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Semi-obnoxious Backup 2-Median Problem on a Tree

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Abstract

In this paper, we discuss the obnoxious and semi-obnoxious version of the backup 2-median problem on a tree. In the obnoxious case of the 2-median problem, all vertices have negative weights, whereas in the semi-obnoxious model the vertices may have either positive or negative weights. In these two problems, we should find the location of two facility servers on the tree so that the sum of minimum weighted distances from vertices in the tree to the set of functioning servers is minimized. In the backup model, each facility server may probably fail. If a facility server fails, the remaining server should serve the clients. Vertex optimality is an important property for the 2-median problem. This property indicates that the set of vertices involves an optimal solution of the 2-median problem. We verify that the vertex optimality holds for the semi-obnoxious backup 2-median problem on a tree network. In the obnoxious 2-median problem, the set of leaves contains an optimal solution, we show that this property does not hold for the obnoxious backup 2-median problem.

Keywords: 2-median, Backup, Obnoxious, Semi-Obnoxious, Positive and negative weight.

1 | Introduction

Location theory is an important branch of transportation and communication. In location theory one looks for sites for one or more facilities that should interact with a set of existing users. These sites should be optimal concerning some performance measure. The p-median problem is a basic problem in this area of investigation. The goal in the p-median problem is finding the location of p facilities on edges or vertices of a given network G, such that the sum of weighted distances from all vertices in G to the closest facility is minimized.

Kariv and Hakimi [1] showed that the p-median problem on general networks is NP-hard. However, when the underlying network is a tree, they showed the solution of this problem can

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be found by an $O(p^2n^2)$-time algorithm. Tamir [2] improved the time complexity by presenting an $O(pn^2)$ algorithm. In the case $p=1$ on trees linear time algorithms is proposed by Hua [3] and Goldman [4]. Gavish and Sridhar [5] investigated an algorithm with time complexity $O(nlogn)$ for the case $p=2$ on trees. Charikar et al. [6] introduced a constant-factor approximation algorithm for the k-median issue. An uncertain model for single facility location problems on networks was presented by Gao [7]. Since the median problem is NP-hard, some heuristic methods were presented for finding the solution to this problem (e. g. see [8] and [9]).

Traditionally it has been assumed that such interaction is beneficial to the users, who want the facility as close as possible to them. This ignores that most facilities also have negative effects on the population, ranging from threat to lifestyles, such as parking troubles and noise around business centers or soccer stadium and then the facility is called obnoxious- to danger for health or life, such as lethal fumes close to chemical plants -and then the facility is called noxious. Although in recent years there has been an increasing interest in considering such obnoxious aspects, most existing models accommodate just one of the two aspects of the problem, and see the facility as either purely attractive or purely obnoxious.

When the facilities are semi-obnoxious, i.e., they provide both services and disservices, the models designed for purely desirable or purely undesirable facilities oversimplify things, and may yield solutions which are unrealistic due to either the enormous economic costs incurred or the social reaction they provoke against them. Instead, one can combine models of both types.

Although, conceptually the resulting model is just the combination of two existing models, things are more complicated from an algorithmic viewpoint. since such models cannot usually be coped with using the basic tools of the original models: to mention a few, semi-obnoxious facility models are as a rule multimodal, thus the standard techniques of convex analysis used for locating desirable facilities in the plane may be trapped in local optimal; moreover, the geometry of these problems is in general rather involved, thus the computational geometry approaches often used for the purely undesirable case may not be successfully adapted to this new context.

However, in recent years, semi-obnoxious facilities, i.e., facilities that desirable for a part of the clients, and undesirable for the other part of clients, have gained increasing interest. Research in the location of multiple semi-obnoxious facilities is limited, though there are a few instances available in the literature (e. g., Berman and Wang [10]; Coutinho-Rodrigues et al. [11]; Eiselt and Marianov [12]; Silva et al. [13]). Other types of the multiple semi-obnoxious facility models found in the literature are either discrete (see e.g., Colmenar et al. [14]; Song et al. [15]) or network-based models presented in [16] and [17].

