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Decomposition of Cost Efficiency, Given the Set of Price and Cost Production Possibilities in Data Envelopment Analysis

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Abstract

To evaluate the performance and estimate the efficiency of Decision-Making Units (DMUs) in Data Envelopment Analysis (DEA), the available data are used. These data are usually divided into two categories of inputs and outputs based on their natures. If the price data is also available for inputs, it is necessary to calculate another type of the efficiency called Cost Efficiency (CE). Since the efficiency of units in such a framework is depended on the both quantities of inputs and outputs and also the prices of inputs, it is important to find the sources of cost inefficiency related to each of the factors and plans to address them. In this paper, we intend to present a new decomposition of CE and observed cost versus optimal cost, which are raised from each of the factors involved in the cost inefficiency, in a non-competitive pricing environment which the input price vector for different DMUs can be different. Moreover, for the first time, in parallel with using the PPS based on input and output quantities and introducing some cost inefficiency factors related to this set, we will introduce new sets called price and cost production sets that the first is based on the prices of inputs and output factors, and the second is based on the optimal vectors of inputs and prices obtained from two previous PPS, and then we will introduce other factors of cost inefficiency in the sets. Accordingly, new decomposition for cost inefficiencies will be presented. Also, in the previous analyzes, congestion inefficiency has not been considered as one of the important factors in cost inefficiency. In this study, we also intend to consider the impact of this factor on CE.

Keywords: Data envelopment analysis, Cost efficiency, Non-competitive environment, Cost PPS, Cost decomposition, Congestion.

1 | Introduction

One of the most important concepts in the performance appraisal by Data Envelopment Analysis (DEA) is efficiency. This score indicates the position of a unit relative to an efficient frontier. Usually, based on the acceptance of a series of basic principles, a set called the Production Possibility Set (PPS) is made, and its boundary is called the efficient frontier. This set is based on the information available to the units. This data is usually classified into two categories: inputs and outputs. The first category is usually the resources used to generate the outputs. If, the input prices are also available, the Cost Efficiency (CE) measure will be considered in performance appraisal.

1.1 | Literature Review

In the CE models, the efficiency of the under evaluation unit is compared to a real or virtual unit of the efficient frontier with a lower cost to produce at least the same output as the unit under evaluation.

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Mombini et al. [1] proposed an approach to attain the sustainability radius of the CE considering interval data. Sarab et al. [2] proposed a two-step procedure to maintaining CE in sugar industries under any fluctuation in input costs. To calculate CE in aquaculture using the case of intensive white-leg shrimp farming in Phu Yen province of Vietnam, Long [3] used a two-stage bootstrapping technique. Focusing on CE and running longitudinal case-based research over six years (2014–2019), Piran et al. [4] applied an internal benchmarking analysis for evaluation of economic efficiency of a broiler production system. Lotfi et al. [5] proposed a method to modify the classic CE DEA model in order to investigate the situations of market discounts. Rezaei Hezaveh et al. [6] introduced a cost based PPS in the non-competitive environment to assess Cost, Revenue and Profit Efficiency. Paleckova [7] used two-stage DEA to estimate CE and its determinants of the Czech and Slovak commercial banks within the period of 2005-2015. Soleimani-Chamkhorami [8], used inverse DEA for preserving cost/revenue efficiency of European and American banks. A new decomposition of CE is given in [9] when DMUs are not price takers. A cost minimizing planning problem of a state government in the US were considered by Shiraz et al. [10] in the framework of economic efficiency measures for stochastic data with known input and output prices. Ghiyasi [11] provides the theoretical foundation of the inverse DEA problem when price information is available. Toloo [12] developed a method for finding the most cost efficient DMU when the prices are fixed. Khanjani Shiraz et al. [13] developed a nonparametric methodology for cost-efficiency based on rough set theory to rank and evaluate DMUs when incorporation of data uncertainty. Fang and Li [14] developed models and a base-enumerating algorithm to calculate the upper and lower bounds of CE for each firm in the case of non-unique law of one price while keeping the industry CE optimal. Mozaffari et al. [15] formulated an original DEA-R cost and revenue efficiency models in the case of same price vector for ratio quantities of inputs to outputs. Sahoo et al. [16] states that in a non-competitive market with different input prices, it would be appropriate to use a value-based technology, in which the performance of units can be evaluated in comparison with it. Fang and Li [17] presented a method which can acquire the Pessimistic CE measure in cases with multiple inputs and outputs using the weight restrictions in the form of input cone assurance to determine the lower bound of CE. Camanho and Dyson [18] presented the idea of economic efficiency as a development for Farrell CE in the non-competitive. Jahanshahloo et al. [19] given a method for CE analysis which deals with ordinal data. Camanho and Dyson [20] proposed a process for estimating the bounds of the CE in situations where only a maximal and minimal bounds of input prices can be determined for each DMU. Jahanshahloo et al. [21] suggested a condensed version of [20]'s model with fewer numbers of restrictions and variables. Tone and Tsutsui [22] decomposed observed total cost into the global optimal (minimum) cost and loss due to technical inefficiency in technical PPS, input price difference and inefficient cost mix, which are measured in the cost based PPS.

1.2 | Research Gap and Main Contricutions

It is known that the cost is a function of the amount of inputs and their price. So any inefficiency in the proper use of inputs and inefficiency in the use of appropriate prices leads to cost inefficiency. In the DEA literature, two types of inefficiencies can be attributed to inputs, one is technical inefficiency and the other is congestion. Under normal conditions, an increased input will lead to the increased output, but if one or more input increase as one or more outputs decrease, or else, if one or more inputs decrease as one or more outputs increase, congestion will be said to exist in the inputs. Congestion, to be sure, might not be necessarily the result of a direct association between each input and output since the above mentioned concept of congestion is more comprehensive than the concept of the congestion in economics. In general, however, congestion is said to exist if the increase of an input factor which have cost nature does cause a decrease of outputs which have an income nature. What is considered in cost studies in DEA is often technical inefficiency and the effect of congestion inefficiency on cost performance is overlooked. If price data are present, one can consider the price-related congestion, since the prices have a nature similar to inputs, and if they increase too much, the input may decrease. On the other hand, in the literature on CE, less attention is paid to fixing cost inefficiencies and improving them by fixing price inefficiencies. It is obvious that such an issue makes sense in the presence of different price data between units. If the prices are the same for all units, it is clear that cost inefficiencies can only be attributed to inefficiencies related to resources or inputs.

Tone [23] and Tone and Tsutsui [22], using the basic principles of DEA, introduced their cost PPS. Their PPS were made by accepting the constant return to scale principle and, in making cost PPS of Tone [23], only the observed prices and outputs were used, and in Tone and Tsutsui [22] modified cost obtained by removing technical inefficiencies in inputs was used in conjunction with outputs, and the modified prices and eliminating other types of technical inefficiencies such as congestion was neglected. Furthermore, if we consider a triple (xj, cj, cjxj) = (observed quantities of inputs, obsereved prices, and observed costs) for CE estimation, then we can also consider a triple (Px, Pc, Pcx) for PPSs. Accordingly, considering the first component of the first triple with the observed outputs leads to the creation of conventional quantity based PPS Px, and also considering the observed costs 'cjxj' along with the observed outputs leads to the creation of the cost PPS of Tone [23]. Therefore, what is expected is to build a PPS using the second component of the first triple, i.e. 'cj' and the observed outputs, and this can be considered our motivation in building the Pc.

