E-ISSN: 2676-6167 | P-ISSN: 2538-5100

Journal of Applied Research on Industrial Engineering

JARIE

www.journal-aprie.com

J. Appl. Res. Ind. Eng. Vol. x, No. x (xx) x–x.

Paper Type: Original Article

The Use of Data Envelopment Analysis (DEA) to Estimate the Educational Efficiency of Brazilian Schools

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Citation:

MUNIZ, Rita de Fátima; ANDRIOLA, Wagner Bandeira; MUNIZ, Sheila Maria; THOMAZ, Antônio Clécio Fontelles. (2021). The use of Data Envelopment Analysis (DEA) to estimate the educational efficiency of Brazilian schools. *Journal of applied research on industrial engineering, Volume* (Issue), PP.

Accept: 01/05/2021 Revised: 09/04/2021 Reviewed: 12/03/2021 Received: 06/01/2021

Abstract

The present article deals with the application of the Data Envelopment Analysis (DEA) methodology to identify the most weighty factors that are associated with student performance on large-scale assessments, amongst them, the Permanent Assessment System for Basic Education (SPAECE) test. The DEA Slacks-Based-Measure (SBM) model was used to estimate the relative efficiency of school units in the city of Sobral (CE), one of the most prominent Brazilian counties in the educational scenario. It was evident that the presence of libraries, computer labs, sports courts and rooms for special care in school units constitutes a significant factor associated with the high performance of students, impacting, therefore, on school efficiency.

Keywords: Data Envelopment Analysis. DEA Slacks-Based-Measure Model. Relative Efficiency. Systems Assessment. Elementary Education.

1 | Introduction

In Brazil, the quality of Basic Education has been measured through the Basic Education Development Index (Ideb), according to the Decree nº 6.094 – Plano de Metas Compromisso Todos pela Educação [1], which, in its article 3º, established that "The quality of Basic Education will be objectively measured, based on the IDEB, calculated and published periodically by the National Institute of Educational Studies and Research Anísio Teixeira (Inep)" [2].

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http://dx.doi.org/10.22105/jarie.2021.308815.1388

Created in 2007, the Ideb measures the quality of student learning using data such as the average proficiency in exams that measure skills in Portuguese and in Mathematics and the pass rates recorded in the School Census [3].

It should be noted, however, that the term "quality" is relative, because what would represent good quality, in a given society, would not necessarily maintain this status if transposed to another reality or historical period [4], [5] and [6]. This idea of quality is a requirement of contemporary society, "whose most visible characteristics are cultural diversity and growing awareness of citizen rights" [7].

It should be clarified that the Federal Government's initiative to create the Saeb promoted the genesis of state assessment systems, which "were designed as a complement to Saeb, which means that its characteristics (curriculum, analysis methodology, item bank, etc.) would be originally compatible or so they were constituted over time" [8](p. 811-812).

According to Bonamino and Sousa [9], since the Saeb, initially, was a sample form, the states felt the need to implement assessments that included all schools in their instances. This situation occurred in Minas Gerais, which created in 1991 the Public Education Assessment System (Simave), and in Ceará, with the Permanent Assessment System for Basic Education (Spaece), created in 1992.

The State of Ceará has been using the skills obtained by public school students at Spaece as one of the mechanisms to guide educational public policies. Through this system, Elementary and High School students are evaluated in the subjects of Portuguese Language and Mathematics, to identify levels of proficiency and evolution of results [5].

It was established in this state the "Grade A+ School Award" (Prêmio Escola Nota Dez), a bonus policy guaranteed by State Law that provides a bonus for the 150 public schools with the best and worst rates in the 2nd and 5th grade classes of the first years of elementary school. Therefore, the "Award" came to be seen as an inducing policy for improving results.

In this scenario, the Ceará county of Sobral stands out. According to the Brazilian Institute of Geography and Statistics - IBGE cities [10], the referred county has one of the highest schooling rates in Brazil: in the age group from 6 to 14 years old, reached 97.9%. Other educational results corroborate the effectiveness of the public Municipal Education system. To illustrate, Sobral obtained the highest scores in the country in the 2017 Ideb; in the early years of Elementary School it obtained 9.1 (the Brazilian index was 5.5) while in the final years it obtained 7.2 (the Brazilian index was 4.4). In a decade, the grade of this city in the 5th year of the initial years of Elementary School took a significant leap: it went from 4.9 (in 2007) to 9.1 (in 2017).

