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# Designing a Resilient Location-Allocation Model for Cell Site Networks with Regional Coverage Enhancement Approach Using Robust Programming-Lagrangian Relaxation

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## Abstract

The development of cell sites as part of the infrastructure of telecommunication technology is playing a unique role in emerging businesses at present. Natural disasters and crises can disrupt communication equipment and create severe challenges in service provisions, especially health and security, by damaging sites. This might lead to traffic congestion in certain network sections, causing chaos and social crises and increasing the commissioning and equipping costs of backup sites for operators. This study developed an integrated location-coverage-allocation model to improve sustainability through maximum coverage, enhanced flexibility, and minimized overhead expense by determining the position of backup sites and mitigating environmental pollution resulting from the establishment of sites. The stochastic robust optimization model was employed to control the effect of nonparametric uncertainty, while acceptable solutions were generated using the Lagrangian relaxation to address complicated model constraints.

**Keywords:** Location, Coverage, Robust, Lagrangian relaxation, Resilient.

## 1 | Introduction

Telecommunication systems, especially mobile phones and the Internet, have spread worldwide in recent years. Recent smartphone functionalities and improved wireless network performance have boosted the demand for mobile multimedia content, leading to the emergence of cellular systems [1]. A cellular system refers to a group of mobile phone subscribers at any location, such as a street, a moving vehicle, a road, or a mountain. A mobile phone network is a cellular network consisting of cells. Generally, a cellular network is connected to a telephone center, which is connected to the public phone network [2]. A telecommunication channel is an environment exploited to transfer information between a transmitter and a receiver. Basic Station Controllers (BSCs) are also embedded to cover users through a central control structure. An area covered by a BSC is called a cell. The

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arrangement of BSCs aims at achieving good telecommunication coverage depending on the surrounding environment (Fig. 1).

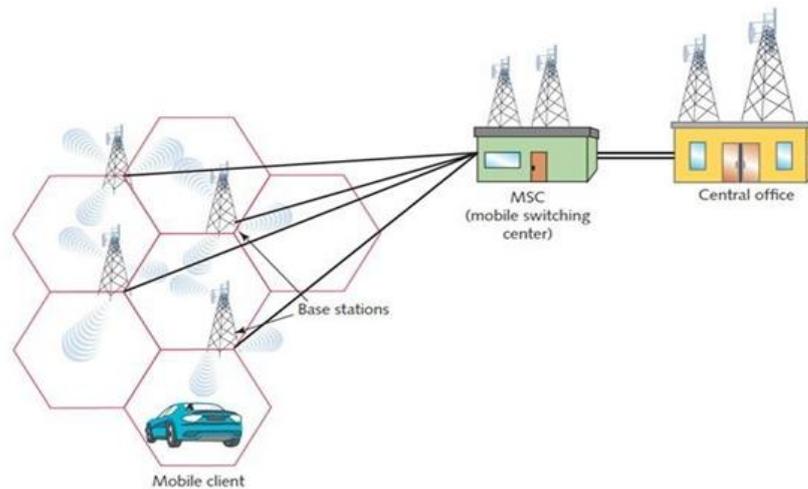


Fig. 1. Design and arrangement of the coverage radii of telecommunication cells.

Nevertheless, crises are inevitable; a crisis refers to a condition, natural or manufactured, that disrupts the routine life patterns and imposes damages and unwanted changes to the human society or the environment. The disruption of life routines, damage to communication systems and emergency services, an outbreak of diseases, destruction of infrastructure, and fatality are devastating outcomes of crises, which require extraordinary emergency management, planning, and measures [3]. During a crisis, many facilities would be unable to provide services due to failure, leading to loss of customer demands and many other unpleasant outcomes. Moreover, inattention to systems reliability can cause tremendous damage in any country. This indicates the necessity of proposing mathematical models that take into consideration the failure of facilities during crises and resulting consequences such as the loss of clients [4]. Based on a study conducted by Ahmadi et al. [5], a method has been proposed that by analyzing GIS information, the optimal areas for the deployment of relief forces in times of crisis have been identified, which can be used to locate telecommunication sites with minimal damage in natural disasters [5]. Telecommunication systems usually communicate via base stations or antennas. These stations should be positioned so that to provide complete services. Evidently, the quantity and position of these facilities (base stations) can significantly affect the quality of their services. In such cases, it isn't easy to increase the capacities of facilities for various reasons, such as fixed capital. Moreover, facilities are always prone to different kinds of natural hazards (earthquakes, hurricanes, and floods), intentional hazards (terrorist attacks, sabotage operations, and labor strikes), and accidental hazards (industrial accidents, fires, and failure of system components); however, effective and resilient systems can be developed to withstand natural disasters by optimal location and improvement of antennas [6].

Communication networks play a crucial role during crises, including providing information, connecting with the victims and relief centers, and the like [7]. The following obstacles may disrupt the services provided by a communication network during crises:

- I. Sudden congestion and heavy traffic in one zone of the network.
- II. Probable damage to the infrastructure on which communication networks rely to provide services.
- III. Physical damage and failure of communication network components.

The positions of base and main stations affect the performance of mobile communication networks; thus, having backup stations to be used at times of failure and natural disasters is of utmost importance [8]. The establishment of backup stations for main sites is a sustainable approach that would provide clients with telecommunication coverage in case of disruptions. The coverage problem is involved determining the position and quantity of facilities at the minimum cost of providing services in demand points [9]. First, the maximum coverage problem was introduced as the objective function maximization, where the number of covered demand points was maximized against a fixed number of facilities [10]. Due to the importance

of providing services for demand points in some cases, researchers have considered the multi-coverage problem, where one demand point might be covered by more than one piece of equipment to enhance service accessibility [11].

With the ever-increasing developments in mobile phone technology, the information and telecommunication transmission industry has gained growing importance. The number of subscribers to this type of communication is increasing daily. The positions of base stations (towers) play a significant role in the performance of a telecommunication network. Therefore, determining their optimal positions is necessary for maximum coverage [12]. At the same time, deciding the places of telecommunication masts and towers is among the long-term strategic decisions, which are less flexible due to heavy financial consequences [11]. These decisions are made to prevent the failure of telecommunication coverage when a crisis emerges [13]. With the widespread establishment of wireless 5G networks, there has been a growth in different Internet-based systems and services [14]. Furthermore, network services based on the geographical positions of individuals and objects are now expanding [15]. It is essential to guarantee access to the data traffic required by various broadband devices such as computers, smartphones, security systems, and healthcare systems. The COUNTERACT technology, for example, is used in cellular network applications to detect Covid 19 contaminated areas by acquiring location information from a mobile device connected with an infected individual [16], [17]. This calls for the development of a resilient model for locating cell sites to increase the maximum profit while ensuring complete coverage so that no demand is lost during crises and disruptions. Fig. 2 illustrates network recovery and flexibility approaches in three phases, namely before, during, and after a crisis. These approaches are divided into two main sections: network resilience design and network recovery design (ITU-T, 2010–2014).