From the considerations above, the contribution of this paper in the literature are given in the following:

I. Investigating the effect of congestion related to input quantities and inputs technical inefficiencies on CE by calculating the relevant inefficiencies and excess costs due to these inefficiencies in Px.
II. Construction a new price-based PPS Pc.
III. Investigating the effect of congestion related to prices and price technical inefficiencies on CE by calculating the relevant inefficiencies and excess costs due to these inefficiencies in Pc.
IV. Composing a new set of cost based PPS using optimal inputs and prices, calculating two types of mix and cost allocative efficiency and finally break down CE based on all previous efficiencies, as well as express the observed cost based on the optimal cost and all excess costs incurred by the unit under assessment due to various inefficiencies.

It should be noted that the cost PPS set will be made in this paper is different from Tone [23] and Tone and Tsutsui [22]. As mentioned before, the observed costs were used in Tone [23]. In Tone and Tsutsui [22], the modified costs obtained by the optimal input vectors and the observed prices are used, while in this paper, both the modified inputs and the modified prices will be used to construct the cost PPS. It can be easily proved that the two previous cost PPS are a subset of the cost PPS will be introduced in this paper, and therefore it can be examined that the method proposed in this paper is able to identify more sources of cost inefficiency. In [22] and [23], all inefficiencies in the input and output quantities based PPS are considered as technical. As a result, only the effect of this type of inefficiency on CE and excess cost is mentioned. However, as far as the authors know, for the first time in this paper, the effect of input congestion on cost inefficiency has been considered independently. Also in [22], reducing inputs is a priority to eliminate cost inefficiencies, rather than reducing prices. In this way, first the sources of quantitiy technical inefficiency are eliminated and then the price inefficiency corresponding to the cost point made from the optimal inputs and observed prices in their proposed set of cost based PPS Pcx are eliminated. Methods in [22] and [23] are not able to respond if the reduction of prices and the selection of optimal prices or the investigation of the cost inefficiency and the excess cost imposed due to the non-selection of the appropriate price are desired. It should be noted that depending on the whether fixing input inefficiencies is a priority for the decision maker or fixing price inefficiencies, two decompositions of CE and observed cost will be presented which are considered as a step-by-step path to eliminate inefficiencies. In both presented decomposition, attention has been paid to the elimination of both types of congestion inefficiency and congestion free technical inefficiencies in separate PPSs of quantities and prices.

The paper is organized as follow. The second part is dedicated to some preliminaries. In Section 3, the proposed method for analyzing CE and observed cost is described in stages. In Section 4, by giving two numerical examples and a practical example, we show the applicability of the proposed method. The final section is devoted to conclusions and suggestions for the future studies.

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2 | Preliminary

2.1 | Data Envelopment Analysis

A pair of multiple-input $X \in R^m$ and multiple-output $Y \in R^s$ is called an activity, and it is expressed by the notation (X,Y).The set of feasible activities which is called the PPS is given as follow:

$$T = \{(X, Y) \mid X \text{ can produce } Y\}.$$

The following properties is postulated in DEA for PPS T:

I. Inclusion of observations: the observed activity (X_j, Y_j) belongs to T for all $j=1,\dots,n$.

II. Convexity: if $(X,Y)\in T$ and $(X',Y')\in T$, then $(\lambda X+(1-\lambda)X', \lambda Y+(1-\lambda)Y')\in T$ for all $\lambda \in (0,1)$.

III. Ray unboundedness (constant return to scale): if $(X,Y)\in T$, then $(kX,kY)\in T$ for any positive scalar k.

IV. Free Disposal (monotonicity): if $(X,Y)\in T$ and $X' \geq X \,\&\, Y' \leq Y$, then $(X',Y')\in T$.

V. Minimum extrapolation: if a PPS T' satisfies before postulates, then $T \subset T'$.

The unique empirical PPS T_V has four properties that is defined *Eqs. (1)*, *(2)*, *(4)* and *(5)* as follows[1]:

$$T_V = \left\{(X,Y) \,\middle|\, X \geq \sum_{j=1}^{n}\lambda_j X_j ,\, Y \leq \sum_{j=1}^{n}\lambda_j Y_j ,\, \sum_{j=1}^{n}\lambda_j = 1 ,\, \lambda_j \geq 0 \,;\, j=1,\dots,n\right\}.$$

Sometimes this set is called the PPS of the BCC model, or the PPS with variable returns to scale technology [24].

The boundary of the PPS is called "efficient frontier". Each DMU on the efficient frontier is called efficient, and the others are inefficient. There are various strategies that can determine the efficiency position of an under evaluation. One of the most important of them aims to minimize inputs while satisfying at least the given output levels. This is called the input-orientation. There is another type called the output-orientation that attempts to maximize outputs without requiring more of any of the observed input values. To evaluate the relative efficiency of DMU_o in T_v in the input orientation, the following BCC model can be solved:

$$\begin{aligned}
&\text{Min} \quad \theta - \varepsilon(1^T S^- + 1^T S^+),\\
&\text{s.t.}\\
&\quad \sum_{j=1}^{n}\lambda_j X_j + S^- = \theta X_o,\\
&\quad \sum_{j=1}^{n}\lambda_j Y_j - S^+ = Y_o,\\
&\quad \sum_{j=1}^{n}\lambda_j = 1,\\
&\quad \lambda_j \geq 0, \quad j=1,\dots,n,\\
&\quad S^- \geq 0,\, S^+ \geq 0.
\end{aligned} \tag{1}$$

[1] It is necessary to explain that by accepting all or some of the principles, and as well as changes in some principles such as ray unboundedness, different PPSs that led to models BCC-CCR, CCR-BCC, FDH, CHD [24] and etc. can be made, which the discussion of which is beyond the scope of this paper.

Where ε is a non-Archimedean value, meaning that it is smaller than any small positive number. For more information about Epsilon [25].