The county of Sobral also stands out for its good skills at Spaece and for the number of schools awarded with the Grade A+ School Award. In the 2017 edition, at Spaece, the students in this county achieved very high proficiencies for the 5th year of elementary school: 290.4 in Portuguese Language and 305.6 in Mathematics. These proficiencies were higher than the state averages, whose result in Portuguese Language was 225.3 and in Mathematics 30.9.

Given the results obtained by students in this municipality, the following research problem arises: *how to identify the most important factors associated with the performance of students in tests to assess the quality of learning offered in the public education system in Sobral?* In order to answer the research problem, the Data Envelopment Analysis (DEA) methodology was used, because it is one of the best mathematical techniques to calculate the overall performance of units under analysis, considering of course, the input and output factors used, according Edalatpanah [11].

The greatest contributions of the study described herein resided in the demonstration that structural factors of schools, such as the existence of computer labs, libraries and rooms for assistance to students with disabilities, significantly contribute to the fact that students in these teaching units have better learning and, as a result, obtain good grades in assessments in large schools, such as the SPAECE, and in the index that measures educational quality in Brazil: the IDEB.

2 | Literature Review

The modeling named *Data Envelopment Analysis* (DEA) seeks to assess the relative efficiency of production units, the *Decision Making Units* (DMU), enabling the comparative evaluation of the efficiency of DMUs, according Moradi and Maghbouli [12]. Therefore, it is imperative that they carry out the same activities, that is, that there is similarity between the DMUs, as well as the resources they employ and the activities they develop. [13]. The DEA method allows you to assess efficiency in two different ways: with input orientation, aimed at minimizing production factors (inputs), maintaining a fixed level of production; and output orientation, which seeks to maximize the level of production, keeping inputs fixed [14]-[15].

Chaves and Thomaz [18] point out that the DEA is a relevant technique to support decision-making of a managerial nature, as it makes it possible to evaluate alternative scenarios, based on the best practices identified through mathematical modeling. Thus, this modeling and evaluation technique allows the choice of best practices, taking into account multiple criteria established a priori [17].

This technique was originated from studies by Farrel [19], which proposed its own innovative method to estimate the efficiency of processes. Posteriorly, Charnes, Cooper and Rhodes (1978) [20], continued Farrel's studies, operating an efficiency estimation method, naming it Data Envelopment Analysis (DEA), later understood as a non-parametric model designed to measure the relative efficiency of DMUs, from multiple inputs and outputs, consonant Edalatpanah [11].

Cavalcante and Leite [15] and Chaves and Thomaz [16] pointed out that since the emergence of the DEA method, several models have been introduced to its original version, specifically, the CCR model [20] which has as its main property the proportionality between inputs and outputs at the border, that is, the increase in the quantity of inputs will cause a proportional increase in the value of the outputs; the BCC model (BANKER; CHARNES; COOPER, 1984 apud CAVALCANTE; LEITE) [15] or VRS (Variable Returns to Scale), which represents a variable return to scale and can reach both input and output orientations.

Therefore, the DEA technique allows estimating the relative efficiency of the production units by comparing the analyzed units (DMUs), in order to identify the most efficient ones, that is, those that obtain greater productivity through the resources available to them. According to Edalatpanah [21], the DEA methodology consists of "a linear programming to measure the relative efficiencies of homogeneous decision-making units (DMUs) without knowing the production functions, only using input and output information". Moradi and Maghbouli [12] emphasize that the efficiency of a DMU is obtained by maximizing the ratio between the weighted sum of its outputs and the weighted sum of its inputs. It should be clarified that this relationship cannot exceed a score of 01 (one) for any units.

Edalatpanah [21], in turn, also clarifies that "The efficiency of a DMU is established as the ratio between the weighted sum of the output and the sum of the weighted input, subject to occurring between one and zero" (p. 342). This represents that these DMUs have reached the 100% efficiency target and, so, reached the efficiency border, represented by the score 1 ($\varrho = 1.0$), while the others were considered less efficient units. However, the methodology indicates that these units are able to improve their efficiency as long as they mirror themselves in their benchmarks.