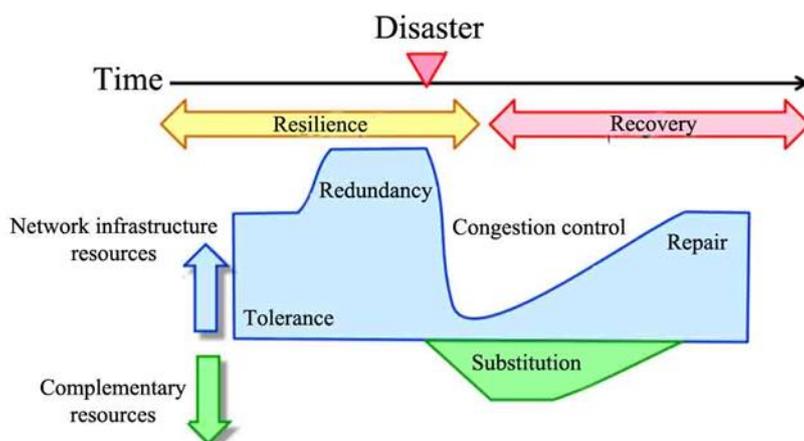


Fig. 2. Network recovery and flexibility approaches.

The costly and challenging task of finding the best positions for mobile phone operators in order to cover all subscribers shall be considered. However, the essential problem is to find appropriate strategies for estimating the positions of subscribers, an area which has interested many researchers due to the necessity of compliance with the Bylaw of the US Federal Committee Organization (ITU-T, 2013) that requires mobile operators to determine the positions of subscribers with an accuracy of 50–100 m in 67% of cases and an accuracy of 150–300 m in 95% of cases. Operators are also to provide the whole city with mobile network coverage, including the interior spaces of buildings, parks, and conventional markets, as part of the condition of their licenses. Therefore, appropriate positioning and maximum coverage are considered essential tasks. The statistics reported by the Communications Regulatory Authority (CRA) of the IR of Iran in the statistical quarterly of the third quarter of 1399 (2020–2021) indicated a penetration rate of 151.8% for mobile phones with more than 127 million SIM cards sold and more than one thousand active sites. Therefore, it is essential to optimize the costs of mobile site designing and upgrading and stabilize communication with the lowest rate of failure. Although the resilient supply chain network might not be the least expensive one, it can compensate for uncertainties and disruptions in the business environment [18].

This paper proposes a mathematical mixed-integer model to maximize coverage in case of failure and locate primary and secondary facilities and towers to maximize coverage, profit, and network resilience by considering design redundancy and backup and substitute sites. The realistic robust optimization approach was employed to solve the model, whereas Lagrangian relaxation was utilized to deal with complex and complicated constraints and reduce the model solution time.

This paper consists mainly of the following sections: the research literature review and the novelty and innovations of the research are presented in Section 2. Section 3 introduces the mathematical model of the problem, which consists of the mathematical model, robust optimization, and Lagrangian relaxation for the rigid constraints of the problem. The solution procedure, numerical results, and sensitivity analysis are presented in Section 4. Finally, Section 5 provides the conclusion and recommendations for future research.

## 2 | Literature Review

A brief literature review revealed some of the most critical studies on facility location and telecommunication site coverage. Andrew [19] proposed maximizing communication and overlap to improve telecommunications. Balakrishnan et al. [20] examined the development of telecommunication networks, assuming that there would be no planning. The problem would focus on installing centralized antennas and expanding communication ranges at the lowest costs in response to the project demand. Therefore, they developed a decomposition method based on the Lagrangian coefficient and dynamic programming. Kremling [21] addressed network development and optimization by giving a challenging insight into the mobile phone industry. He discussed the very complicated non-optimal expansion of network topology that could cause serious problems. Arguing that the future mobile phone networks would need higher levels of reliability. Kremling stated that costs should be controlled at the same time. Using mathematical models, also proposed linear programming for network topology optimization. Buys et al. [22] proposed a novel location layout for mobile phone towers and their coverage in terms of population. Vinu et al. [23] discussed the maximum use of ICT tools (mobile phones and the Internet).

Wilson [24] proposed a location model using telecommunication tools (mobile phones) and considering the maximum communication between origin and destination points. Dorn and Reuven [25] analyzed the minimum distance between origin and destination points (transmitter and receiver) for the coverage of mobile phone towers. They determined the locations of service centers, assuming that demand points can receive indefinite services. Lemamou et al. [26] proposed a model for mobile programming networks to minimize installation costs by considering network maintenance operations while maximizing its durability. They also mentioned a few constraints on the received signal's allocation, capacity, and quality.

It is crucial to determine the locations of designated stations to provide appropriate services for all mobile phone subscribers in terms of coverage. Subscriber demand is uncertain given the different coverage radii and geographical conditions of every region. Khalafi and Tavakoli Moghaddam [27] employed a novel approach to the location of mobile phone telecommunication stations with two-level services and stochastic demand as well as the set coverage model in which the coverage strategy was adopted for all demanded regions.

Peppanen et al. [28] proposed a technique with novel dimensions for the wireless WiMAX technology, providing a new level and more accurate outcomes compared to conventional methods. They also introduced the optimization framework for programming the WiMAX network in three steps: 1) network dimension definition, 2) initial segmentation, and 3) final network configuration. The proposed framework is employed to simultaneously solve two major problems, i.e., cellular programming and frequency programming, using a simulation algorithm.

Lee et al. [29] analyzed a two-level location-allocation problem in a tree topology for the design of access networks in order to find the optimal location of a switch and allocate demand to minimize the total cost of switches and fiber cables by meeting the constraints on switch ports, switch capacity, and to route with no division. This problem was formulated as a mixed-integer programming model in which the substitute formulas developed a MIP-based method by using the re-linearization technique and the tree structure based on tree partitioning to find high-quality solutions to large-scale problems.

Furthermore, Lee [30] addressed the location problem in a 3D space and introduced a 3D coverage location model for Wi-Fi Access Points (APs) in an interior environment. For this purpose, a public radio transmission was first installed in the environment by assuming the ordinary obstacles of a building in the standard coverage model through the 3D Euclidean distance. The proposed model was then introduced for an efficient AP installation program in a multistory building. A practical modeling framework was proposed to integrate the positioning model with the wireless network design in a 3D environment.

Tohidinasab et al. [31] proposed a coverage-positioning model for antennas and towers using the robust optimization approach. They modeled the telecommunication networks on certain cells, each of which would cover a specific region. Different coverage radii of stations, their locations, and geographical conditions made the location of stations an NP-hard optimization problem.

Asghar et al. [32] proposed a 5G coverage model by offering a solution for presenting and preparing the new generation of the Internet. Akpakwu et al. [33] conducted a comprehensive review of newly emerged robust technologies by focusing on the 5G mobile networks predicted to support the exponential traffic growth upon IoT activation. They also proposed the open-ended research challenges and paths regarding the establishment of extensive and vital IoT applications as well as an efficient mechanism for network traffic congestion control.