The output orientation version of BCC is:

$$\begin{aligned}
&\text{Max} \quad \varphi+\varepsilon(1^T S^- + 1^T S^+),\\
&\text{s.t.}\\
&\sum_{j=1}^{n} \lambda_j X_j + S^- = X_o,\\
&\sum_{j=1}^{n} \lambda_j Y_j - S^+ = \varphi Y_o,\\
&\sum_{j=1}^{n} \lambda_j = 1,\\
&\lambda_j \geq 0, \quad j=1,\ldots,n,\\
&S^- \geq 0, \quad S^+ \geq 0.
\end{aligned} \tag{2}$$

2.2 | Congestion

Noura et al. [26] presented a relatively simple approach for finding congestion. Using this method, the input congestion of DMU_o will be diagnosed as follows:

The set E of efficient units of *Model (2)*, will be considered as $E=\{j|\varphi_j^*=1\}$. Among the efficient units of the E set, the units with the highest values in at least one input will be selected and the input related to that unit will be specified with x_i^{max}. There is congestion in DMU_o if and only if there is at least one of the following conditions in the optimal solution of *Model (2)*, $(\theta^*, \lambda^*, s^{-*}, s^{+*})$:

I. $\varphi_o^* > 1$ and at least in one component, $x_{io} > x_i^{max}$.

II. At least in one component, $s_r^{+*} > 0$ and at least in one component, $x_{io} > x_i^{max}$.

In this case, the congestion value will be $S_i^c = x_{io} - x_i^{max}$ in the i^{th} input.

3 | Methodology

This section consists of two subsections. In the first part, we will explain the steps of the proposed method. In order to maintain the integrity of the proposed algorithm structure, we present in the second section some discussions about the PPSs, the models used, as well as some theoretical material that scientifically supports the proposed method.

3.1 | Steps of the Proposed Method

Step 1. Calculating the observed cost for DMUo with the input x_o and the observed price C_o:

$$C_o = \sum_{i=1}^{m} c_{io} x_{io}.$$

Step 2. Constructing the PPSs P_x and P_c:

$$P_x = \{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j=1,\ldots,n\}.$$

$$P_c=\{(C,Y)\mid C\geq\sum_{j=1}^{n}\lambda_jC_j,\ Y\leq\sum_{j=1}^{n}\lambda_jY_j,\ \sum_{j=1}^{n}\lambda_j=1,\lambda_j\geq 0, j=1,\ldots,n\}.$$

Step 3. Calculations in P_x and P_c :

Part 1: Calculations in P_x :

I. Calculating the input congestion S_i^c of the i^{th} input for (x_o, y_o) using *Model (2)*.

II. Calculating the cost of the congestion-free input $x_{io}{}^c = x_{io} - s_{io}^c\ (i=1,\ldots, m)$, with the observed price, c_o^1 as follows:

$$C_o{}^1=\sum_{i=1}^{m}c_{io}x_{io}^c.$$

III. Calculating the excess cost caused by congestion in the inputs:

$$L_o^{\text{Input congestion}}=C_o-C_o{}^1.$$

IV. Calculating the input congestion efficiency:

$$\text{Input congestion efficiency}=\frac{C_o{}^1}{C_o}.$$

V. Finding the input technical efficiency of the unit (x_o^c, y_o) with the components of congestion-free x_o^c vector, using *Model (1)*.

VI. Obtaining the technical efficient point (input projection point) $x_o^*=\theta_x^* x_o^c - s_x^{-*}$ using the optimal solution $(\theta_x^*, \lambda_x^*, s_x^{-*}, s_x^{+*})$ of *Model (1)*.

VII. Calculating the cost of the technical efficient point x_o^*, with the observed price c_o as:

$$C_o{}^2=\sum_{i=1}^{m}c_{io}x_{io}^*.$$

VIII. Calculating the excess cost caused by technical inefficiency in the inputs:

$$L_o^{\text{Tech. input}}=C_o{}^1-C_o{}^2.$$

IX. Definition of the (free congestion) input technical efficiency as:

$$\text{Input technical efficiency}=\frac{C_o{}^2}{C_o{}^1}.$$

Part 2: Calculations in P_c :

I. Calculating the input congestion S_i^c of the i^{th} price for (c_o, y_o) using *Model (2)*.

II. Calculating the cost of the congestion-free price $c_{io}{}^c = c_{io} - s_{io}^c$ (i = 1,…, m), with the observed input, x_o as follows:

$$C_o{}^3=\sum_{i=1}^{m}c_{io}^c x_{io}.$$

III. Calculating the excess cost caused by congestion in the prices:

$$L_o^{\text{Price congestion}} = C_o - C_o^{\,3}.$$

IV. Calculating the price congestion efficiency:

$$\text{Price congestion efficiency} = \frac{C_o^{\,3}}{C_o}.$$

V. Finding the price technical efficiency of the unit $\left(c_o^c, y_o\right)$ with the components of congestion-free c_o^c vector, using *Model (1)*.

VI. Obtaining the price efficient point (price projection point) $c_o^* = \theta_c^* c_o^c - s_c^{-*}$ using the optimal solution $\left(\theta_c^*, \lambda_c^*, s_c^{-*}, s_c^{+*}\right)$ of *Model (1)*.

VII. Calculating the cost of the price efficient point c_o^*, with the observed input x_o as:

$$C_o^4 = \sum_{i=1}^{m} c_{io}^* x_{io}.$$

VIII. Calculating the excess cost caused by price inefficiency in the inputs:

$$\text{Price technical efficiency} = \frac{C_o^{\,4}}{C_o^{\,3}}.$$

IX. Definition of the (free congestion) price technical efficiency as:

$$L_o^{\text{Tech. Price}} = C_o^3 - C_o^4.$$

Step 4. Considering the various combinations of x_i^c, x_i^*, c_i^c and c_i^*:

Part 1: Calculating the costs.

I. The cost value of congestion-free input x_i^c with the technical efficient price c_i^* will be as:

$$C_o^{\,5} = \sum_{i=1}^{m} c_{io}^* x_{io}^c.$$

II. The cost value of technical efficient input x_i^* with the congestion-free price c_i^c will be as follows:

$$C_o^{\,6} = \sum_{i=1}^{m} c_{io}^c x_{io}^*.$$

III. The cost value of technical efficient input x_i^* with the technical efficient price c_i^* will be as follows:

$$C_o^{\,7} = \sum_{i=1}^{m} c_{io}^* x_{i0}^*.$$

Part 2: Calculating the excesses.

I. Calculating the excess cost caused by price congestion when the inputs are free of inefficiency (optimal inputs):

$$L_o^{\overline{\text{Price congestion}}} = C_o^{\,2} - C_o^{\,6}.$$

II. Calculating the excess cost caused by price inefficiency when the inputs are free of inefficiency (optimal inputs):

$L_o^{\overline{\text{Tech price}}} = C_o^6 - C_o^7$.

III. Calculating the excess cost caused by input congestion when the prices are free of inefficiency (optimal prices):

$L_o^{\overline{\text{Input congestion}}} = C_o^4 - C_o^5$.

IV. Calculating the excess cost caused by input inefficiency when the prices are free of inefficiency (optimal prices):

$L_o^{\overline{\text{Tech. input}}} = C_o^5 - C_o^7$.

The sign is to emphasize that the inefficiencies of one of the factors have been eliminated or the optimal value of one of the factors has been considered.

Part 3: Defining the efficiencies.