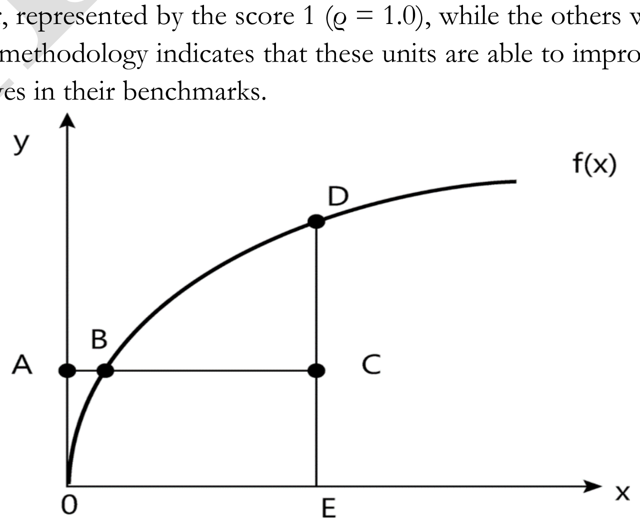

Figure 1 - Representation of the Efficiency Border resulting from the use of the DEA.

Source: Souza Júnior and Gasparini (2006)

The Figure 1 illustrates a production function, in which the input is represented by the variable X and the output is represented by the variable Y. The maximum amount of product that X can generate is represented by f(x), that is, the production function. It is observed that points B and D reached the efficiency border. They are, therefore, the efficient units. Point D, for example, used the same amount of inputs as C, but obtained superior results. Point C, on the other hand, was located below the border, implying less efficiency, because although C used the same amount of resources (inputs) as D, it got lower results (outputs).

According to Rosano-Peña [18] DEA application requires a sequence of steps. First, the DMUs are selected. It is important to emphasize that the selected productive units must be homogeneous, that is, they must produce the same goods and services, using equivalent inputs. In the scope of this article, the DMUs will be represented by the municipal public schools of Sobral. The next step is the classification of inputs and products, that is, which factors should be selected to act as inputs and outputs. Here, factors relevant to the functioning of these school units were considered. Moradi and Maghbouli [12] clarify that the traditional DEA model considers a set of DMUs with several inputs and outputs and each DMU consumes a set of inputs to produce outputs.

Thus, structural factors were chosen, such as libraries, reading rooms, service rooms for students with disabilities, computer labs, sports courts, for considering them extremely important inputs for an educational institution, as they demonstrate a high association with student learning [7]. The outputs were also selected, that is, the students' proficiencies resulting from the students' performance in tests of Portuguese Language and Mathematics components of the SPAECE.

Thus, the data obtained and organized were submitted to statistical analysis using the DEA software of Multinational Company SAITECH Solver, in the professional version 7.0.

Another fundamental procedure to be clarified is the choice of DEA modeling. In this article, we opted for DEA SBM (this last acronym for Slacks-Based-Measure), a slack-based measure. Given its potentialities, among which, the model contemplates the minimization of the weights of both input factors and output factors, thus representing less waste [22] and [23], the SBM-O-V model was opted, which is the DEA method with measurements based on slacks, oriented by the products (The outputs) and with variable returns of scales (V), according to Wilhelm [22].

Also, Wilhelm [22] (p. 39) clarifies that "the SBM measure is based on excess consumption and production gaps in relation to the observed quantities". In this sense, in addition to identifying the degree of efficiency of the organizations under analysis, this measure provides inefficient organizations with guidance to identify efficient production plans, eliminating existing excesses and slacks. The slacks are, therefore, the extra quantities to be reduced in inputs for the producer to reach the efficiency set, after all inputs have been reduced to reach the isoquant.

Hereafter, the equations of the Efficiency Assessment DEA linear programming model entitled SBM (Slacks-Based-Measure), oriented to inputs or outputs, whose mathematical modeling has the mathematical formalization are presented below.

$$(SBM): \min_{\lambda s^- s^+} \rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m} si^-/x_{i0}}{1+\frac{1}{s}\sum_{r=1}^{s} sr^+/y_{r0}} = FuncObj; \qquad \textit{(Eq. 1)}$$

Subject a:

$$x_0 = X\lambda = s^- \qquad \textit{(Eq. 2)}$$

$$y_0 = Y\lambda = s^+ \qquad \textit{(Eq. 3)}$$

$$\lambda \geq 0; s^{-} \geq 0; s^{+} \geq 0; \text{(Eq. 4)}$$

In the objective function FuncObj (1) described above, the ϱ parameter represents the efficiency index of the DEA Model, which will result in a numeric value between 0 and 1. Its maximum efficiency value (ϱ = 1) will only be achieved when the slacks due to excess inputs, which are the s+ variables, and the gaps due to lack of outputs, which are the s– variables, are null. Paying attention to the mentioned specificities that affect the SBM Model, the present study adopted the aforementioned analytical technique, the results of which will be presented in the next section.