Akhtar [34] analyzed and compared 2G to 5G mobile phone networks. In addition to exploring different services that can be or are provided on every network, he suggested the exciting and unbelievable growth of users' interest in using online videos to be among the main reasons for the need for developing next generations. Santhi et al. [35] analyzed the performance of 4G networks and their improvement in comparison with the third generation and the fundamental concepts of networks, spectra, technologies, standards, terminals, and services. They concluded that 4G networks would be unable to provide the necessary bandwidth in the future. Goodarzian et al. [36] proposed an incomplete model which considered just coverage maximizing under crises and natural disasters with three target functions. In the mentioned paper, a novel Mixed-Integer Non-Linear Programming (MINLP) model is proposed to best cover and assign base mobile communications rigs in various locations under critical situations.

In a study entitled A Model for Improving and Enhancing the Cell Phone Tower Coverage for Service during Natural Disasters, Akbarpour et al. [12] proposed a model to maximize mobile phone coverage in different areas, minimize the damage caused by the average failure of towers, and maximize coverage in the worst-case scenario of failure in mobile phone towers at the time of natural disasters. Their computational results indicated the importance and efficiency of the proposed model in real decision-making because the construction and enhancement of main stations and antennas improved the radio transmission coverage, especially in low-signal areas and high-traffic stations. The proposed model aimed at supporting the system in the worst- and moderate-case scenarios of loss to maximize the number of clients that the network would cover after the failure of facilities.

In addition to analyzing the principles and concepts, Janevski et al. [37] regarded 5G networks as user-oriented rather than operator-oriented. They believed that the need for operator response was the main reason for developing new technologies in previous networks. In contrast, in 5G networks, this is motivated by the demands of the users.

Pokorny et al. [1] optimized the costs of establishing and maintaining services in wireless networks, aiming to optimize the number of service centers to cover locations selected by customers based on requirements. This need for optimization is observed primarily in 5G networks and cellular systems characterized by many interconnected devices, which are usually difficult to control by wireless systems. Currently, the network infrastructure planning tools used in the industry include the Atoll Radio Planning Tool and Radio Planner, which do not provide the automatic selection of establishment position for specific nodes of gNodeB in a particular region with the predefined requirements. They intended to develop novel mathematics and propose models with the emerging scenarios adaptable to wireless network installation and maintenance.

Soleimani et al. [38] considered backup hubs with diverse objective functions to respond to interruption and uncertainties. In their research, backup hubs are chosen for each major hub to deal with interruptions and natural calamities and avoid delays. Then, to cope with uncertainty, a resilient possibilistic strategy is provided. Two metaheuristic algorithms are used to solve the problem: a non-dominated sorting genetic algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO).

Khalili Damghani et al. [39] suggested a bi-level two-echelon mathematical model reduce pre-disaster expenditures while increasing post-disaster aid coverage. The model uses a geographic information system (GIS) to categorize the disaster region and estimate the ideal number and location of distribution centers while reducing the inventory costs of relief goods. The objective of this research is multifold: 1) to recognize vulnerable urban infrastructures in sequential disaster events, 2) to prioritize urban areas using a GIS due to the severity of cascade disasters, and 3) to develop a bi-objective multi-echelon multi-supplies mathematical model for the location, allocation, and distribution of humanitarian supplies under uncertainty.

In a recent study entitled Location-Allocation Model in Telecommunication Technology and Presentation of a Novel Solution, Dinu and Ciucur [40] proposed a location-allocation model in a telecommunication network called the "Capacitated Concentrator Location-Aligning Problem" (CCLA). It is based on a general network location-allocation model focused on analyzing clients, demand, and facilities. Like in a location-allocation model, every client node has demand traffic that should be serviced, while facilities can respond to the requests within their capacity ranges. The CCLA problem was proposed as a single-source location-allocation model in this study. The optimization goal was to determine the minimum network costs, including the fixed costs of developing centralized places, execution costs, and the costs of allocating terminals. This problem is known as a hybrid NP-hard optimization problem requiring robust solutions. The research approach proposes a fuzzy genetic algorithm with a local search method for calculating the optimal values of location and allocation variables.

*Table 1* compares a few of the recent studies regarding the coverage problem in the telecommunication industry and their proposed models. They considered four main categories:

- *Demand coverage strategy.*
- *Objective function.*
- *Mathematical model type.*
- *Network structure.*

## 2.1 | Novelty and Innovation

Studies on the resilience of communication networks are scarce, despite the use of 5G generation being necessary for places or at least in the proximity of places where the previous generations of networks were established, given disruptions during the network traffic transmission and occurrence of natural disasters, which leads to loss of active subscribers. The literature review revealed a substantial research gap regarding the comprehensive decision-making models that take into account the robustness and sustainability of

communication networks at all times through broad criteria for reducing costs and environmental pollutants, increasing profit, and maximizing coverage.

**Table 1. Review of the literature on coverage problems, comparison of models, and the research gap.**

References	Year of Publication	Demand Coverage Strategy					Objective Function		Model Type		Network Structure		Problem Approach		
		Complete	Maximal	Full Coverage	Incomplete Coverage	Average Coverage	Cost/Profit	Environmental	Deterministic	Stochastic	Resilient	Non-Resilient	Location	Allocation	Coverage
[19]	2004	*		*		*	*	*				*			*
[20]	2006	*			*		*		*			*			*
[10]	2006	*			*		*		*			*			*
[21]	2007	*		*			*		*			*			*
[22]	2009	*					*		*			*	*		*
[34]	2009	*			*	*	*		*			*	*		*
[37]	2009	*				*	*		*			*	*		*
[23]	2011	*			*		*		*			*			*
[24]	2012	*		*			*		*			*	*		*
[25]	2013	*				*	*		*	*		*	*		*
[28]	2015	*		*	*		*		*			*	*		*
[58]	2015	*					*		*			*	*		*
[32]	2017	*		*		*	*		*			*	*		*
[55]	2017	*			*		*		*			*	*		*
[57]	2018		*									*	*		*
[56]	2020											*	*		*
[40]	2021	*		*	*	*	*		*			*	*		*
[36]	2021		*						*			*	*	*	*
This study	2021	*	*	*	*	*	*	*	*	*	*	*	*	*	*

### 2.1.1 | Research limitations

- I. How the geographical distribution of facilities in a system plays a vital role in the vulnerability of this system to disruptions, while in the establishment of telecommunication sites, we have to have a different geographical distribution.
- II. Although choosing a place to install equipment faces various challenges of ownership and beautification of urban space, planning can provide the most suitable place to build and strengthen the coverage of sites and increase the number of subscribers covered by sites. Costs should also be reduced.
- III. Considering environmental and health considerations of citizens in designing and locating telecommunication sites to optimize energy consumption in sites requires spending more money to purchase up-to-date equipment, which of course, this comprehensive mathematical model pays attention to both aspects.
- IV. Deploying facilities at remote distances reduces the likelihood of identifying and destroying facilities. Although the BTS coverage radius in rural areas is usually larger than in urban areas, this model is considered a fixed coverage radius for the entire study area.
- V. In an unstable economy with repeated exchange rate changes, design and deployment costs are different and inconsistent but do not affect the proposed model.

### 2.1.2 | Innovations

The innovations of this study and its significant distinction from other studies are as follows:

- I. Proposing a multi-objective location-allocation model for the maximum coverage of mobile phone sites before and after the emergence of a crisis.
- II. Offering a green model for the system.
- III. Providing multiple support facilities in critical conditions.
- IV. Considering general disturbances and possible scenarios for uncertainty parameters.
- V. Presenting a model through the robust approach to solve the uncertainty problem.
- VI. Proposing a Lagrangian relaxation solution to eliminate hard and complicated constraints.

