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A compromise solution for neutrosophic multiobjective linear programming problems and its application in transportation problems

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Abstract

Neutrosophic set theory plays an important role in dealing with the impreciseness and inconsistency in data encountered in solving real-life problems. The current paper focuses on the neutrosophic fuzzy multiobjective linear programming problem (NFMOLPP), where the coefficients of the objective functions, constraints and right-hand side parameters are single-valued trapezoidal neutrosophic numbers (NNs). From the viewpoint of complexity of the problem, a ranking function of NNs is proposed to convert the problem into equivalent MOLPs with crisp parameters. Then suitable membership functions for each objective are formulated using their lowest and highest value. With the aim of linear programming techniques, a compromise optimal solution of NFMOLPP is obtained. The main advantage of the proposed approach is that it obtains a compromise solution by optimizing truth-membership, indeterminacy-membership, and falsity-membership functions, simultaneously. Finally, a transportation problem is introduced as an application to illustrate the utility and practicality of the approach.

Keywords: Multiobjective linear programming problem, Neutrosophic set, single-valued trapezoidal, neutrosophic number, indeterminacy membership functions, compromise solution, transportation problem.

1 | Introduction

Optimizing more than one commensurable and/or conflicting objective function under a set of well-defined constraints is termed as multiobjective programming problems (MOLPs). Most often, many real-world applications, such as transportation, supplier selection, inventory control, supply chain planning, etc., take the form of MOLPs. While dealing with multiple objectives, it is not always possible to obtain a single solution that optimizes each objective function, efficiently. However, a compromise solution is possible that satisfies each objective, simultaneously. Therefore, the concept of compromise solutions is an important aspect and leads in search of the global optimality criterion. In the past few decades, a tremendous amount of research has been presented in the context of multiobjective optimization techniques.

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Table 1: List of abbreviations

Abbreviation	Description
MOLP	Multiobjective programming problems
NFMOLPP	Neutrosophic fuzzy multiobjective linear programming problem
NN	Neutrosophic number
SVTN	Single-valued trapezoidal neutrosophic
SVNS	Single-valued neutrosophic set

A major disadvantage of fuzzy sets is its inability to efficiently represent imprecise and inconsistent information as it considers only the truth membership function [29]. Intuitionistic fuzzy set is a modification of fuzzy sets, which considered both the truth and falsity membership functions [4]. But it still had some drawbacks in depicting human-like decision making. In the last decade, a large number of studies on fuzzy and Intuitionistic fuzzy multi-objective optimization techniques have been presented. Among these studies, we can mention the works of Mahajan and Gupta [18], Borovika [6], Ahmadini et al. [3], Yu et al. [26], and Rizk-allah et al. [20].

In 1998, a new type of sets called the neutrosophic set was introduced by Smarandache [22] to deal with decision making problems which involved incomplete, inconsistent and indeterminate information. Here indeterminacy is considered as an independent factor, which has a major contribution in decision making. Neutrosophic set helps in human-like decision making by considering truth, falsity and indeterminacy membership functions.

Ye et al. [24] presented some new operations of NNs to make them suitable for engineering applications. They proposed a neutrosophic function involving NNs. Then, they used it to solve neutrosophic linear programming problems [24,25]. Ye et al. [23] analysed joint roughness coefficient taking the help of NN functions. NN generalized weighted power averaging operator formulated by Liu [17] and it is applied to multi-attribute group decision making in NN environment. Maiti et al. proposed a goal programming strategy to solve multi-level multi-objective linear programming problem with NNs [19].

Recently, Deli and Uba [10] suggested a novel ranking method for single-valued neutrosophic numbers and they applied it to multi-criteria decision-making problems. Ahmad et al. [1] discussed the energy-food-water nexus security management through neutrosophic modeling and optimizing approaches. Also, they presented a study on supplier selection problem with Type-2 fuzzy parameters and solved it using an interactive neutrosophic optimization algorithm [2]. Wang et al. proposed a novel method to solve multiobjective linear programming problems with triangular neutrosophic numbers [28]. Das et al. presented a novel lexicographical-based method for linear programming problems with trapezoidal neutrosophic numbers [8]. Their method uses a lexicographical order.