3 | Research Results

As this is a study focused on results, the DEA BCC-O modeling was used, that is, output-oriented. However, the DEA SBM O-V (Slacks-Based Measure) modeling was also used, output-oriented and with variable returns from scales. The intention was to identify the model that best explains the reality under study. The Table 1 presents the results from the two models: DEA BCC-O and DEA SBM O-V.

Table 1 –Analysis results of SBM – O-V and DEA BCC-O models. Table 1 –Analysis results of SBM – O-V and DEA BCC-O models.

Methodology DEA	**SBM-O-V**	**BCC-O**
Nº of DMUs	34	34
Arithmetic average	0,941187	0,947849
Standard deviation (DP)	0,061398	0,055449
Maximum Value	1	1
Minimum Value	0,8249	0,83859
Nº of DMUs analyzed	34	34
Nº of DMUs inappropriate	0	0
Nº of DMUs efficient	16	16
Nº of DMUs inefficient	18	18

Source: DEA-Solver (2019).

As shown in Table 1, there was no significant difference in the mean efficiency obtained by the school units in the analyzes carried out by the two models, since both presented mean values around 0.94. This difference is more noticeable in relation to the standard deviation, which ranged from 0.06 in the SBM model to 0.05 in the BCC model. Despite this subtle difference, there is homogeneity between the analyzed units. In the comparison of the analyzes with the two models, it is also noticed that there were no DMUs unsuitable for analysis, that is, no school units with inappropriate or inconsistent data were identified. It is also evident that the number of DMUs evaluated was 34, as well as the quantity of DMUs considered efficient (16 units) and inefficient (18 units) in both models.

Regarding the average efficiency, its value can range from 0 to 1, with 1 being the highest score to be achieved by the analyzed units. According to Table 1, the Maximum Value, which is the maximum efficiency achieved by the DMUs, was 1, which represents 100% efficiency in both models. As for the Minimum value, which is the lowest efficiency obtained by DMUs, there was a slight difference between the models, ranging from 0.82 in SBM to 0.83 in BCC. In Table 2 there is the nominal relation of the 16 efficient DMUs, depending on the results of the two Analytical Models.

Table 2 - Efficient Production Units, corresponding on SBM Models – O-V and DEA BCC-O.

SBM-O-V Model		**BCC-O Model**	
DMUs	**Score**	**DMUs**	**Score**
Vicente Antenor	1	Vicente Antenor	1
Antônio Custódio	1	Antônio Custódio	1
Odete Barroso	1	Odete Barroso	1
Antônio Mendes Carneiro	1	Antônio Mendes Carneiro	1
Araújo Chaves	1	Araújo Chaves	1
Massilon Saboia	1	Massilon Saboia	1
Manoel Marinho	1	Manoel Marinho	1
José Leo	1	José Leoncio	1
Elpidio Ribeiro	1	Elpidio Ribeiro	1
José Ermínio de Morais	1	José Ermínio de Morais	1
Francisco Aguiar	1	Francisco Aguiar	1
Francisco Mon	1	Francisco Monte	1
Frederico Auto Correi	1	Frederico Auto Correia	1
Jacira Men	1	Jacira Mendes	1
Joaquim Barreto Lima	1	Joaquim Barreto Lima	1
José Arimatéia	1	José Arimatéia	1

Source: DEA-Solver (2019).

It was observed that the efficient school units were the same, regardless of the model used. Because there is no significant difference between both and because of the potential of the SBM-O-V model, this one was chosen, as it employs slacks, that is, it points out excess inputs in the DMUs so that the efficiency border is reached. Graphic 1 contains the Relative Efficiency values (ϱ) of the 34 DMUs (school units) analyzed.

Source: DEA-Solver (2019).