### 3 | Mathematical Model

#### 3.1 | The Mathematical Model

The mathematical model presented in this section seeks to locate the sites in the first place. The best optimal locations must be selected from various options, taking into account the probability of failures and problems in providing services. The radiation pollution of towers must be minimized and kept within a safe range set by the national standard of non-ionizing rays-radiation limits. These radio and telephone radiations are among the non-ionizing low-energy rays. This energy is measured in terms of density, i.e., the amount of energy radiated per square centimeter of human skin, irrespective of the distance from the telecommunication tower. The safe energy reception limit of non-ionizing rays is 0.45 mW/cm. The safe electric field intensity is also considered up to 28 V [41]. Based on these location requirements, the second step in the proposed model is to allocate a site to a group of clients to maximize the regional coverage.

#### Assumptions

- I. All facilities (sites) have limited capacities.
- II. The demand is an uncertain scenario.
- III. The costs of establishing sites are limited.
- IV. The capacities and costs of all sites are considered separately and differently.
- V. Every site has a backup set for support and substitution in case of failure.
- VI. The probability of failure is different for various sites.
- VII. The costs of different generations of mobile phone technology are additional.
- VIII. The capacities of different generations of mobile phone technology are various.
- IX. Maximum coverage is considered.
- X. The failure of facilities is probable.

#### Sets

i	demand points set; $i = 1, 2, \dots, m$ .
j	potential locations for the installation of sites (towers); $j = 1, 2, \dots, n$ .
k	different generations of networks; $k = 1, 2, \dots, en$ .
S	set of scenarios for demand; $S = 1, 2, \dots, se$ .
$a_{ij}$	$\begin{cases} 1, & \text{Site (tower) } j \text{ covers client } i, \\ 0, & \text{Otherwise.} \end{cases}$

#### Parameters

$W_{is}$	the demand of client $i$ under scenario $S$ .
$e_{jk}$	cost of setting up the $j$ th tower for the $k$ th generation of Internet.
$q_j$	pollution level of setting up tower $j$ and support.
price	maximum budget considered for setting up a tower.
cap	maximum capacity of a tower for coverage.
$d_{ij}$	distance between client $i$ and tower $j$ .

$P_{ik}$	probability of tower failure in location $i$ for the Internet generation $k$ .
$RT_j$	the maximum radius of tower $j$ .
$BigM$	a big number.
$R_j$	A percentage of coverage radius of tower $j$ in which all signals are received.

**Decision variables**

$x_{ij}$	$\begin{cases} 1, & \text{if client } i \text{ is covered by tower } j, \\ 0, & \text{Otherwise.} \end{cases}$
$Y_j$	$\begin{cases} 1, & \text{potential site (tower } j) \text{ is activated,} \\ 0, & \text{Otherwise.} \end{cases}$
$z_{ij}$	$\begin{cases} 1, & \text{if client } i \text{ is covered by backup tower } j, \\ 0, & \text{Otherwise.} \end{cases}$
$Y'_j$	$\begin{cases} 1, & \text{if client } i \text{ is covered by backup tower } j, \\ 0, & \text{Otherwise.} \end{cases}$
$k_{ij}$	$\begin{cases} 1, & \text{if distance between client } i \text{ and} \\ & \text{tower } j \text{ is shorter than the coverage radius,} \\ 0, & \text{Otherwise.} \end{cases}$
$v_i$	$\geq 0$ the lost (unallocated) sites if the facility fails at location $l$ .
$O$	the number of active sites (towers) and accessible backups.
$r_j$	the variable radius of site $j$ .

**Modeling**

$$MAX Z1 = \sum_i^m \sum_j^n \sum_s^{se} x_{ij} w_{is} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij}. \tag{1}$$

$$MAX Z2 = \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\}. \tag{2}$$

$$MAX Z3 = \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n z_{ij} w_{is} - \sum_i^m \sum_k^{en} v_i p_{ik}. \tag{3}$$

$$MAX Z4 = \sum_j^n e_{jk} \sum_k^{en} Y_j + \sum_j^n \sum_k^{en} e_{jk} Y'_j. \tag{4}$$

$$MIN Z5 = \sum_j^n q_j Y_j + \sum_j^n q_j Y'_j. \tag{5}$$

$$\sum_j^n Y_j + \sum_j^n Y'_j = O. \tag{6}$$

$$x_{ij} \leq (1 - Z_{ij}). \tag{7}$$

$$\forall i j \quad x_{ij} \leq Y_j. \tag{8}$$

$$\sum_j^n x_{ij} \leq 1 \quad \forall i. \tag{9}$$

$$a_{ij} Y_j \geq x_{ij} \quad \forall i j. \tag{10}$$

$$\sum_j^n \sum_k^{en} e_{jk} Y_j + \sum_j^n \sum_k^{en} e_{jk} Y'_j \leq \text{price}. \tag{11}$$

$$\sum_i^m \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \leq \text{cap}(Y_j + Y'_j) \quad \forall j. \tag{12}$$

$$x_{ij} \leq Y_j \quad \forall i j. \tag{13}$$

$$z_{ij} \leq Y_j \quad \forall i j. \tag{14}$$

**Objective functions**

$$V \geq v_i \quad \forall I. \tag{15}$$

$$d_{ij} \leq R_j RT + (1 - K_{ij}) \text{Big } M. \tag{16}$$

$$r_j \leq RT_j. \tag{17}$$

$$r_j \geq 0. \tag{18}$$

$$x_{ij}, y_j, z_{ij}, y'_j, k_{ij} \in \{0, 1\}. \tag{19}$$

The first objective function, *Eq. (1)*, is for coverage maximization of the design of the primary and backup sites. *Eq. (2)* addresses the coverage maximization of towers in the worst-case coverage scenarios of sites for clients at the time of failure. The difference between the total allocated towers and the maximum number is the set of unallocated towers. To linearize the objective function,  $v = \max\{v_i\}$  Considered. In this case, *Constraint (20)* is added:

$$V \geq v_i \quad \forall I. \tag{20}$$

*Eq. (3)* considers the average coverage of sites. In other words, a backup tower is destructed randomly. If the failure probability of the backup site at location  $l$  is  $\frac{1}{O} P_i$ , then the anticipated failure is obtained from  $v_i = \frac{1}{O} \sum_i^m v_i \sum_i^m p_i$ . This objective function expresses the coverage maximization of sites in the average failure at the time of natural disasters. It also maximizes the difference between the allocated sites and the sites covered even once. The fourth objective function tries to minimize the costs of setting up the main and backup sites. For this purpose, a constant cost is considered for both sites, which the objective function tries to minimize. *Eq. (5)* aims to reduce a constant pollution value for primary and backup sites, whereas *Eq. (6)* indicates a limited number for setting up the centers. According to *Eq. (7)*, if the main tower fails to respond to the client's need, the backup tower should do the task. *Eq. (8)* expresses allocation; a site should first be set up to be allocated to a client. This constraint is considered only for the main tower. *Eq. (9)* indicates that every client receives services from at least one of the activated sites. *Eq. (10)* denotes the necessity of providing client demand coverage by sites within the coverage radius. It also guarantees that every client's request to activate potential centers and sites should be allocated. *Eq. (11)* indicates the maximum budget for setting up sites, whereas *Eq. (12)* denotes the coverage capacity. According to *Eq. (13)*, only if a site is set up its allocation to one of the clients is justifiable. *Eq. (14)* indicates allocation for the backup site, whereas *Eq. (15)* results from the linearization of the objective function that adds a constraint to the problem. *Eq. (16)* indicates that client  $l$  receives all signals. According to *Eq. (17)*, the variable radius should be shorter than the maximum radius, whereas *Eq. (18)* indicates that the variable radius should be more significant than zero. According to *Eq. (19)*, the analyzable values can only be zero and one.