As a special instance of linear programming problems, many authors focused on solving transportation problems in fuzzy environment, such as the multi-objective case [11, 14], the case with fractional objectives [13, 16], the inverse version [12], the problem with heptagonal and pentagonal fuzzy numbers [9,15], and the problem with fuzzy variables [7].

The main results of this paper are as follows:

- The paper formulates the multiobjective linear programming problem with single-valued trapezoidal neutrosophic (SVTN) parameters.
- A new method is proposed to find a compromise optimal solution of NFMOLP problem.
- In the proposed method, the accuracy function is used to transfer the NFMOLPP into equivalent crisp MOLPP.
- The approach is applied for a transportation problem to show its utility and performance.

The rest of this paper is organized as follows: In Section 2 basic concepts and algebra operations of neutrosophic numbers are reviewed. Section 3 deals with modelling multiobjective linear programming problems with neutrosophic fuzzy parameters. In Section 4, a solution method for obtaining a compromise solution of NFMOLPP is introduced. In Section 5, the entire solution procedure is summarized in the form of an algorithm. In Section 6 a transportation problem as an application is presented. Finally, some concluding remarks are reported in Section 7.

2 | Preliminaries

In this section, some basic concepts and definitions on neutrosophic sets and single-valued trapezoidal neutrosophic numbers are reviewed from the literature.

Definition 1. [13] Let X be a nonempty set. A neutrosophic set (NS) $\tilde{A}^N$ is defined as:

$$\tilde{A}^N = \{\langle x: T_A(x), I_A(x), F_A(x)\rangle, x \in X\}, T_A(x), I_A(x), F_A(x) \in]0^-, 1^+[\,\},$$

where $T_A(x), I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively, and there is no restriction on the summation of them, so $0^- \le \mathrm{T_A}(\mathrm{x}) + I_A(\mathrm{x}) + F_A(\mathrm{x}) \le 3^+$, and $]0^-, 1^+[$ is non-standard unit interval.

Since it is difficult to apply NSs to practical problems, Wang et al. [27] introduced the concept of a single valued neutrosophic set (SVNS), which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2. Let $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \in [0,1]$ then a single-valued trapezoidal neutrosophic number (SVTNN) $\tilde{\mathrm{a}}^{\mathrm{N}} = \langle(\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$ is a special NS on the real numbers $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} T_{\tilde{a}}\left(\dfrac{x - a_1}{a_2 - a_1}\right) & a_1 \le x \le a_2 \\ T_{\tilde{a}} & a_2 \le x \le a_3 \\ T_{\tilde{a}}\left(\dfrac{x - a_4}{a_4 - a_3}\right) & a_3 < x \le a_4 \\ 0 & oherwise, \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + I_{\tilde{a}}(x - a_1)}{a_2 - a_1} & a_1 \le x \le a_2 \\ I_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{x - a_3 + I_{\tilde{a}}(a_4 - x)}{a_4 - a_3} & a_3 < x \le a_4 \\ 1 & oherwise, \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + F_{\tilde{a}}(x - a_1)}{a_2 - a_1} & a_1 \le x \le a_2 \\ F_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{x - a_3 + F_{\tilde{a}}(a_4 - x)}{a_4 - a_3} & a_3 < x \le a_4 \\ 1 & oherwise, \end{cases}$$

where, $T_{\tilde{a}}$, $I_{\tilde{a}}$ and $F_{\tilde{a}}$ are the maximum truth, minimum indeterminacy, and minimum falsity membership degrees, respectively.

Definition 3. Let $\tilde{\mathrm{a}}^{\mathrm{N}} = \langle(\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$ and $\tilde{\mathrm{b}}^{\mathrm{N}} = \langle(\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4); T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}\rangle$ be two arbitrary SVTNNs and $\gamma \ne 0$ be any real number, then