Burkard and Krarup [18] were the first authors that assigned the positive and negative weights to desirable and undesirable clients in semi-obnoxious facility location models, respectively. They presented a $O(n)$ time algorithm for the semi-obnoxious 1-median problem on a cactus. A cactus is a connected graph with no two cycles has more than one vertex in common. Burkard et al. [19] proposed two models for the semi-obnoxious 2-median problem on trees. One model tries to minimize the sum of the Minimal Weighted Distances (MWD) from clients to servers, whereas the second one minimizes the sum of the Weighted Minimum Distance (WMD) from clients to the facilities. For the MWD-model on trees, they presented some properties of the problem to propose an $O(n^2)$ algorithm. They also, developed an $O(nlogn)$ algorithm for star graphs, and a linear time algorithm for paths. The WMD problem is harder than MWD to solve. Burkard et al. [19] find an $O(n^3)$ algorithm for WMD on trees. They showed that if the medians should locate only on the nodes, the complexity will be reduced to $O(n^2)$. These results were improved by Benkoczi et al. [20]. They showed that the MWD model of 2-median on a tree with positive and negative weights can be solved in $O(nlogn)$ time. For the WMD model they developed an $O(nhlog^2n)$ algorithm, where h is the height of the tree. In the case that all vertices of the tree have negative weights, Burkard et al. [21] showed that an optimal solution of the MWD model of 2-median problem can be solved in a linear time. In the case that the underlying network is a cycle, Burkard and Hatzl [22] developed a linear and $O(n^2)$ algorithm for the MWD and WMD models of 2-

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median problem, respectively. In the case $p=3$ Burkard and Fathali [23] gave an $O(n^5)$ algorithm for the WMD model on a tree and an $O(n^3)$ algorithm when medians are restricted to nodes. Zaferanieh and Fathali [24] considered the case of finding the semi-obnoxious median path in a tree.

In the classical models of location problems, the facility weights and their availability are known precisely. However, in most real-life emergencies, a customer (demand) is likely not to receive immediate service if his designated nearest emergency service (facility) is engaged in serving other patients at the time of incident (see, Karatas et al. [25]). This is especially common in large scale emergency location problems or in severely supplies restricted systems. Cases, where this is observed, include emergency ambulance operations, repair crews responding to critical equipment failures, utilization of a limited number of expensive medical devices, etc. (see, Weaver and Church [26]). Keeping in mind that the difference between life and death can be sometimes measured in minutes (or even seconds), decision makers should take into account the issue of congestions and queues and should consider providing backup supplies for demands. When a demand nearest (primary) facility is busy, a common approach is to serve the demand with an available backup facility.

Several cases of uncertainty have been defined and studied. Snyder and Daskin [27] investigated the reliability location model where servers may fail, and the clients allocated to these servers have to request service from other active servers. Wang et al. [28] developed a reliability-based formulation for the 2-median problem. In their model, each server may fail with a given probability and when a server fails, the other server should serve the clients. They called this model as the backup 2-median problem and presented an $O(nlogn)$ algorithm for finding the solution of this problem. Cheng et al. [29] developed an $O(nlogn+m)$ algorithm for the backup 2-median problem on block graphs, where m is the number of edges of the given tree. Fathali [30] investigated the backup multifacility location problem on the plane, and proposed an iterative method for solving this problem. Nazari and Fathali [31] and Nazari et al. [32] considered the reverse and inverse of backup 2-median problem on trees, respectively.

The utilization of backup service concept in emergency service systems continued in subsequent studies, including recent papers such as Araz et al. [33], Chanta et al. [34], Curtin et al. [35], Janosikova et al. [36] and Kordjazi and Kazemi [37], which mostly adopt a multi-objective optimization approach.

In this paper we investigate the backup semi-obnoxious and obnoxious 2-median problems with MWD objective function on trees. The vertex optimality verified for these models.

In what follows we describe the backup semi-obnoxious and obnoxious 2-median problems in Section 2. Some properties of these models on trees are given in sections 3 and 4, respectively.