**3.2| Robustification**

Generally, optimization models consist of two separate sections: the structural section that is constant and lacks any input data volatility and the control section that is a function changed by unreliable and volatile data. The LP optimization model is defined as below

$$\text{Min } z = c^T x + d^T y. \tag{21}$$

$$\text{Subject to } Ax = b. \tag{22}$$

$$Bx + Cy = e. \tag{23}$$

$$x, y \geq 0. \tag{24}$$

X is the decision variable of certain parameters, and Y indicates the control variables. The LP model includes structural constraints, whose coefficients are constant (i.e., reliable coefficients), and control constraints, whose coefficients include unreliable cases. Moreover, a finite set of scenarios  $\varphi = \{1, 2, \dots, S\}$  is assumed for uncertain parameters. Based on each scenario  $se\varphi$ ,  $\{d_s, B_s, C_s, e_s\}$  is defined as the realization of its performance. In addition,  $P_s$  refers to the occurrence probability of each scenario, and  $\sum P_s = 1$ .

According to the multiple scenarios, the objective function  $C = c^T x + d^T$  is a random variable that takes  $C_s = c^T x + d_s^T y_s$  with the probability of  $P_s$ . The multiple-criteria decision-making (MCDM) concept performs the exchange between solution and model robustness. The above robust optimization model can measure this exchange. Furthermore,  $\sigma_0$  is considered a nonlinear expression. The model is based on a stochastic nonlinear programming model [42], [43]. The expression  $\sigma(x, y_1, \dots, y_s)$  includes the average value  $\sigma_0$  and the constant value  $\lambda$  multiplied by its variance

$$\sigma(x, y_1, \dots, y_s) = \sum_{s \in S} P_s C_s + \lambda \sum_{s \in S} P_s \{C_s - \sum_{s \in S} P_s C_s\}^2. \tag{25}$$

Since the above equation has an expression by the power of 2, making it a quadratic equation, it was formulated as below by [42]:

$$\sigma(x, y_1, \dots, y_s) = \sum_{s \in S} P_s C_s + \lambda \sum_{s \in S} P_s |C_s - \sum_{s \in S} P_s C_s|. \tag{26}$$

While this objective function is still nonlinear, it can be transformed into a linear function through Yu and Li's approach [44] by adding nonnegative deviation variables. The two deviation variables are minimized concerning constraints instead of reducing the reference of absolute deviations from the mean of the above two functions:

$$\text{Min } z = \sum_{s \in S} P_s C_s + \lambda \sum_{s \in S} P_s [(C_s - \sum_{s \in S} P_s C_s) + 2\theta_s]. \tag{27}$$

$$C_s - \sum_{s \in S} P_s C_s + \theta_s \geq 0. \tag{28}$$

$$\theta_s \geq 0. \tag{29}$$

The robust optimization approach is employed to deal with the uncertainty of parameters through a set of possible scenarios by transforming the particular model into a robust model.

### 3.2.1| General form of robust optimization model

The research problem is modeled using the method proposed by Mulvey et al. [42] as the following:

$$\begin{aligned} \text{Max } z_1 = & \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \right) + \\ & \lambda \sum_s P_s \left[ \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \right) - \sum_{s'} P_{s'} \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \right. \right. \\ & \left. \left. \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \right) + 2\theta_s \right] + \omega \sum_i \sum_j \sum_s P_s \delta_{ijs}. \end{aligned} \tag{30}$$

$$\begin{aligned} \text{Max } z_2 = & \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\} \right) + \\ & \lambda \sum_s P_s \left[ \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\} \right) - \right. \\ & \left. \sum_{s'} P_{s'} \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\} \right) + 2\theta_s \right] + \\ & \omega \sum_i \sum_j \sum_s P_s \delta_{ijs}. \end{aligned} \tag{31}$$

$$\begin{aligned} \text{Max } z_3 = & \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \sum_i^m \sum_k^{en} v_i p_{ik} \right) + \\ & \lambda \sum_s P_s \left[ \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \sum_i^m \sum_k^{en} v_i p_{ik} \right) - \right. \\ & \left. \sum_{s'} P_{s'} \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \sum_i^m \sum_k^{en} v_i p_{ik} \right) + 2\theta_s \right] + \\ & \omega \sum_i \sum_j \sum_s P_s \delta_{ijs}. \end{aligned} \tag{32}$$

$$\begin{aligned} & \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \right) - \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \right. \\ & \left. \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} \right) + \theta_s \geq 0 \quad \forall s. \end{aligned} \tag{33}$$

$$\sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\} - \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \max\{v_i\} \right) + \theta_s \geq 0 \quad \forall s. \tag{34}$$

$$\left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \sum_i^m \sum_k^{en} v_i p_{ik} \right) - \sum_s P_s \left( \sum_i^m \sum_j^n \sum_s^{se} w_{is} x_{ij} + \sum_i^m \sum_j^n \sum_s^{se} w_{is} z_{ij} - \sum_i^m \sum_k^{en} v_i p_{ik} \right) + \theta_s \geq 0 \quad \forall s. \tag{35}$$

$$\theta_s \geq 0. \tag{36}$$

### 3.2.2 | Solving the problem using lagrangian relaxation

The main idea of Lagrangian relaxation is to relax complicated constraints by multiplying them by Lagrangian coefficients and adding them to the objective function of a problem. This method is employed to simplify the complicated constraints to make the problem easier to solve. For every constant Lagrangian coefficient, the optimal solution to the relaxed minimization problem is a lower bound of the main problem. In other words, each solution to the relaxed problem is bound to the solution to the main problem. Due to the elimination of some constraints and the expansion of the feasible area, the relaxed problem is solved more easily than the main one [45], [46].

Many of the large-scale linear programming models have successfully been solved through Lagrangian relaxation. Examples include studies conducted by Hamdan and Diabat [47], Heidari-Fathian and Pasandideh [48], Diabat and Richard [49], and Pakravan and Behnamian [50]. Some researchers have also adopted the Lagrangian relaxation method to relax complicated constraints and achieve an acceptable approximate solution to the robust problem [51]-[53].