Graphic 1 informs the 16 DMUs that reached the efficiency border (ϱ = 1), that is, that reached 100% efficiency with the inputs available. These school units had better productivity due to the resources used, that is, they achieved the best possible result based on the inputs they had. It is also evident that school units with less efficiency do not have such low averages, since the lowest Score (ϱ) was 0.824. Despite not reaching 100% efficiency, these 18 DMUs reached levels between 82% and 94%, which can be understood as a high and very significant school efficiency.

The results also show that Sobral is managing to induce some equity among its educational units regarding the use of the available inputs, as shown in Graphic 2. This last information is very relevant, because it allows us to infer that the learning of students in the subjects of Portuguese

Language and Mathematics is developing and consolidating in practically all the analyzed municipal schools. There is no school with very different results in relation to the others, although there are units that need to mirror their benchmarks, which are the efficient schools.

In Table 3, the schools that were most referenced by the Analytical Models are presented, being considered benchmarks for the other DMU.

Table 3 – Reference frequency of the main DMUs.

School Unit	**Reference frequency to other DMUs**
Francisco Aguiar	9
Araújo Chaves	8
Francisco Monte	4
Elpidio Ribeiro	2

Source: DEA-Solver (2019).

The Reference Frequency represents the number of times the analyzed DMUs were referenced during the modeling. According to the information in Table 3, the DMUs Francisco Aguiar was referenced nine times, leading it to be a model for other schools. This is, therefore, one of the potentialities of this mathematical modeling to identify school units that serve as a model (benchmark) for the set of schools in the educational system analyzed.

4 | Case Study

In this section, the application of DEA SBM modeling for the case study of the DMUs Francisco Aguiar and Dinorah Ramos will be highlighted, because the model also makes it possible to view the projections between the chosen factors (inputs), so that school units can improve their efficiency. This information is made available by the DEA on the Projection tab, according to the data provided in Table 4.

Table 4 - Projection of the Factors of two DMUs in Sobral.

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Input/Output	**Data**	**Projection**	**Difference**	**%**
DMU Francisco Aguiar	**ϱ = 1,0**			
Nº_School_Employee	66	66	0	0,00%
Nº_Enrollm	634	634	0	0,00%
Enrollment_5th grade ES	74	74	0	0,00%
Special Education	54	54	0	0,00%
Special Service Room	1	1	0	0,00%
Library	1	1	0	0,00%
Reading room	0,001	0,001	0	0,00%
Computer_lab	1	1	0	0,00%
Sports court	1	1	0	0,00%
Proficiency_Spaece LP_5thgrade	327,1	327,1	0	0,00%
Proficiency_Spaece MT_5thgrade	334,7	334,7	0	0,00%
DMU Dinorah Ramos	**ϱ = 0,82**			
Nº_School_Employees	49	47,55932	-1,440678	-2,94%
Nº_Enrollment	521	398,6102	-122,3898	-23,49%

Enrollment_5th grade ES	42	42	0	0,00%
Special Education	47	30,13559	-16,86441	-35,88%
Special Service Room	1	1	0	0,00%
Library	1	1	0	0,00%
Reading room	0,001	1,00E-03	0	0,00%
Computer_lab	1	0,458169	-0,541831	-54,18%
Sports court	1	1	0	0,00%
Proficiency_Spaece LP_5thgrade	254,9	319,5068	64,60678	25,35%
Proficiency_Spaece MT_5thgrade	284	332,5847	48,58475	17,11%

Source: DEA-Solver (2019).

According to Table 4, the DMUs Francisco Aguiar was the most referenced efficient unit, constituting a benchmark for less efficient units.

It is therefore appropriate to observe the factors (inputs) that had the greatest impact on the results obtained in terms of student proficiency (outputs), so that other schools can orient themselves. Taking the data in Table 4 as an example, it is observed that all inputs used were well used by DMUs Francisco Aguiar. The DEA did not point out the need to make any changes to the factors (inputs), because the school appropriately appropriated the inputs that were made available to it.

At the opposite extreme is DMUs Dinorah Ramos, which obtained the lowest Score (ϱ = 0,82) between the analyzed units. There are differences between the actual value and the projected values for some of the factors (inputs) of DMUs Dinorah Ramos. For example, with regard to the number of employees, the DEA-Solver projected a 3% reduction in the value of this factor, without, probably, compromising school results.