I. Definition of the price congestion efficiency when the inputs are free of inefficiency (optimal inputs):

$$\overline{\text{Price congestion efficiency}} = \frac{C_o^6}{C_o^2}.$$

II. Definition of the price technical efficiency when the inputs are free of inefficiency (optimal inputs):

$$\overline{\text{Price technical efficiency}} = \frac{C_o^7}{C_o^6}.$$

III. Definition of the input congestion efficiency when the prices are free of inefficiency (optimal Prices):

$$\overline{\text{Input congestion efficiency}} = \frac{C_o^5}{C_o^4}.$$

IV. Definition of the input technical efficiency when the prices are free of inefficiency (optimal prices):

$$\overline{\text{Price congestion efficiency}} = \frac{C_o^6}{C_o^2}.$$

Note that one can consider the combination of x_i^c and c_i^c, which has been neglected in this paper.

Step 5. Constructing the PPS P_{cx} :

After removing the congestion and the price technical inefficiencies in P_x and P_c, and finding the optimal inputs x^* and prices c^* for DMUs, the new PPS P_{cx} which is a cost based technology is produced as follows:

$$P_{cx} = \{(CX, Y) \mid CX \geq \sum_{j=1}^{n} \lambda_j x_j^* c_j^*, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n\}.$$

Step 6. Calculations in P_{cx} :

I. Obtaining the input of projection point $x_o = \theta_{cx}^{\ *} c_o^* x_o^* - s_c x^{-*}$ using the optimal solution $(\theta_{cx}^*, \lambda_{cx}^*, s_{cx}^{-*}, s_{cx}^{+*})$ from *Model (1)* for $(c_o^* x_o^*, y_o)$.

II. Calculating the cost for the technical efficient input $\hat{x}_o$:

$$C_o^8 = \sum_{i=1}^{m} \hat{x}_{io}.$$

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III. Defining the Mix CE as:

$$\text{Mix Cost efficieny}=\frac{C_o^{\,8}}{C_o^{\,7}}.$$

IV. Calculating the excess cost caused by mix cost inefficiency:

$$L_o^{Mix}=C_o^{\,7}-C_o^{\,8}.$$

V. Finding the unit with the least cost C_o^9 in P_{cx} using the following *Model (3)*:

$$
\begin{aligned}
&C_o^{\,9}=\min e\hat{x},\\
&s.t.\\
&\quad \sum_{j=1}^{n}\mu_j x_{ij}+t_i^-=x_i, \quad i=1,\dots,m,\\
&\quad \sum_{j=1}\mu_j y_{rj}-t_r^+=y_r, \quad r=1,\dots,s,\\
&\quad \sum_{j=1}\mu_j=1,\\
&\quad t_i^-\geq 0, \quad i=1,\dots,m,\\
&\quad y_r, t_r^+\geq 0, \quad r=i,\dots,s\ ,\\
&\quad \mu_j\geq 0, \quad j=1,\dots,n.
\end{aligned}
\tag{3}
$$

VI. Defining the allocative efficiency as:

$$\text{Allocative efficiency}=\frac{C_o^{\,9}}{C_o^{\,8}}.$$

VII. Defining the excesses of cost caused by allocative inefficiency:

$$L_o^{Allocative}=C_o^{\,8}-C_o^{\,9}.$$

Step 7. Decompositions of cost and CE:

Strategy 1. Preferring the input reduction on price reduction.

$$C_o=L_o^{\text{input congestion}}+L_o^{\text{tech. input}}+L_o^{\overline{\text{price congestion}}}+L_o^{\overline{\text{tech. price}}}+L_o^{Mix}+L_o^{Allocative}+C_o^{\,9}.$$

$$\text{Cost eff.}=\text{Input congestion eff.}\times\text{Input tech. eff}\times\overline{\text{Price congestion eff.}}$$
$$\times\overline{\text{Price tech. eff.}}\times\text{Mix ineff.}\times\text{Allocative eff.}$$

Strategy 2. Preferring the price reduction on input reduction.

$$C_o=L_o^{\text{price congestion}}+L_o^{\text{tech. price}}+L_o^{\overline{\text{input congestion}}}+L_o^{\overline{\text{tech. input}}}+L_o^{Mix}+L_o^{Allocative}+C_o^{\,9}.$$

$$\text{Cost eff.}=\text{Price congestion eff.}\times\text{Price tech. eff.}\times\overline{\text{Input congestion eff.}}$$
$$\times\overline{\text{Input tech. eff.}}\times\text{Mix eff.}\times\text{Allocative eff.}$$

As it can be seen, the CE is dependent to different factors such as the input and price congestion, the input and price efficiency, the mix and the allocative efficiency. *Fig. 1* shows the steps of the proposed cost and CE decomposition method.

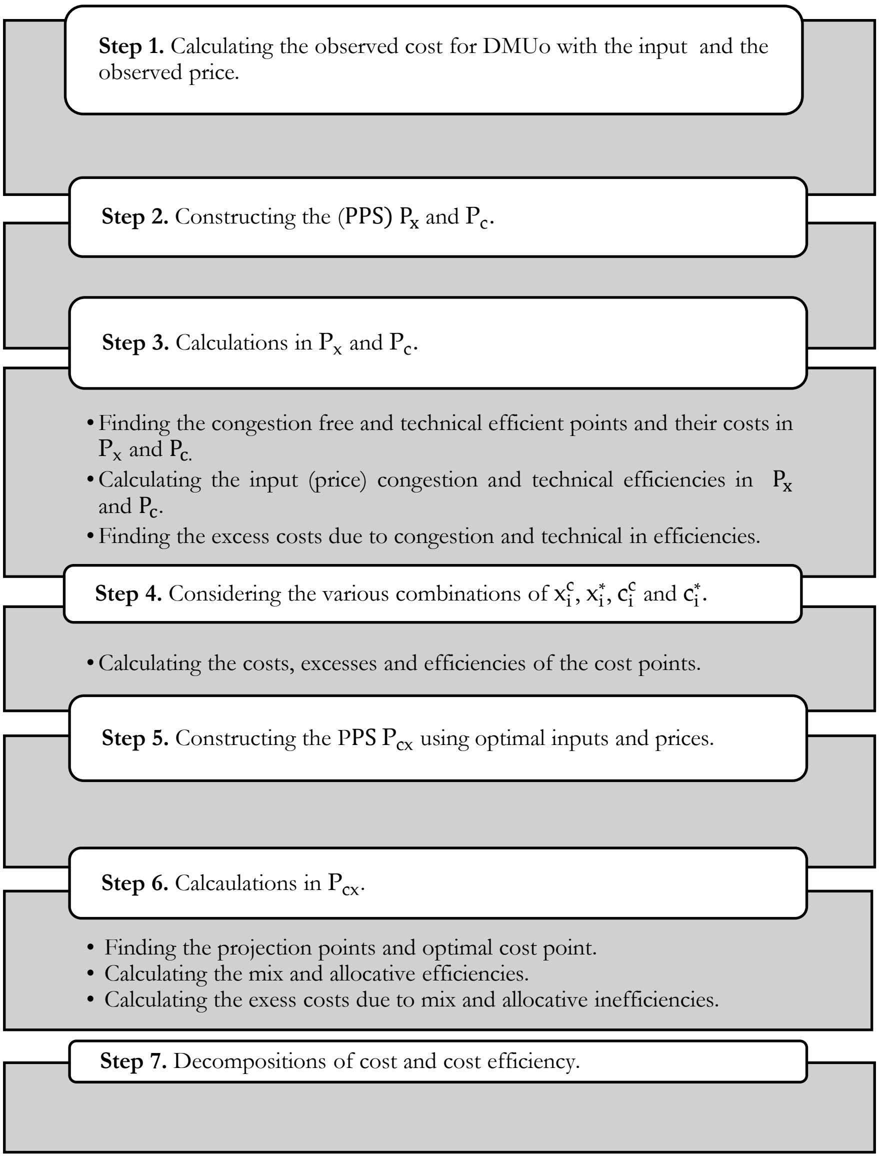

Fig. 1. Overview of the proposed method.