- $\tilde{\mathrm{a}}^{\mathrm{N}} + \tilde{\mathrm{b}}^{\mathrm{N}} = \langle(\mathrm{a}_1 + \mathrm{b}_1, \mathrm{a}_2 + \mathrm{b}_2, \mathrm{a}_3 + \mathrm{b}_3, \mathrm{a}_4 + \mathrm{b}_4); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}}\rangle$,
- $\tilde{\mathrm{a}}^{\mathrm{N}} - \tilde{\mathrm{b}}^{\mathrm{N}} = \langle(\mathrm{a}_1 - \mathrm{b}_4, \mathrm{a}_2 - \mathrm{b}_3, \mathrm{a}_3 - \mathrm{b}_2, \mathrm{a}_4 - \mathrm{b}_1); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}}\rangle$,

- $\gamma \tilde{a}^N = \begin{cases} \langle(\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4);\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle & \gamma > 0, \\ \langle(\gamma a_4, \gamma a_3, \gamma a_2, \gamma a_1);\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle & \gamma < 0. \end{cases}$

Definition 4. Let $\tilde{a}^N = \langle(a_1,\ a_2,\ a_3,\ a_4);\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$ be a SVTNN. Then, the score function $S(\tilde{a}^N)$ and accuracy function $A(\tilde{a}^N)$ of a SVTNN are respectively defined as follows:

- $S(\tilde{a}^N) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4)(T_{\tilde{a}} + (1 - I_{\tilde{a}}) + (1 - F_{\tilde{a}}))$
- $A(\tilde{a}^N) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4)(T_{\tilde{a}} + (1 - I_{\tilde{a}}) - (1 - F_{\tilde{a}}))$

Definition 5. Suppose $\tilde{a}^N = \langle(a_1,\ a_2,\ a_3,\ a_4);\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$ and $\tilde{b}^N = \langle(b_1, b_2, b_3, b_4);\ T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}\rangle$ be any two SVTNNs. Then, we define a ranking method as follows:

- If $S(\tilde{a}^N) > S(\tilde{b}^N)$ then $\tilde{a}^N > \tilde{b}^N$.
- If $S(\tilde{a}^N) = S(\tilde{b}^N)$ and $A(\tilde{a}^N) > A(\tilde{b}^N)$ then $\tilde{a}^N > \tilde{b}^N$.
- If $S(\tilde{a}^N) = S(\tilde{b}^N)$ and $A(\tilde{a}^N) < A(\tilde{b}^N)$ then $\tilde{a}^N < \tilde{b}^N$.
- If $S(\tilde{a}^N) = S(\tilde{b}^N)$ and $A(\tilde{a}^N) = A(\tilde{b}^N)$ then $\tilde{a}^N = \tilde{b}^N$.

3 | Problem formulation

The general form of a multi-objective linear programming problem (MOLPP) with k objectives can be described as follows:

$$\begin{aligned} \min \quad & Z(x) = [Z_1(x), Z_2(x), \dots, Z_r(x)] \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}x_j \geq b_i, \quad i = 1, 2, \dots, m_1 \\ & \sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = m_1 + 1,\ m_1 + 2, \dots, m_2 \\ & \sum_{j=1}^{n} a_{ij}x_j = b_i, \quad i = m_2 + 1,\ m_2 + 2, \dots, m \\ & x_j \geq 0, \quad j = 1, 2, \dots, n, \end{aligned} \tag{1}$$

$where\, Z_k(x) = \sum_{k=1}^{r} c_{kj}\, x_j, k = 1, 2, \cdots, r$ is the k-th objective function.

Definition 6. Let S be the set of all feasible solutions for (1). A point x^* is said to be an efficient or Pareto optimal solution of (1) if there does not exist any $x \in S$ such that, $Z_k(x^*) \geq Z_k(x)$ for every k, and $Z_k(x^*) > Z_k(x)$ for at least one k.

If all the parameters of problem (1) are uncertain, and they can be represented by SVTNNs, then problem (1) becomes a neutrosophic fuzzy multi-objective linear programming problem (NFMOLPP) as follows:

$$\begin{aligned} \min \quad & \tilde{Z}^N(x) = [\tilde{Z}_1^N(x), \tilde{Z}_2^N(x), \dots, \tilde{Z}_r^N(x)] \\ s.t. \quad & \sum_{j=1}^{n} \tilde{a}_{ij}^N x_j \geq \tilde{b}_i^N, \quad i = 1, 2, \dots, m_1 \\ & \sum_{j=1}^{n} \tilde{a}_{ij}^N x_j \leq \tilde{b}_i^N, \quad i = m_1 + 1,\ m_1 + 2, \dots, m_2 \\ & \textstyle\sum_{j=1}^{n} \tilde{a}_{ij}^N x_j = \tilde{b}_i^N, \quad i = m_2 + 1,\ m_2 + 2, \dots, m \\ & x_j \geq 0, \quad j = 1, 2, \dots, n, \end{aligned} \tag{2}$$

where $\tilde{Z}_k^N = \sum_{j=1}^{n} (\tilde{c}_{kj})^N x_j, k = 1, 2, \ldots, r.$

Using accuracy function which is linear, problem (2) is converted into the following crisp MOLPP:

$$
\begin{aligned}
\min \quad & Z'(x) = [Z'_1(x), Z'_2(x), \ldots, Z'_r(x)] \\
s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \geq b'_i, \quad i = 1, 2, \ldots, m_1 \\
& \sum_{j=1}^{n} a'_{ij} x_j \leq b'_i, \quad i = m_1 + 1, m_1 + 2, \ldots, m_2 \\
& \sum_{j=1}^{n} a'_{ij} x_j = b'_i, \quad i = m_2 + 1, m_2 + 2, \ldots, m \\
& x_j \geq 0, \quad j = 1, 2, \ldots, n,
\end{aligned} \tag{3}
$$

Where $Z'_k(x) = A\left(\tilde{Z}_k^N(x)\right) = \sum_{j=1}^{n} A\left((\tilde{c}_{kj})^N\right) x_j, \ \forall k = 1, 2, \ldots, r; \ b'_i = A(\tilde{b}_i^N)$ and $a'_{ij} = A(\tilde{a}_{ij}^N)$ for all $i = 1, \ldots, m, j = 1, \ldots, n$.

Theorem 1. [21] An efficient solution for crisp MOPP (3) is an efficient solution for NFMOLPP (2).

Thus, solving the NFMOLPP model (2) is equivalent to solving the crisp MOLPP model (3).

4 | Solution method

In this section, we restrict our attention to NFMOLPP and present an approach to solve it.

By using the definition of the fuzzy decision proposed by Bellman and Zadeh [5], we can characterize the fuzzy decision set D as follows:

$$D = Z \cap C$$

where Z and C are fuzzy goals and fuzzy constraints, respectively.

In a similar manner, we also introduce the neutrosophic decision set D^N, which consider neutrosophic goals and constraints as follows:

$$D^N = \left(\bigcap_{k=1}^{r} Z_k\right) \cap \left(\bigcap_{i=1}^{m} C_i\right)$$

where

$$
\begin{aligned}
\mu_{D^N}(x) &= \min\{\mu_{Z_1}(x), \ldots, \mu_{Z_r}, \mu_{C_1}, \ldots, \mu_{C_m}, \forall x \in X\}, \\
\lambda_{D^N}(x) &= \max\{\lambda_{Z_1}(x), \ldots, \lambda_{Z_r}, \lambda_{C_1}, \ldots, \lambda_{C_m}, \forall x \in X\}, \\
\nu_{D^N}(x) &= \max\{\nu_{Z_1}(x), \ldots, \nu_{Z_r}, \nu_{C_1}, \ldots, \nu_{C_m}, \forall x \in X\},
\end{aligned}
$$

are the truth, indeterminacy and the falsity membership functions of neutrosophic decision set D^N, respectively.

In the sequal, we solve the multi-objective programming problem by considering one objective function at a time and ignoring the others. Then, we find minimum and maximum values of each objective function. Let L_k be the minimum value and U_k be the maximum value of Z_k, i.e.,

$$U_k = \max\ [Z_k(x)] \qquad and \qquad L_k = \min\ [Z_k(x)] \qquad \forall k = 1, 2, \dots, r. \tag{4}$$

The bounds for the k-th objective function under the neutrosophic environment can be obtained as follows: $U_k^{\mu} = U_k, L_k^{\mu} = L_k$ for truth membership (5)

(6)

for indeterminacy membership

(7)