2 | Problem Description

Let $T=(V,E)$ be a tree with n vertices. Each vertex v_i in the tree T has an arbitrary weight w_i. The weight w_i also called the demand at v_i. Let $d(x,y)$ be the length of shortest path between two points x and y in T. The 2-median problem asks to find a set of 2 vertices, $X_2=\{x_1,x_2\}$ called facilities, so that the sum of the weighted distances from the vertices to the closest facility in X_2 is minimized, i.e.,

$$\min F(X_2)=\sum_{i=1}^{n} w_i d(X_2, v_i). \quad (1)$$

For every edge $e(v,u)\in T$ let T_v be the subtree of T obtained by deleting edge $e(v,u)$ so that $v\in T_v$, and let $T_u=T\setminus T_v$. Moreover, for each subtree $T'\subseteq T$ we define $W(T')=\sum_{v_i\in T'} w_i$.

Let ϱ_1 and ϱ_2 be the failure probabilities of the two servers in the backup semi-obnoxious 2-median problem. In this paper, we suppose that $\varrho_1=\varrho_2=\varrho$. The backup semi-obnoxious 2-median problem, asks to find the location of servers such that the expected sum of distances from all vertices to the set of functioning servers is minimized, i.e., we want to minimize:

$$F(v_1,v_2)=(1-\varrho)\sum_{v\in V}\min_{\{1\leq i\leq 2\}}w(v)d(v,v_i)+\varrho\left[\sum_{v\in V}w(v)d(v,v_1)+\sum_{v\in V}w(v)d(v,v_2)\right]. \tag{2}$$

Let $\{u_1,u_2\}$ be a solution of the considered problem. Denote by m the midpoint of $P(u_1,u_2)$, where $P(u_1,u_2)$ is the shortest path between u_1 and u_2. In fact, m is a point in T such that $m\in P(u_1,u_2)$ and $d(u_1,m)=d(u_2,m)$. Let $e(x,y)$ be the edge containing m in $P(u_1,u_2)$. Without loss of generality, suppose that vertex x is on the path between u_1 and m. Moreover, if m is a vertex of T, we assume that it coincides with x.

Since $d(u_1,v_i)\leq d(u_2,v_i)$ and $d(u_1,v_j)>d(u_2,v_j)$ for any vertices $v_i\in T_x$ and $v_j\in T_y$, then all nonnegative weighted vertices in T_x and all negative weighted vertices in T_y are allocated to u_1. The rest of vertices are allocated to u_2. (If $m=x$ we assume that x and all other vertices at the same distance from u_1 and u_2 are allocated to u_1.)

Therefore, we can partition the set of vertices in V into two subsets V_1 and V_2 as follows:

$$V_1=\{v|d(v,x)\leq d(v.y),w(v)\geq 0\}\cup\{v|d(v,x)\geq d(v.y),w(v)\leq 0\}, \tag{3}$$

$$V_2=V\setminus V_1. \tag{4}$$

Such that the vertices in V_1 are served by u_1 and the remaining vertices are served by u_2. Let

$$D(V,u)=\sum_{v\in V}w(v)d(v,u). \tag{5}$$

Then the backup semi-obnoxious 2-median problem can be written as follows:

$$\begin{aligned}F(v_1,v_2)&=(1-\varrho)[D(V_1,v_1)+D(V_2,v_2)]\\&+\varrho[D(V_1,v_1)+D(V_2,v_1)+D(V_1,v_2)+D(V_2,v_2)]\\&=D(V_1,v_1)+\varrho D(V_2,v_1)+\varrho D(V_1,v_2)+D(V_2,v_2).\end{aligned} \tag{6}$$

Note that, the obnoxious model is a special case of semi-obnoxious model, in which all vertices have negative weights. Therefore, in this case

$$V_1=\{v\in V|d(v,x)\geq d(v.y)\}. \tag{7}$$

3 | The Properties on Trees

The vertex optimality property is a basic property of the classical 2-median problem on a network with nonnegative weights. This property states that there exists an optimal solution on the set of vertices of T [38]. In the case that vertices may have either positive or negative weights, the vertex optimality does not hold as shown in [19]. However, on the tree networks, this property holds for MWD model due to Burkard et al. [19]. In the following lemma we show that this property also holds for the semi-obnoxious and obnoxious backup 2-median problem on trees.