3.2 | Theoretical Discussion

During the steps of the proposed algorithm, 3 PPS were used, the first set (P_x) being the same PPS with variable return to scale Tv. As stated in the background, this set is made by accepting a series of principles and taking into account the observed input and output quantities of the units. As mentioned before, Tone [23] and Tone and Tsutsui [22], using the basic principles of DEA, introduced their cost PPS. Their PPS were made by accepting the constant return to scale. In contrast, this paper is cautious about accepting the principle of constant return to scale in the presence of prices and therefore costs, and PPSs are based on variable return to scale.

Using the same principles for observed prices and outputs, the Pc set is created. During the proposed algorithm, in Part 1 of Step 3 and the first sentence of Part 2 of Step 3, to investigate the presence of congestion in P_x and Pc, the method of Noura et al. [26] is used, which it uses *Model (2)* itself. Clearly *Model (2)* is always feasible. But the question is whether *Model (1)* is feasible in evaluating virtual units with free of congestion inputs and prices in the fifth sentence of Part 1 of Step 3 and the fifth sentence of Part 2 of Step 3 of the proposed method?

Theorem 1. *Model (1)* is always feasible in evaluating units (x_o^c, y_o) and $\left(c_o^c, y_o\right)$.

Proof: we consider the feasibility of *Model (1)* for the evaluation of (x_o^c, y_o). the proof is the same for $\left(c_o^c, y_o\right)$.

Suppose $(\theta_o^*, \lambda_o^*, s_o^{-*}, s_o^{+*})$ is the optimal solution of *Model (1)* in the evaluation $\left(x_o, y_o\right)$. Then

$$\sum_j \lambda_{oj}^* x_j + s_o^{-*} = \theta_o^* x_o,\ \sum_j \lambda_{oj}^* y_j - s_o^{+*} = y_o,\ \sum_j \lambda_{oj}^* = 1, \lambda_{oj}^* \geq 0.$$

On the other hand as $x_o{}^c = x_o - s_o^c$, then $x_o = x_o{}^c + s_o^c$, and therefore $\sum_j \lambda_{oj}^* x_j + s_o^{-*} = \theta_o^* x_o = \theta_o^*(x_o{}^c + s_o^c)$.

As a result $\sum_j \lambda_{oj}^* x_j + s_o^{-*} - \theta_o^* s_o^c = \theta_o^* x_o{}^c$. It is trivial that $s_o^{-*} - \theta_o^* s_o^c \geq 0$. Since on contrary supposing $s_o^{-*} - \theta_o^* s_o^c < 0$ results $s_o^{-*} < \theta_o^* s_o^c$ which is in contrast with max slack of s_o^{-*} in evaluating $\left(x_o, y_o\right)$ using *Model (1)*. So $(\theta_o^*, \lambda_o^*, s_o^{-*} - \theta_o^* s_o^c, s_o^{+*})$ is a feasible solution for the evaluation of (x_o^c, y_o) using *Model (1)*.

After obtaining points with no congested inputs in P_x and prices with no congested prices in P_c, projection points are obtained for such points in the sixth sentence of Part 1 of Step 3 and the sixth sentence of Part 2 of Step 3. To obtain projections, the optimal solution of *Model (1)* in evaluating these points has been used. It can be easily proved that the optimal value is always positive and therefore *Model (1)* will have a finite optimal value in evaluating these points. Therefore, such points are always accessible. As a result, input and price technical efficiencies can be calculated based on them. Also in 1-6, the optimal solution obtained from *Model (1)* in the cost unit evaluation $(c_o^* x_o^*, y_o)$ in Pcx is used to obtain a projection point, which can be concluded with a similar argument that these points are also available. Therefore, mix efficiencies can be calculated based on them. On the other hand, in 6-5 we used *Model (3)* to find the lowest cost point in Pcx. Since $\hat{x}_i = \hat{x}_{io}$, $y_r = y_{ro}$, $\mu_o = 1$, $\mu_j = 0\left(j \neq o\right)$, $t_i^- = 0(\text{for all } i)$ and $t_r^+ = 0(\text{for all } r)$ apply in the model, so this model is also always feasible.

In all steps of the algorithm, different efficiencies are introduced based on the defined cost values. It can be easily shown that these values are in the range (0, 1] and also the excess costs defined in the algorithm are all non-negative.

Theorem 2. The following efficiencies are all in the range (0, 1].

- *Input congestion efficiency.*
- *Input technical efficiency.*
- *Price congestion efficiency.*
- *Price technical efficiency.*
- *Mix efficiency.*
- *Allocative efficiency.*

Proof: We express the proof for input congestion efficiency. It is the same for others and we refrain from expressing them. By definition Input congestion efficiency is C_o^1/C_o. On the other hand $C_o^1 = \sum_{i=1}^m c_{io} x_{io}^c = \sum_{i=1}^m c_{io}\left(x_{io} - s_{io}^c\right) = \sum_{i=1}^m c_{io} x_{io} - \sum_{i=1}^m c_{io} s_{io}^c$. As $c_{io} \geq 0$,for all i and $s_{io}^c \geq 0$,for all i, So, $C_o^1 = \sum_{i=1}^m c_{io} x_{io} - K = C_o - K$. Then $C_o^1 \leq C_o \Rightarrow \frac{C_o^1}{C_o} \leq 1$. On the other hand, always $C_o^1 > 0$ (obviously), therefore $C_o^1 / C_o > 0$.This completes the proof. So input congestion efficiency in the range (0, 1].

Theorem 3. The following excess cost values are all non-negative.

- *Excess cost caused by congestion in the inputs* ($L^{Input\ congestion}$).
- *Excess cost caused by technical inefficiency in the inputs* ($L^{Tech.\ input}$).
- *Excess cost caused by congestion in the prices* ($L^{Price\ congestion}$).
- *Excess cost caused by technical inefficiency in the prices* ($L^{Tech.\ price}$).
- *Excess cost caused by cost mix inefficiency* (L^{Mix}).
- *Excess cost caused by cost allocative inefficiency* ($L^{Allocative}$).