Furthermore, the solution to the relaxed problem is an upper bound if feasible in the main problem. For this purpose, a heuristic algorithm is usually proposed to create a feasible solution (an upper bound) out of the solution of the lower bound. Therefore, a better lower bound is achieved by maximizing the minimum obtained from the relaxed problem, and the resultant solution can be approximated to the solution of the main problem in iterations. For this purpose, the gradient method is employed to solve the Lagrangian duality problem. The Lagrange function optimization problem with the duality variable (Lagrangian coefficients) is called the Lagrangian duality problem [54]. Consider the following optimization problem

$$\begin{aligned} & \text{Max } c^T x + \lambda^T (b_2 - A_2 x), \\ & A_2 x \leq b_2, \\ & A_1 x \leq b_1, \\ & x \in \mathbb{R}^n, A \in \mathbb{R}^{m,n}, \end{aligned} \tag{37}$$

where  $\lambda = (\lambda_1, \dots, \dots, \lambda_{m_2})$  represents nonnegative weights. If *Constraint (2)* is violated, there will be a fine, and if it is satisfied, there will be a reward. A practical feature of this solution is that the result of Lagrangian optimization will not be smaller than the optimal result of the main problem for every constant set of  $\lambda$ . The problem constraints can be analyzed and rewritten as below:

$$\begin{aligned} c^T x & \leq \widehat{c^T x + \lambda^T}, \\ b_2 - A_2 \hat{x} & \leq c^T x + c^T \hat{x} + \widehat{\lambda^T} b_2 - A_2 \hat{x}. \end{aligned} \tag{38}$$

Evidently, both inequalities are correct because  $\hat{x}$  is accepted in the main problem and  $\hat{x}$  is the optimal solution to the Lagrangian relaxation. In fact, if the maximum value obtained from the relaxation problem is minimized, a more robust constraint will be obtained in the objective value of the main problem. Hence, we can analyze the main problem instead of analyzing a partial duality problem. As a result, a Lagrangian simplification algorithm searches for a range of  $\lambda$  values in order to minimize the result generated by the internal problem P. Every value returned by P is a candidate for the upper bound of the problem, the smallest value of which is considered the best. In addition, if a heuristic function is employed, the problem can be repeated to find the best upper bound, and the cost of the best acceptable solution is close to the

main solution as much as desired. The enhanced Lagrangian method is entirely similar to the Lagrangian simplification method in terms of structure; however, the former adds one expression to the latter to update the dual parameters of  $\lambda$  in a more standard way.

In the integer variable planning problem, the constraints with binary variables, which make the problem difficult to solve, are generally called complicated constraints. The most complicated constraints should be transferred to the objective function to solve a relaxed problem. According to the model evaluation, Eq. (39) is among the complicated, hard constraints. The Lagrangian relaxation problem is created by multiplying this constraint by the Lagrangian coefficient  $\lambda_{ij} \geq 0$  and adding it to the objective function.

$$d_{ij} \leq R_j RT + (1 - K_{ij}) \text{Big M}. \tag{39}$$

$$\sum_i^m \sum_j^n \lambda_{ij} (d_{ij} - R_j RT - (1 - K_{ij}) \text{Big M}). \tag{40}$$

Eq. (39) is deleted from the constraints, and Eq. (40) is added to the objective function.

## 4 | Numerical Calculations and Sensitivity Analysis

Since the proposed model is a multi-objective one, it was solved using LP metric method. The objective functions were first solved separately for every proposed constraint and then weighted to create a single objective function, which can be solved as a single-objective problem. The computations were performed in GAMS v24.8.2 in a small-scale local model in Tehran.

### 4.1 | Definitions of Data and Scenarios

The problem is first modeled by applying scenarios to create a single-objective model. Values obtained before and after the robust solution were then compared, and the results were analyzed. In case of disruptions at telecommunication towers, various scenarios can occur. In this study, demand is considered uncertain. Three scenarios can happen that are presented in Table 2.

**Table 2. Demand for every scenario and values of parameters.**

Occurrence Probability	Descriptions										Scenario
0.20	$w_{is=1} = (1-0.25)w_{is}$ Demand is 25% lower than the average prediction value.										1
0.50	$w_{is=1} = 50$ The average prediction value for demand										2
0.30	$w_{is=1} = (1+0.25)w_{is}$ Demand is 25% higher than the average prediction value.										3
BigM	$R_j$	$RT_j$	$p_{ik}$	$d_{ij}$	Capacity	Price	$q_j$	$e_{jk}$	$w_{is}$	Parameter	
$10^{10}$	0-10	0-30	0-1	Uniform (0-20)	100	10000	0-0.5	2000-5000	0-100	Value	

Table 2 reports the demand values. Moreover, as discussed earlier, the values of parameters were obtained from the analysis of an urban district in Tehran and extracted from the relevant papers.

### 4.2 | Model Analysis

The model was solved, and the values of five objective functions were determined by solving all constraints under the scenario (Table 3).

Table 3. The values of objective functions with LP metrics.

<b>W</b>	0.01	0.13	0.21	0.33	0.41	0.53	0.61	0.73	0.81	0.93	1
<b>Z1</b>	1710	3254	4522	3390	4910	6101	6501	7410	7560	7836	7963
<b>Z2</b>	540	850	1950	3250	4501	4360	4510	4952	5320	5620	5852
<b>Z3</b>	650	1100	1501	1650	1952	2410	2785	3265	3654	3950	4020
<b>Z4</b>	22450	20650	17840	16980	15450	13020	10523	9100	8410	8023	4750
<b>Z5</b>	1002	650	325	569	320	215	198	102	98	91	81

The value of W in every objective function increases from 0.01 to 1. The maximum coverage decreases as W increases from Z1 to Z3, whereas the setup cost and pollution decrease by increasing the LP-metric weight. The robust model was employed to acquire a near-optimal solution to every feasible scenario. Moreover, the proposed solutions were expressed for analysis by considering  $\omega = 1$  and  $\lambda = 1$ . Table 4 reports the values of variables obtained from the solution per scenario for the single-objective and the robust model.

Table 4. The values of objective functions for different  $r_j$ ,  $O$ ,  $v_i$ ,  $y_j$ , and  $y'_j$ .

Values	$r_j$					$O$	$v_i$	$y_j$					$y'_j$				
	$j_1$	$j_2$	$j_3$	$j_4$	$j_5$			$j_1$	$j_2$	$j_3$	$j_4$	$j_5$	$j_1$	$j_2$	$j_3$	$j_4$	$j_5$
<b>Metric</b>	115	139	129	126	115	13	7	1	1	1	1	1	1	1	1	1	1
<b>Obj5</b>	115	150	123.12	140	150	10	5	1	1	1	1	1	0	1	1	0	1
<b>Obj4</b>	1500.52	1200	1390	1400	1290	31	2	0	0	0	1	0	0	1	0	1	1
<b>Obj3</b>	150	139	129	121.7	170	12	7	1	0	1	0	1	0	0	0	0	1
<b>Obj2</b>	137.5	122	115	129	139	50	3	1	0	1	1	1	0	0	0	1	1
<b>Obj1</b>	139	151	139	139	170	25	5	1	1	0	0	1	1	1	1	1	0

### 4.3 | Sensitivity Analysis

In this section, we investigated the amount of change in the objective function with the changes in the value of parameters to determine the sensitivity of these parameters to the objective function. The most important parameter of the sensitivity analysis with the greatest effect is "demand." Therefore, the demand values were changed through a loop in GAMS, and the observations were analyzed. Table 5 presents the values of the objective function for 0–100 demands (the demand was changed from 100 to 200).