As for the number of enrollments registered in the School Census, the DEA-Solver projected a reduction of 23.5%, while the number of enrollments in Special Education would have to be reduced by almost 36%. Logically, these numbers regarding the number of enrollments would deserve to be analyzed and associated with other student indicators, such as the number of transfers, failures and dropouts, in order to have a valid diagnosis of the Dinorah Ramos School.

Regarding the Computer Laboratory, the DEA-Solver pointed out the need to reduce, by almost 54%, the inputs that make up (number of computers), perhaps because they are not working satisfactorily, or because they deserve more careful and systematic maintenance. It is more beneficial to student development to have access to fewer computers, but all working perfectly, than having machines unfit for educational use.

After the idealized adjustments are made on these inputs, in theory and with high probability, better results (outputs) could be obtained with regard to the proficiencies of the students. Thus, the modeling points out that there could be an increase in the proficiencies of students (5^{th} year) around 25% in Portuguese and almost 17% in Mathematics.

Certainly, it is not being asserted that the changes made to the inputs will have an impact on the educational results of the Dinorah Ramos School, as it has already been mentioned that DEA modeling allows to draw a probabilistic scenario for managers to make decisions with greater precision.

5 | Conclusion

The study reported here reveals some trends worth mentioning. For example, the results of the DEA SBM-O-V modeling indicated that, in practically all schools with lower efficiency, the total number of students with disabilities needs to be reduced, given the considerable number of students enrolled in these classes. It should be emphasized, as appropriate, that, pursuant to Article 58 of the Law of Guidelines and Bases of Education (LDB nº 9.394/96), Special Education is understood as the modality of Education offered preferentially in the regular teaching network, for students with disabilities, pervasive developmental disorders and high abilities or giftedness [24].

To deal with these students, the faculty must have specific training for such needs. If this does not happen, school efficiency will likely be affected, especially when there is an excess of students with disabilities in relation to the total number of students enrolled in the school unit, as seems to be the case for schools with less efficiency. On the other hand, by implementing classrooms for special assistance in practically all the schools surveyed, Sobral is taking an enormous leap to ensure inclusion through Special Education. Therefore, the aim is to guarantee the access and permanence of these students in schools, with special attention to their adaptive needs. From this perspective, inclusion brings benefits to all social actors involved, and not only for people with disabilities, as its effects are felt throughout society.

The model pointed out as the most important factors, those of structural order, such as the specialized educational service room, in which students with special educational needs rely on the support of specialized professionals to assist them in their teaching-learning process, the presence of libraries in schools, reading rooms, computer labs and sports courts in these educational units, as factors that contribute for schools to perform their work well, and as a result, stand out in external evaluations, such as the Spaece, and in the indexes of Ideb. Furthermore, by ensuring that educational institutions have these factors at their disposal, municipal managers are moving towards educational equity, a vital aspect to society.

The potential of using the DEA was evidenced as a foundation for planning actions aimed at improving school management. The research provided evidence of a surplus of students enrolled in an educational institution, as well as the existence of well-functioning computer labs and service rooms for students with disabilities. These are very important factors and directly influence student results on performance tests such as the SPAECE. Therefore, this information is relevant to guide municipal managers, school directors and Education Secretaries in making appropriate decisions, thus influencing the efficiency of school institutions. According to Edalatpanah [21], the advent of technology, the complexity and the volume of information encourage school managers to appropriate scientific methods to determine and increase the productivity of the organization under their jurisdiction. Thus, appropriating tools such as the DEA enables action plans to increase the productivity of schools in a rational, scientific and precise manner.

The study presented an application of DEA modeling to assess the efficiency of educational units in Sobral, as it is considered important for educational managers to know the results arising from this analytical tool, in order to adopt appropriate solutions for less efficient school units, with the aim of to induce advances in relation to the proficiencies of the students. Although it seems audacious, it is possible for the municipality of Sobral to make even more progress in relation to the learning of students in its Education network. As the Greek philosopher Heraclitus of Ephesus asserted (540 to 470 a.C.), considered the Father of Dialectics: great results require great ambitions.

Funding

The National Council for Scientific and Technological Development (CNPq) financed part of the research.

Conflicts of Interest

There is no conflict of interest in connection with this paper.

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