Proof: we show that $L^{Input\ congestion} \geq 0$. The argument is similar for the rest. By definition $L^{Input\ Congestion} = C_o - C_o^1$. Given that in the process of proving *Theorem 2* it was shown that $C_o^1 \leq C_o$, then $C_o - C_o^1 \geq 0$, thus $L^{Input\ congestion} \geq 0$.

According to the above theorems, it can be concluded that all efficiencies, excess values and projection points are all calculable and well-defined, and therefore CE and observed cost can be decomposed based on these values.

4 | Examples

Example 1. *Table 1* shows the set of 6 DMU, which have one input and one output, and the price of the input unit has been specified.

Table 1. Data of 6 DMUs.

Dmu	Input x_j	Output y_j	Price of Input c_j	Observed Cost x_jc_j
A	2	1	4	8
B	3	3	1.5	4.5
C	4	4	2.5	10
D	6	4	3	18
E	7	3	3	21
F	5	2	5	25

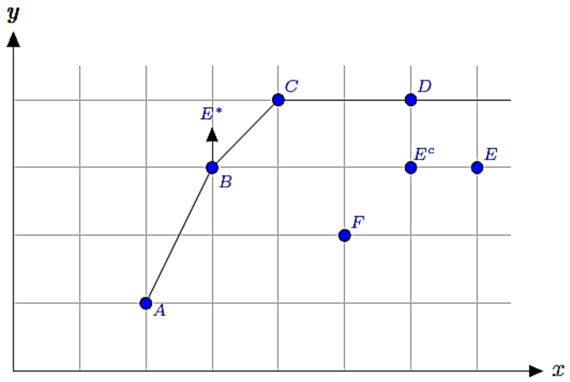

Fig. 2. PPS made by inputs and outputs of DMUs (P_x).

Fig. 2 shows the PPS P_x resulting from the inputs and outputs quantities for these 6 DMUs assuming variable return to scale technology. Clearly, unit's E and F in this set are technically inefficient, and the other units are technically efficient. Consider unit E. One can see that unit E has a congestion in its input. By decreasing one unit of congestion from the input of unit E, the E^c point with value 6 is obtained (See *Fig. 2*). After removing the technical inefficiency of the congestion free point E^c, the can reach to the frontier point E^*.

In the second column of *Table 2*, the congestion free input value for 6 DMUs can be seen. In the third column, there is cost of these inputs. In the two last columns of the *Table 2*, the efficient technical inputs and their costs can be seen which are obtained by removing input technical inefficiencies. The information in this table is derived from some Part 2 calculations of the third step of the proposed algorithm. The values related to the excesses costs and efficiencies of each of the points without congestion and technical inefficiencies in P_x will be presented in *Table 4* to *Table 7*.

Table 2. The congestion free and technical efficient inputs and their costs in P_x.

DMU	x_j^c	$x_j^c c_j$	x_j^*	$x_j^* c_j$
A	2	6	2	6
B	3	4.5	3	4.5
C	4	10	4	10
D	6	18	4	12
E	6	18	3	9
F	5	25	2.5	12.5

It can be noted that in the calculation of the costs in *Table 2*, the observed prices are used. So it may there is some inefficiencies in the prices. On the other hand, considering the prices and outputs, one can form the P_c (*Fig. 3*).

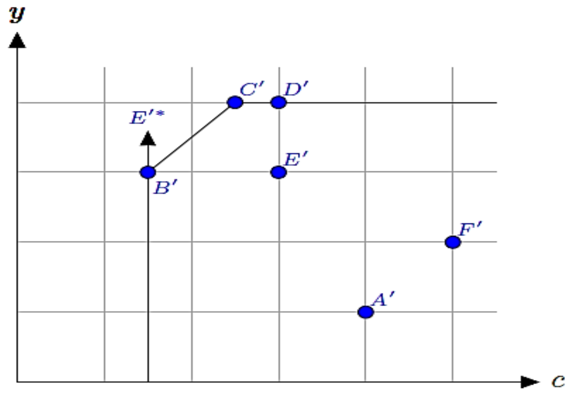

Fig. 3. PPS made by prices and outputs of DMUs (P_c).

Table 3, the first column, shows the prices after congestion removed (A'^c and F'^cin *Fig. 3*).

This time, we consider the process in Part 2 of the third step to obtain congestion and technical inefficiency free prices. It is clear that units A' and F' have a congestion of 2 and 1 units, respectively, while other units E' and D' have only technical inefficiencies in the first input price. Therefore, congestion free points corresponding to price based units A', F', E'and D' will have value 3 in their input. The values are given in column 2 of *Table 3*. In the third column of this table, the corresponding prices for the points for which price congestion inefficiency has been eliminated are obtained using the observed input vector corresponding to each unit. Columns 4 and 5 of *Table 3* show the optimal prices and corresponding costs. As can be seen from *Fig. 3*, there is no price inefficiency for units B'and C', as they are on the Pc efficiency frontier. Therefore, their input price vector is in the optimal position. However, the values of the second and fourth columns for other units have changed, indicating the existence of costly technical inefficiencies after the elimination of congestion inefficiencies (if any). In the last three columns of *Table 3*, the cost values for the various combinations of modified inputs and outputs, which correspond to the first part of Step 4 of the proposed algorithm, are given.

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Table 3. The congestion free prices, technical efficient prices and their corresponding costs in P_c.

DMU	c_j^c	$x_j c_j^c$	c_j^*	$x_j c_j^*$	$x_j^* c_j^c$	$x_j^c c_j^*$	$x_j^* c_j^*$
A'	3	6	1.5	3	6	3	3
B'	1.5	4.5	1.5	4.5	4.5	4.5	4.5
C'	2.5	10	2.5	10	10	10	10
D'	3	18	2.5	15	12	15	10
E'	3	21	1.5	10.5	9	9	4.5
F'	3	15	1.5	7.5	7.5	7.5	3.75

The values in the last column of *Table 3*, obtained using the inputs and optimal prices of Step 1 to Step 3, are the basis for constructing the P_{cx} cost PPS. The corresponding units for each of the A-F units in P_{cx} are indicated by the $\bar{}$ symbol in *Fig. 4*. As can be seen in the figure, and of course it was expected in advance, since most of the inefficiencies of the units in the two sets P_x and P_c have been eliminated, all the units in this space are on the efficient frontier and therefore do not have Mix inefficiencies. In this set, unit $\bar{A}$ is clearly a frontier unit with the lowest cost, and therefore it can be introduced as a cost-efficient point, and the allocative efficiencies of other units $\bar{B}-\bar{F}$ can be found according to that unit. The operations related to investigating the existence of mix inefficiencies, finding the CE point and calculating the allocative efficiencies are performed based on Step 5 of the proposed algorithm.

