}^- and s_{rpt}^+ in *Model (7)* equals 0.

5. Case Study

In order to examine the utility of the proposed method which introduces adjustment variables for interconnecting activities in dynamic framework, the model was applied to evaluate the efficiency of ten branches of an Iranian bank during three consecutive terms. Branches at each term employ total deposits and operating expenses in order to produce total loans and net profit. At the end of each term, net profit is divided into two parts, one part is distributed among shareholders and the other part is carried to the next term as loanable funds or reinvestment. The most important characteristic of this study is to determine the optimum value of the net profit that should be distributed or carried according to the efficient frontier that the adjusted dynamic SBM model generates. The evaluation index system of bank branch performance is shown in *Table 1*.

Table. 1. *Evaluation index system.*

Factors	Name of Index	Unit of Index
Inputs	*Total deposits*	*10'000'000'000 Riyal*
	Operating expenses	*10'000'000 Riyal*
Output	*Total loans*	*1'000'000'000 Riyal*
Carry-Over	*Net profit*	*100'000'000 Riyal*

The overall efficiency score of each branch over three consecutive terms is calculated based on the *Model (4)*. Non-oriented adjusted dynamic SBM model was used to measure inefficiencies in both inputs and outputs concurrently. The results of adjusted dynamic model and reference units are summarized in *Table 2*. According to the characteristics of adjusted dynamic SBM model, directions for higher efficiency at each term can be recognized through the slacks and adjustment variables. Slacks and adjustment variables values are summarized in *Table 3* and *Table 4*, respectively. In order to show the applicability of the proposed model, the slacks and adjustment variables for an inefficient DMU (DMU4) were analyzed.

Efficiency will improve for DMU4 at the first term when the operating expenses have been reduced by $s_{21}^- = 90.7$ units and the total loans have been increased by $s_{11}^+ = 397.3$ units. Due to adjustment variables, net profit as an intermediate output of term1 should be increased by $o_{11}^- = 162$ units, which is carried totally to the next term ($i_{12}^- = 162$ units). In the second term efficiency will improve when the total deposits and operating expenses have been decreased by $s_{12}^- = 8.1$ units and $s_{22}^- = 87.7$ units, respectively. Net profit that is an intermediate input to the second term should be decreased by $s_{z12}^- = 150$ units, this is the amount that should be distributed among shareholders and the remaining value ($162 - 150 = 12$ units) should be carried from term 1 to term 2 as an input. Also, the net profit that is produced in the second term should be increased by $o_{12}^- = i_{13}^- = 11.2$ units. In the last term, total deposits and operating expenses should be reduced by $s_{13}^- = 9.9$ units and $s_{23}^- = 78.1$ units, respectively. Also, the net profit that is an intermediate output from term 2 and intermediate input to term 3 should be decreased by $s_{z13}^{-} = 2.4$ units, which is the distributed amount and the remaining value ($11.2 - 2.4 = 8.8$ units) is carried to the last term.

Table 2. *Adjusted dynamic efficiency evaluation and reference.*

	Adjusted Dynamic Efficiency	Reference Units in Term 1	Reference Units in Term 2	Reference Units in Term 3
DMU1	*1*	*1*	*1*	*1*
DMU2	*0.35*	*7,9*	*6,9*	*6,8*
DMU3	*0.24*	*7,9*	*6,9*	*1,6*
DMU4	*0.23*	*7,9*	*6,7*	*5,6,7*
DMU5	*0.76*	*1,7,9*	*6,7,9*	*5*
DMU6	*0.86*	*7,9*	*6*	*6*
DMU7	*1*	*7*	*7*	*7*
DMU8	*0.89*	*1,7,9*	*8*	*8*
DMU9	*1*	*9*	*9*	*9*
DMU10	*0.34*	*7,9*	*6,7,9*	*6,7*