Lemma 1. For the tree T, there exists an optimal solution of the semi-obnoxious and obnoxious backup 2-median problem on vertices of T.

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Proof. Since the obnoxious model is an especial case of semi-obnoxious model, we proof the lemma for the semi-obnoxious case.

Suppose, to the contrary, there isn't any optimal solution contains vertices of T. Let $\{m_1,m_2\}$ be an optimal solution of the considered problem on T, and m_1 be a non-vertex of the tree. Let $e(v_r,v_q)$ be the edge of the tree containing m_1. Let V_1 be the set of vertices of T that allocated to m_1. Consider the following function:

$$F(m_1,m_2)=D(V_1,m_1)+\varrho D(V_2,m_1)+\varrho D(V_1,m_2)+D(V_2,m_2). \quad (8)$$

Where $V_2=V\setminus V_1$. We have:

$$\begin{aligned}
&D(V_1,m_1)+\varrho D(V_2,m_1)\\
&=\sum_{v\in V_1\cap T_{v_r}} w(v)\big(d(v,v_r)+d(v_r,m_1)\big)+\sum_{v\in V_1\setminus T_{v_r}} w(v)\big(d(v,v_r)-d(v_r,m_1)\big)\\
&+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)\big(d(v,v_r)+d(v_r,m_1)\big)+\sum_{v\in V_2\setminus T_{v_r}} w(v)\big(d(v,v_r)-d(v_r,m_1)\big)\right]\\
&=\sum_{v\in V_1\cap T_{v_r}} w(v)d(v,v_r)+\sum_{v\in V_1\setminus T_{v_r}} w(v)d(v,v_r)\\
&+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)d(v,v_r)+\sum_{v\in V_2\setminus T_{v_r}} w(v)d(v,v_r)\right] \qquad (9)\\
&+\left(\sum_{v\in V_1\cap T_{v_r}} w(v)-\sum_{v\in V_1\setminus T_{v_r}} w(v)+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)-\sum_{v\in V_2\setminus T_{v_r}} w(v)\right]\right)d(v_r,m_1)\\
&=D(V_1,v_r)+\varrho D(V_2,v_r)\\
&+\left[2\left(\sum_{v\in V_1\cap T_{v_r}} w(v)+\varrho\sum_{v\in V_2\cap T_{v_r}} w(v)\right)-w\big(T_{V_1}\big)-\varrho w\big(T(V_2)\big)\right]d(v_r,m_1).
\end{aligned}$$

Let

$$K=\left[2\left(\sum_{v\in V_1\cap T_{v_r}} w(v)+\varrho \sum_{v\in V_2\cap T_{v_r}} w(v)\right)-w\left(T_{V_1}\right)-\varrho w\left(T(V_2)\right)\right]. \quad (10)$$

So

$$D(V_1,m_1)+\varrho D(V_2,m_1)=D(V_1,v_r)+\varrho D(V_2,v_r)+Kd(v_r,m_1). \quad (11)$$

Now if $K\geq 0$ since $d(v_r,v_i)<d(v_r,m_1)$ for every point v_i in $P(v_r,m_1)$ then $F(v_i,m_2)<F(m_1,m_2)$, which contradicts the optimality of $\{m_1,m_2\}$.

If $K<0$ since $d(v_r,v_j)>d(v_r,m_1)$ for every point v_j in $P(m_1,v_q)$ then $F(v_q,m_2)<F(m_1,m_2)$ which v_q and m_2 are vertices. But this contradicts with our assumption that $\{m_1,m_2\}$ is an optimal solution.

An efficient method for 2-median problem on a tree is edge deletion method. This method developed for the problem with positive and negative weights by Burkard et al. [19]. The edge deletion method has been also applied to find the solution of backup 2-median problem with positive weights on a tree by Wang et al. [28]. However, this method does not work for the semi-obnoxious 2-median problem. We can find the optimal solution, using vertex optimality property, in $O(n^3)$ time.

In the next section we investigate the obnoxious case where the weights of all vertices are negative.