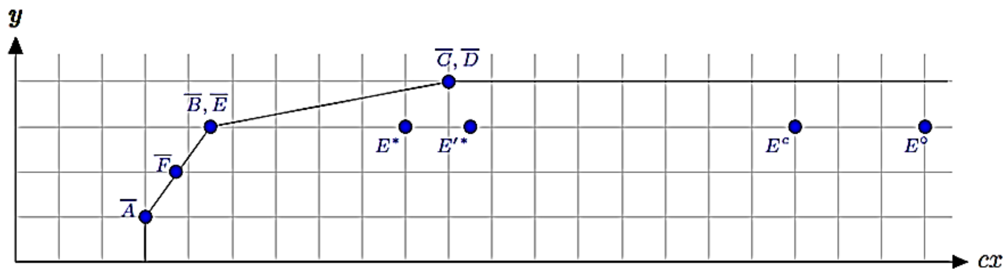

Fig. 4. PPS P_{cx}.

In this case, all cost inefficiencies of other units can be calculated based on a comparison with this cost efficient unit. Also, the amount of excess costs of all units can be analyzed in comparison with this point. *Table 4* shows the excess costs imposed due to various inefficiencies, assuming that input correction takes precedence over price correction. *Table 5* presents the excess costs imposed due to various inefficiencies, assuming that price correction takes precedence over input correction.

Table 4. Excess costs when input reduction is preferred.

DMU	$L_o^{\text{Input congestion}}$	$L_o^{\text{Tech.input}}$	$L_o^{\text{Price congestion}}$	$L_o^{\text{Tech price}}$	L_o^{Mixed}	$L_o^{\text{Allocatice}}$
A	2	0	0	3	0	0
B	0	0	0	0	0	1.5
C	0	0	0	0	0	7
D	0	6	0	2	0	7
E	3	9	0	4.5	0	1.5
F	0	12.5	5	3.75	0	0.75

Table 5. Excess costs when price reduction is preferred.

DMU	$L_o^{\text{Price congestion}}$	$L_o^{\text{Tech. price}}$	$L_o^{\text{Input congestion}}$	$L_o^{\text{Tech.input}}$	L_o^{Mixed}	$L_o^{\text{Allocatice}}$
A	2	3	0	0	0	0
B	0	0	0	0	0	1.5
C	0	0	0	0	0	7
D	0	3	0	5	0	7
E	0	10.5	1.5	4.5	0	1.5
F	10	7.5	0	3.75	0	0.75

Table 6, in turn, shows the decompositions of inefficiencies when the priority is the removal of the input inefficiencies, and *Table 7* shows those when one tries to remove the price inefficiencies as the first priority.

Table 6. Efficiency decomposition when input reduction is preferred.

Dmu	Input Congestion EFF.	Input Tech. EFF.	Price Congestion EFF.	Price Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.75	1	1	0.50	1	1	0.38
B	1	1	1	1	1	0.67	0.67
C	1	1	1	1	1	0.30	0.30
D	1	0.67	1	0.83	1	0.30	0.17
E	0.86	0.50	1	0.50	1	0.67	0.14
F	1	0.50	0.60	0.50	1	0.80	0.12

Table 7. Efficiency decomposition when price reduction is preferred.

Dmu	Price Congestion EFF.	Price Tech. EFF.	Input Congestion EFF.	Input Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.75	0.50	1	1	1	1	0.38
B	1	1	1	1	1	0.67	0.67
C	1	1	1	1	1	0.30	0.30
D	1	0.83	1	0.67	1	0.30	0.17
E	1	0.50	0.86	0.43	1	0.67	0.14
F	0.60	0.50	1	0.50	1	0.80	0.12

Fig. 3 shows the cost decomposition of unit E. If the priority is the removal of input efficiencies, the E^o point will be the cost of observed input with observed price, E^c will be the cost of congestion-free with observed price, E^* will be the cost of technical efficient input with the observed price, and $\bar{E}$ will be the cost of technical efficient input with the technical efficient price. In this condition, the decomposition of costs will be as follows:

$$C_E = 21 = 0 + 10.5 + 1.5 + 4.5 + 0 + 1.5 + 3.$$

Moreover, if the priority is to remove the price efficiency, the E^o point will be the cost of observed input with the observed price and also the cost of observed input with the congestion-free price (since there is no congestion in the E price), $E^{*\prime}$ will be the cost of observed input with the technical efficient price, E^* will be the cost of congestion-free input with the technical efficient price, and $\bar{E}$ will be the cost of technical efficient input with the technical efficient price. In this case, the decomposition of costs will be as follows:

$$C_E = 21 = 3 + 9 + 0 + 4.5 + 0 + 1.5 + 3.$$

In the above example, the points obtained the multiples of technical projections of inputs and prices in the P_{cx} space are efficient in this space, and as it was said, this is because all the input and price efficiencies have been already eliminated in the P_x and P_c spaces. In some cases, these points may not be efficient in the P_{cx} space and the remaining inefficiency might not be caused by inefficient inputs or prices, and this type of inefficiency referred to as mix efficiency in the present study. In the following example, this type of inefficiency can be seen.

Example 2. *Table 8* shows the set of six DMU which has two inputs and one output, 1 for all the units, along with the input prices. *Table 9* shows the cost decomposition when the priority is to decrease the input. However, *Table 10* shows the cost decomposition when the priority is to decrease the price. As it can be seen, in this example, after the technical and congestion inefficiencies are removed in the input space and price space, some inefficiencies will continue to exist, and *Tables 9* and *10* show the cost shortages caused by these Mix inefficiencies in the sixth column.

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Table 8. Data of DMUs (Example 2).

DMU	Input 1	Input 2	Output	Price of Input 1	Price of Input 2
A	3	1	1	2	3
B	2	2	1	4	2
C	4	3	1	0.5	2
D	5	1	1	1	3
E	3	4	1	1.5	1
F	6	2	1	2	4

Table 9. Cost decomposition of DMUs in Example 2, when input reduction is preferred.

DMU	$L_o^{input\ congestion}$	$L_o^{tech.input}$	$L_o^{price\ congestion}$	$L_o^{tech\ price}$	L_o^{Mix}	$L_o^{allocatice}$
A	0	0	0	4.5	0	0.73
B	0	0	0	7	0.83	0.4
C	0	3.43	0	0	0	0.8
D	0	2	0	2.23	0	0
E	0	3.5	0	0	0.83	0.4
F	0	10	0	5.84	0	0.93

Table 10. Cost decomposition of DMUs in Example 2, when price reduction is preferred.