Table 5. The values of the objective function with demand change.

<b>w(i,s)-2</b>	100	150	170	100	130	170	158	190	187	120	165	105	150	140	140
<b>Z1-2</b>	1800	2500	4650	4850	3650	5050	5120	6590	6980	6970	6970	7980	7850	7900	8150
<b>w(i,s)-1</b>	90	100	25	30	12	95	23	10	15	6	87	16	95	30	15
<b>Z1-1</b>	1710	2340	4401	4522	3250	4910	5610	5970	6101	6301	6501	7410	7403	7836	7955

Fig. 3 compares values of the objective function recorded after the demand was changed from 0 to 100 and then from 100 to 200. In fact, by increasing the demand by 100 units, the first objective function increased in some points and decreased in fewer ones. The objective function rises after a specific moment when the demand increases by a higher ratio because this would definitely increase the need for maximum coverage. As a result, the coverage increases when the demand increases.

As is shown in Fig. 3, increasing the demand increases the values of Objective Function 1 and 2 with a mild slope. The second and third objective functions indicate coverage in different cases. The results revealed an increase in both following the increase in the demand. Analysis of  $\lambda$  for each function suggested that increasing this coefficient when the probability is constant leads the model to adapt strategies characterized by A) maximum coverage, B) minimum cost and C) minimum pollution as managerial implications of the findings (Fig. 4). In other words, the more significant these coefficients, the more risk-evasive the decision-maker, which finally affects the expected coverage costs

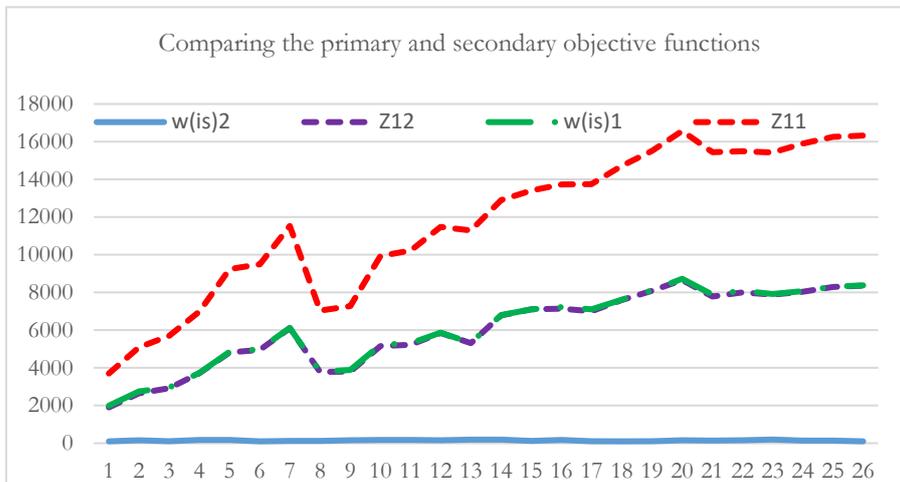


Fig. 3. Comparing the primary and secondary objective functions in terms of changes in the demand.

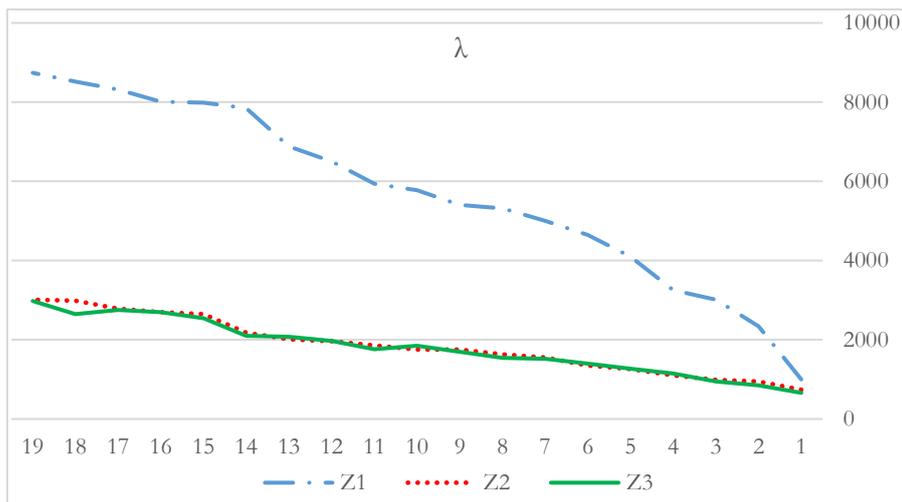


Fig. 4. Changes in the objective function by changes in  $\lambda$ .

Fig. 4 demonstrates three objective functions per the robustification coefficient. As  $\lambda$  increases, the values of all three parts increase. However, Z1, which denotes the maximum complete coverage, grows with a steeper slope than the second and third objective functions, indicating incomplete coverage scenarios. Therefore, the first objective function stands above the other two.

Fig. 5 shows that the first, second, and third objective functions improved in the robust scenario instead of the non-robust one. In other words, they yielded larger values. Furthermore, the third objective function, i.e., the average coverage of sites, is more optimal in the robust scenario than the first and second objective functions, i.e., the maximum coverage and coverage during crises. In addition, according to the sensitivity analyses of the fourth and fifth functions (Fig. 5), changes in demand were relatively ineffective on the primary and secondary functions. In general, they proved to descend

Constraint 16 of the primary model is complicated and complex; therefore, the Lagrangian relaxation method eliminated its complexity. While  $d_{ij}$  was determined as the lower bound, an upper bound was obtained in every iteration of the Lagrangian method. The algorithm ended when the upper bound was equal to the lower bound, which meant that the optimal solution was obtained (Fig. 6).

The distance ( $d_{ij}$ ) between Uniform (0-20) and Uniform (20-40) was prolonged, and the objective function behavior was analyzed.