4 | The Obnoxious Backup 2-Median

Note that the obnoxious problem is a special case of semi-obnoxious model, in which we don't have any vertex with positive weight. Therefore, the vertex optimality holds for obnoxious backup 2-median problem. In this section we consider the obnoxious model and discuss some properties for this case.

The obnoxious backup 2-median problem is equivalent to the backup 2-maxian problem. In the backup 2-maxian problem the weights of all vertices in tree are positive, i.e., $w(v)\geq 0$ for $v\in V$, and the following objective function should be maximized

$$F_1(v_1,v_2)=(1-\varrho)\sum_{v\in V} \max_{i=1,2} w(v)d(v,v_i)+\varrho\left[\sum_{v\in V} w(v)d(v,v_1) + \sum_{v\in V} w(v)d(v,v_2)\right]. \quad (12)$$

Due to Burkard et al. [21] the solution of 2-maxian problem on trees is the two end vertices of the longest path of tree. In Example 1, we show this property does not hold for the backup 2-maxian problem.

Example 1. Consider the tree depicted in *Fig. 1*. In the case $\varrho=0$ we have the 2-maxian problem, therefor, the two ends of longest path i.e., $\{v_1,v_4\}$, is the optimal solution with the value of objective function $F_1(v_1,v_4)=45.5$. However, in the case $\varrho>0$ the objective function for this solution can be obtained as follow

$$F_1(v_1,v_4)=(1-\varrho)45.5+\varrho(25.5+25.5)=45.5+5.5\varrho. \quad (13)$$

Now consider the solution $\{v_1,v_3\}$. The value of objective function for this solution is

$$F_1(v_1,v_3)=(1-\varrho)43+\varrho(25.5+36.5)=43+19\varrho. \quad (14)$$

If $0.185<\varrho\leq 1$ then $F_1(v_1,v_3)<F_1(v_1,v_4)$. Therefore, the longest path is not the optimal solution in this case. *Fig. 2* shows the value of objective functions for three solutions $\{v_1,v_4\}$, $\{v_1,v_3\}$ and $\{v_4,v_3\}$ for varying values of ϱ.

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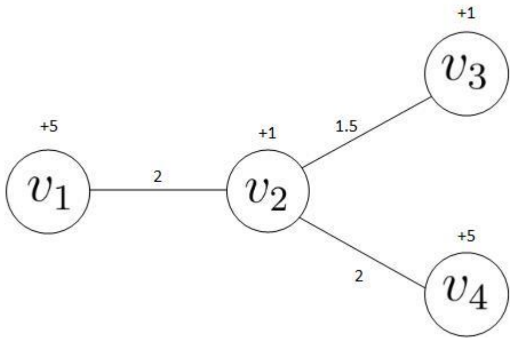

Fig. 1. The optimal solution of backup 2-maxian problem contains no both two ends of longest path of the tree.

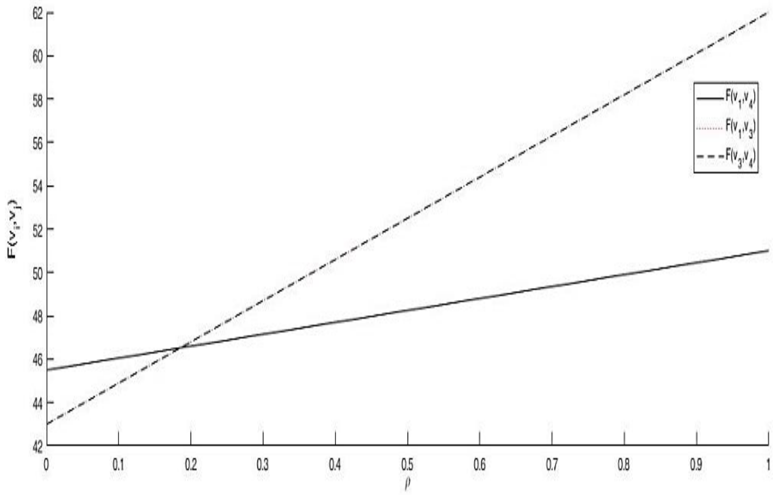

Fig. 2. The value of objective functions in example 1 respect to $0 \leq \rho \leq 1$.