DMU	$L_o^{price\ congestion}$	$L_o^{tech.price}$	$L_o^{input\ congestion}$	$L_o^{tech.input}$	L_o^{Mixed}	$L_o^{allocative}$
A	0	4.5	0	0	0	0.73
B	0	7	0	0	0.83	0.4
C	0	0	0	3.43	0	0.8
D	0	2.97	0	1.26	0	0
E	0	0	0	0.5	0.83	0.4
F	0	11.68	0	4.16	0	0.39

Example 3. In order to show the practicability of this decomposition, we use the example presented by Tone and Tsutsui [22]. *Table 11* shows the data of 12 hospitals, consisting of two inputs (physicians and nurses) and the price of each input unit (wages of each physician and nurse) and two outputs (number of hospitalized patient and outpatient). *Table 12* and 14 shows the decomposition of cost and efficiency for the case in which the priority is to remove the input inefficiencies or decrease the input, i.e. to decrease the number of physicians and nurses, than to decrease their wages. *Tables 13* and *15* shows the decomposition of cost and efficiency for the case in which the priority is to remove the price inefficiencies, i.e., to decrease the wages of physicians and nurses, than to decrease the workforce consisting of physicians and nurses. Unit E, with the best performance among all the units, has only the technical inefficiency, i.e., Hospital E just has to decrease the workforce in order to achieve efficiency.

Table 11. Data of 12 hospitals.

	Inputs					Outputs	
DMU	Number of Inpatient	Number of Outpatient	PER Nurse's FEE	Number of Nurses	Per Doctor's FEE	Number of Doctors	Number of Inpatient
A	90	100	100	151	500	20	90
B	50	150	80	131	350	19	50
C	55	160	90	160	450	25	55
D	72	180	120	168	600	27	72
E	66	94	70	158	300	22	66
F	90	230	80	255	450	55	90
G	88	220	100	235	500	33	88
H	80	152	85	206	450	31	80
I	100	190	76	244	380	30	100
J	100	250	75	268	410	50	100
K	147	260	80	306	440	53	147
L	120	250	70	284	400	38	120

Table 12. Cost decomposition of DMUs in real example, when input reduction is preferred.

DMU	$L_o^{input\ congestion}$	$L_o^{tech.input}$	$L_o^{price\ congestion}$	$L_o^{tech\ price}$	L_o^{Mixed}	$L_o^{allocatice}$
A	0	0	4220	2610	0	2720
B	0	0	0	1577.9	0	2.1
C	0	2998.8	1650	2625.15	0	2826.05
D	0	0	11040	3165	0	6605
E	0	2110	0	0	0	0
F	900	5552.1	434.3	4688.27	0	18025.33
G	0	0	6680	4165	0	13605
H	0	6330.4	1070.4	3007.4	0	5501.8
I	0	1906.84	0	1765.53	0	10721.63
J	0	0	0	1840	0	23210
K	0	0	0	0	0	32250
L	0	0	0	0	0	19530

Table 13. Cost decomposition of DMUs in real example, when price reduction is preferred.

DMU	$L_o^{price\ congestion}$	$L_o^{tech.price}$	$L_o^{input\ congestion}$	$L_o^{tech.input}$	L_o^{Mixed}	$L_o^{allocative}$	$L_o^{price\ congestion}$
A	23012	2610	0	0	0	2720	23012
B	0	1577.9	0	0	0	2.1	0
C	1850	2975	0	2448.95	0	2826.05	1850
D	11040	3165	0	0	0	6605	11040
E	0	0	0	2110	0	0	0
F	550	5455.1	774.36	4795.21	0	18025.33	550
G	6680	4165	0	0	0	13605	6680
H	1340	3765	0	5303.2	0	5501.8	1340
I	0	1904.28	0	1768.09	0	10721.63	0
J	0	1840	0	0	0	23210	0
K	0	0	0	0	0	32250	0
L	0	0	0	0	0	19530	0

Table 14. Efficiency decomposition of DMUs in real example when input reduction is preferred.

DMU	Input Congestion EFF.	Input Tech. EFF.	Price Congestion EFF.	Price Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	1	1	0.83	0.88	1	0.85	0.62
B	1	1	1	0.91	1	0.99	0.91
C	1	0.88	0.93	0.88	1	0.85	0.61
D	1	1	0.70	0.88	1	0.70	0.43
E	1	0.88	1	1	1	1	0.88
F	0.98	0.87	0.99	0.88	1	0.46	0.34
G	1	1	0.83	0.88	1	0.53	0.39
H	1	0.80	0.96	0.88	1	0.74	0.49
I	1	0.94	1	0.94	1	0.59	0.52
J	1	1	1	0.95	1	0.40	0.38
K	1	1	1	1	1	0.33	0.33
L	1	1	1	1	1	0.44	0.44

Table 15. Efficiency decomposition of DMUs in real example when price reduction is preferred.

DMU	Price Congestion EFF.	Price Tech. EFF.	Input Congestion EFF.	Input Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.83	0.88	1	1	1	0.85	0.62
B	1	0.91	1	1	1	0.99	0.91
C	0.93	0.88	1	0.88	1	0.85	0.61
D	0.70	0.88	1	1	1	0.70	0.43
E	1	1	1	0.88	1	1	0.88
F	0.99	0.88	0.98	0.88	1	0.46	0.34
G	0.83	0.88	1	1	1	0.53	0.39
H	0.96	0.88	1	0.80	1	0.74	0.49
I	1	0.94	1	0.94	1	0.59	0.52
J	1	0.95	1	1	1	0.40	0.38
K	1	1	1	1	1	0.33	0.33
L	1	1	1	1	1	0.44	0.44

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5 | Conclusion

One of the main strengths of DEA compared to other methods of evaluating the performance of DMUs, is that in addition to providing a performance score, it also provides factors affecting inefficiencies, so unit decision makers understand the reasons for the weakness and can plan to fix them. If, in addition to the input and output data of DMUs, price information is also available, various factors influencing cost inefficiency can be determined. If the prices for the units are not the same, it is possible that the units have been inefficient in providing resources at a reasonable price. Therefore, in addition to inefficiency factors related to the quantities of inputs and outputs, inefficiency factors in prices must also be determined. In this paper, for the first time, three PPS was used to provide a decomposition for CE. Accordingly, congestion and technical inefficiencies in the PPS based on input and output quantities $\left(P_x\right)$, and based on input and output prices $\left(P_c\right)$ were considered. In addition, two other types of inefficiencies, mix and allocative inefficiencies, were introduced in the third PPS $\left(P_{cx}\right)$, which was the result of input and optimal price vectors resulting from operations in the first two sets. Since each type of cost inefficiency leads to an excess cost, we express the cost of observation as the sum of the optimal cost and excess costs. Based on whether in the process of fixing inefficiencies, fixing inefficiencies related to inputs is a priority or fixing price inefficiencies, we presented two types of analysis for CE as well as observed cost.

In the present study, the proposed efficiency decomposition is apt for a case in which the input price data are available. If the output prices, in addition to the input price data, are available, one can analyze the profit and income efficiencies by developing the proposed method. Due to the existence of inaccurate and ambiguous data in the amount of inputs and costs, and the importance of fuzzy DEA, the researchers are refered to [27]–[32] for future studies. Moreover, the proposed method can be used when the structure of the DMUs is in the form of a network. In the evaluation, Moreover, the units were considered to be quite independent, and there was not any discussion concerning the hierarchical and group structures, not for a case in which the units are under a centralized or original decision-maker with the possibility of resource displacements, which can be considered in future studies.

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Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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