Fig. 7 shows that the substitute objective function resulting from Lagrangian relaxation had an undefined behavior (sometimes ascending and sometimes descending) after the distance was increased, while it proved ultimately climbing after a certain point compared to the primary function

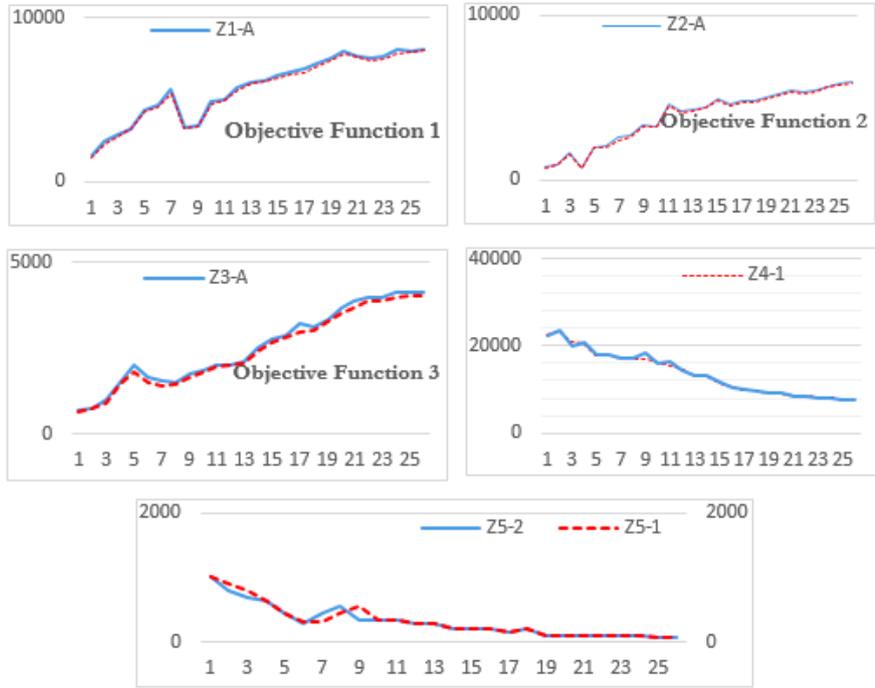


Fig. 5. Comparing the objective functions before and after robustification (UN: before – A: after).

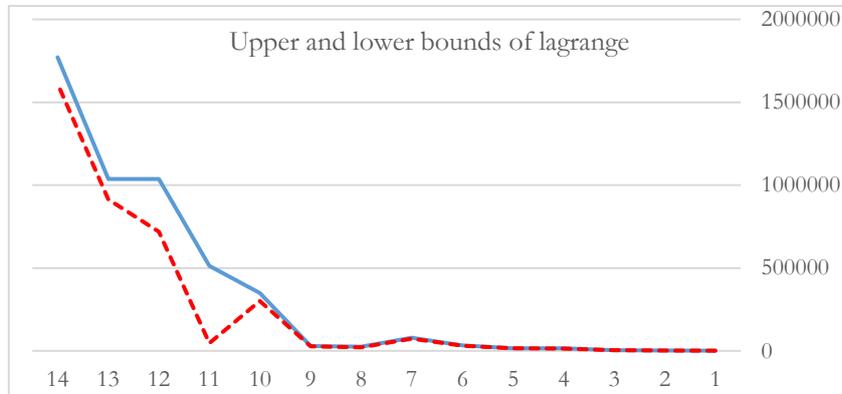


Fig. 6. Convergence of upper and lower bounds of the Lagrange algorithm after relaxing a hard constraint.

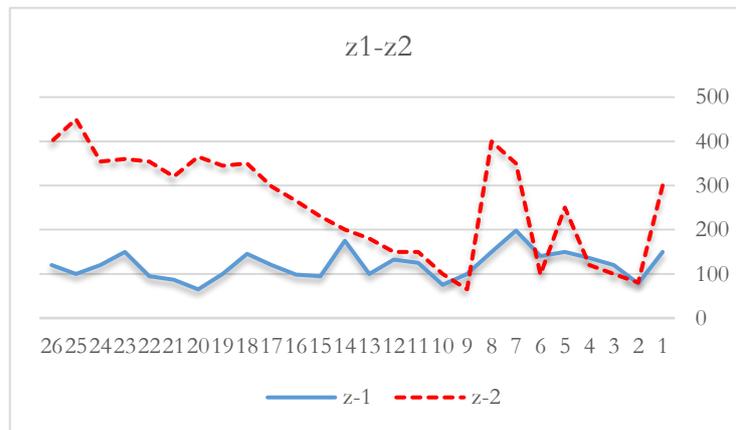


Fig. 7. Comparing the objective function before and after the distance is prolonged (lagrangian relaxation).

## 5 | Conclusion and Recommendations

As shown in the tables, as the value of  $\lambda$  increases when the scenario probability is constant, the model adopts strategies that are characterized by maximum coverage, minimum cost, and minimum pollution. In other words, the larger this coefficient, the more elusive the decision-maker, which eventually affects the expected costs of the coverage.

Mobile phones and the Internet play crucial roles in the modern world. Men perform most of their activities via the Internet at present. For instance, they use their mobile phones and the Internet to chat, use social media, send and receive documents, and do many other things. Even most of the material used for writing this manuscript was extracted from the Internet. Hence, the Internet and smartphones are indispensable components of human life in the 21st century. The perfectionist human being always seeks optimal and advanced use of tools. The Internet has been developed from Generation 0 to Generation 5 so far. In Iran, mobile phone operators have always been trying to provide the best services and attract more subscribers. This would be more challenging during accidents and disasters when disruptions happen. Hence, this paper aims to propose an optimal location-allocation model for the maximum coverage of client demand and the provision of better services.

In today's world, our daily lives depend significantly on communication networks in a tangible way. Any performance disruption in critical situations can have irreversible consequences in many sectors affected by communication, especially providing relief and security services for the health of citizens. Moreover, businesses that are based on mobile phone networks can be disrupted during crises. Therefore, it is necessary to address the design of mobile phone networks and the Internet in terms of resilience and resistance in such situations. This is considered an indispensable area of knowledge in operations research studies. Literature review showed that few models were proposed that comprehensively analyzed coverage maximization as well as minimization of environmental pollutants by considering backup sites for the network resilience. Due to particular political and geographical situations, Iran has always been prone to natural and artificial disasters. In recent years, a great deal of damage has been sustained by telecommunication networks in the floods of Golestan, Khuzestan, and Ilam Provinces and the earthquake in East Azerbaijan Province. Therefore, investigation of this problem was proven crucial in Iran. From an environmental point of view, the installation conditions of BTS antennas in Iran are practically based on information approved by the international scientific community. If these conditions are applied correctly during design and deployment, people can benefit from this technology without disrupting communication. This study has several suggestions for the development of communication technology for future research, as follows:

- I. Using a mobile phone with existing guidelines requires a set of low-risk, green solutions. Therefore, creating environmental and health standards should be a priority.
- II. For future research, the location of restricted areas can be considered in possible shapes in rectangular or circular shapes, and the use of several limited regions instead of one local area can bring the model closer to reality.
- III. The parameters of the proposed model in this study can follow other distributions, such as the binomial pattern and fuzzy conditions that can be used to improve the relevant problems.
- IV. Due to the changeable nature of urban elements, it is necessary to expand the model for use in developing networks over networks under construction. Therefore, it is helpful to pay close attention to the number of rigs when installing new rigs in an area and increase, decrease, or maintain the capacity of the rigs after reinforcement.

## 5.1 | Recommendations

- I. Considering ideal planning in the model.
- II. Considering a more significant number of uncertain parameters (using the fuzzy approach).
- III. Considering the hierarchical coverage instead of partial coverage.
- IV. Using metaheuristic methods for location-allocation.
- V. Proposing novel and operational strategies for dealing with the demand points lost in critical conditions.

### 5.1.2| Future research areas in robust optimization

- I. Robust optimization based on the average case.
- II. Nonlinear and discrete robust optimization.
- III. Proposing a solution to complicated robust optimization problems, especially when the uncertainty set has a general form.

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