Ting [39] showed that the optimal solution of 1-maxian problem is on the leaves of the tree and the optimal solution of 2-maxian, as mentioned previously in this section, is the two end vertices of the longest path. However, the following example shows that an optimal solution is not necessarily on the set of leaves of the tree anymore.

Example 2. Consider the path depicted in *Fig. 3*. The value of objective function for the solution $\{v_1,v_5\}$ is $F_1(v_1,v_5)=(1-\varrho)(7+52)+\varrho(15+53)=59+9\varrho$. On the other hand, the value of objective function for $\{v_2,v_5\}$ can be calculated as:

$$F_1(v_2,v_5)=(1-\varrho)(5+52)+\varrho(18+53)=57+14\varrho. \quad (15)$$

Therefore, if $59+9\varrho\leq57+14\varrho$ or $\frac{2}{5}<\varrho\leq1$, then $F_1(v_2,v_5)>F_1(v_1,v_5)$. Thus, the end vertex v_1 is not a part of solution anymore.

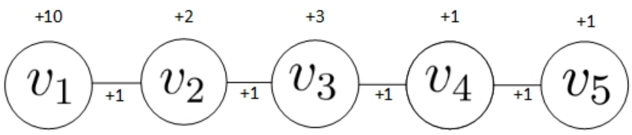

Fig. 3. A path with 5 vertices.

Fig. 4 shows the value of objective functions for the solutions $\{v_1,v_5\}$ and $\{v_2,v_5\}$ for varying values of ϱ.

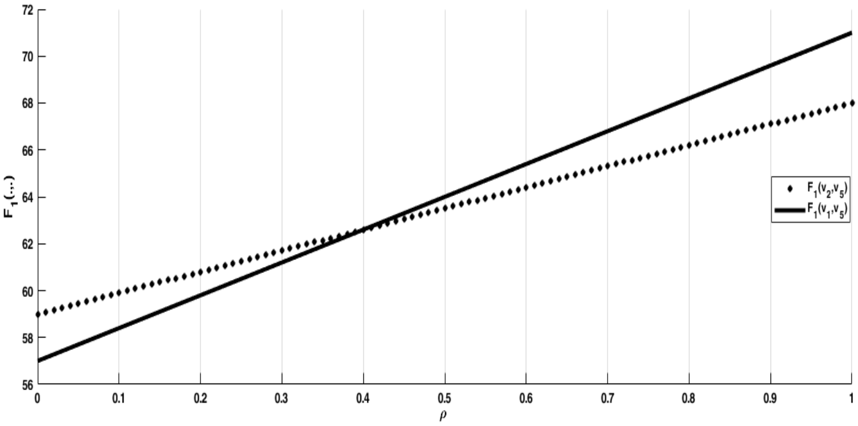

Fig. 4. The value of objective functions in example 2 respect to $0 \leq \rho \leq 1$.

To find the optimal solution of backup 2-maxian problem, we should compare the objective function of $\binom{n}{2}$ solution. Thus, the time complexity is $O(n^3)$.

In summary, using the obtained results in this paper, we will see that just the vertex optimality property holds for the both obnoxious and semi-obnoxious backup 2-median problems on a tree. However, the optimal solution of these problems can be found in polynomial times.

5 | Summary and Conclusion

In this paper we studied the semi-obnoxious and obnoxious versions of the backup 2-median problem on trees. In the classical 2-median problem we should find the location of two servers on the tree so that the sum of the minimum weighted distances from all vertices to the closest severs is minimized. In the semi-obnoxious case, every vertex has either a positive or a negative weight, whereas in the obnoxious model all vertices have negative weights. In these two problems the vertices with positive and negative weights are allocated to the closest and farthest servers, respectively. In the backup models the servers may fail and if a server fails, the functioning server should serve the clients. We investigated the vertex optimality for the semi-obnoxious and obnoxious backup 2-median problem on trees. For the obnoxious model it has been shown that the optimal solution may not happens on the leaves of the tree.

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