Table 3. *Inefficiency slacks.*

	Slacks													
DMUs	**Term 1**				**Term 2**					**Term 3**				
	s_{11}^{-}	s_{21}^{-}	s_{11}^{+}	s_{z11}^{+}	s_{z12}^{-}	s_{12}^{-}	s_{22}^{-}	s_{12}^{+}	s_{z12}^{+}	s_{z13}^{-}	s_{13}^{-}	s_{23}^{-}	s_{13}^{+}	s_{z13}^{+}
DMU1	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU2	*0.0*	*46.6*	*488.2*	*0.0*	*46.6*	*0.0*	*99.0*	*335.3*	*0.0*	*124.7*	*3.7*	*46.6*	*0.0*	*0.0*
DMU3	*0.0*	*137.7*	*632.1*	*0.0*	*166.6*	*8.1*	*152.1*	*201.9*	*0.0*	*0.0*	*9.2*	*137.5*	*0.0*	*0.0*
DMU4	*0.0*	*90.7*	*397.3*	*0.0*	*150.0*	*7.3*	*87.7*	*0.0*	*0.0*	*2.4*	*9.9*	*78.1*	*0.0*	*0.0*
DMU5	*0.0*	*32.6*	*0.0*	*0.0*	*17.8*	*2.7*	*23.2*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU6	*3.9*	*49.1*	*0.0*	*0.0*	*11.5*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU7	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU8	*6.8*	*37.9*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU9	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU10	*0.0*	*16.0*	*325.0*	*0.0*	*73.1*	*0.0*	*69.2*	*126.9*	*0.0*	*0.0*	*2.5*	*46.2*	*188.5*	*0.0*

Also base on ratio data approach, the advantages of proposed DEA-R models be illustrated in *Table 5*. Based on new approach, the data set for the formulation of the DEA-R model is the set of all ratios between inputs and outputs in each stage: Total deposit/ Net profit and Operating expenses/Net profit and finally Net profit/ Total loans. Since the ratios are on the input side of the ratio DEA model so the input orientation is applied to calculate virtual ratios and targets for inefficient units. The efficiency of the bank branches is evaluated by dynamic DEA-R models and the results of this evaluation are summarized in *Table 5* as follows.

Table 4. *Adjustment variables values.*

	Adjustment Variables							
	Term 1		**Term 2**				**Term 3**	
	$\mathbf{o_{11}^{+}}$	$\mathbf{o_{11}^{-}}$	$\mathbf{I_{12}^{+}}$	$\mathbf{I_{12}^{-}}$	$\mathbf{o_{12}^{+}}$	$\mathbf{o_{12}^{-}}$	$\mathbf{I_{13}^{+}}$	$\mathbf{I_{13}^{-}}$
DMU1	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU2	*0.0*	*190*	*0.0*	*190*	*0.0*	*147*	*0.0*	*147*
DMU3	*0.0*	*200*	*0.0*	*200*	*0.0*	*14.7*	*0.0*	*14.7*
DMU4	*0.0*	*162*	*0.0*	*162*	*0.0*	*11.2*	*0.0*	*11.2*
DMU5	*0.0*	*28.2*	*0.0*	*28.2*	*0.0*	*0.0*	*0.0*	*0.0*
DMU6	*0.0*	*11.5*	*0.0*	*11.5*	*0.0*	*0.0*	*0.0*	*0.0*
DMU7	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU8	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU9	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
DMU10	*0.0*	*100*	*0.0*	*100*	*0.0*	*14.6*	*0.0*	*14.6*

Table 5. *Efficiency evaluation by dynamic DEA-R models.*

	Overall Efficiency of Units	**Efficiency of Units in Term 1**	**Efficiency of Units in Term 2**	**Efficiency of Units in Term 3**
DMU1	*0.26501*	*0.23837*	*0.26357*	*0.38143*
DMU2	*0.44051*	*0.55293*	*0.58492*	*0.33052*
DMU3	*0.29458*	*0.35597*	*0.3379*	*0.28807*
DMU4	*0.42305*	*0.57044*	*0.58035*	*0.25938*
DMU5	*0.42233*	*0.57186*	*0.4672*	*0.3687*
DMU6	*0.62455*	*0.56842*	*0.93572*	*0.5777*
DMU7	*0.227*	*0.27255*	*0.27417*	*0.20993*
DMU8	*0.39387*	*0.38373*	*0.39898*	*0.53019*
DMU9	*0.71761*	*0.91755*	*0.87977*	*0.59472*
DMU10	*0.28473*	*0.34169*	*0.32762*	*0.27981*

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