$U_k^{\nu} = U_k^{\mu}, \quad L_k^{\nu} = L_k^{\mu} + t_k$ for falsity membership

where $s_k, t_k \in (0, 1)$ are predetermined real numbers prescribed by decision-makers. For each objective function, consider truth membership function $\mu_k(Z_k(x))$, indeterminacy membership function $\lambda_k(Z_k(x))$ and falsity membership function $\nu_k(Z_k(x))$ as the following functions:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & Z_k(x) \le L_k^{\mu} \\ \frac{U_k^{\mu} - Z_k(x)}{U_k^{\mu} - L_k^{\mu}} & L_k^{\mu} \le Z_k(x) \le U_k^{\mu} \\ 0 & Z_k(x) \ge U_k^{\mu}, \end{cases} \tag{8}$$

$$\lambda_k(Z_k(x)) = \begin{cases} 0 & Z_k(x) \le L_k^{\nu} \\ \frac{Z_k(x) - L_k^{\lambda}}{U_k^{\lambda} - L_k^{\lambda}} & L_k^{\lambda} \le Z_k(x) \le U_k^{\lambda} \\ 1 & Z_k(x) \ge U_k^{\nu}, \end{cases} \tag{9}$$

$$\nu_k(Z_k(x)) = \begin{cases} 0 & Z_k(x) \le L_k^{\nu} \\ \frac{Z_k(x) - L_k^{\nu}}{U_k^{\nu} - L_k^{\nu}} & L_k^{\nu} \le Z_k(x) \le U_k^{\nu} \\ 1 & Z_k(x) \ge U_k^{\nu}, \end{cases} \tag{10}$$

Since, decision maker wants to maximize the range of acceptance and to minimize the range of rejection, we are looking for a solution with the maximum degree of membership and the minimum degree of nonmembership.

In this regard, according to the concept of fuzzy decision set [5], an optimal compromise solution can be selected as the design for which it maximizes the minimum truth degree (acceptance) and minimize the maximum indeterminacy (rejection up to some extent) and a falsity (rejection) degree by taking all objectives, simultaneously. Therefore, according to the fuzzy decision of Belman and Zadeh [16], we have to solve the following multiobjective programming problem:

$$\begin{aligned} &Maximize \quad (\min\{\mu_1(Z_1(x)), \dots, \mu_r(Z_r(x))\}) \\ &Minimize \quad (\max\{\lambda_1(Z_1(x)), \dots, \lambda_r(Z_r(x))\}) \\ &Minimize \quad (max\{\nu_1(Z_1(x)), \dots, \nu_r(Z_r(x))\}) \\ &\text{s.t.} \qquad \text{all the constraints of (3).} \end{aligned} \tag{11}$$

Suppose that $\alpha = \min_{k=1,\dots,r} \mu_k(Z_k(x))$, $\beta = \max_{k=1,\dots,r} \lambda_k(Z_k(x))$ and $\gamma = \max_{k=1,\dots,r} \nu_k(Z_k(x))$.

Therefore, problem (11) can be rewritten in the form of

$$
\begin{array}{lll}
Maximize & \alpha & \\
Minimize & \beta & \\
Minimize & \gamma & \\
s.t. & \mu_k(Z_k(x)) \geq \alpha & k=1,\dots,r \\
 & \lambda_k(Z_k(x)) \leq \beta & k=1,\dots,r \\
 & \nu_k(Z_k(x)) \leq \gamma & k=1,\dots,r \\
 & \alpha \geq \beta, \quad \alpha \geq \gamma, \quad \alpha+\beta+\gamma \leq 3 & \\
 & \alpha,\beta,\gamma \in (0,1) & \\
 & \text{all the constraints of (3).} &
\end{array} \tag{12}
$$

Using the weighted sum method and by setting the relations (8), (10) and (9), the problem (12) can be formed into the following equivalent problem.

$$
\begin{array}{lll}
Max & w_1\alpha - w_2\beta - w_3\gamma & \\
s.t. & Z_k(x) + (U_k^{\mu} - L_k^{\mu})\alpha \leq U_k^{\mu}, & k=1,\dots,r \\
 & Z_k(x) - (U_k^{\lambda} - L_k^{\lambda})\beta \leq L_k^{\lambda}, & k=1,\dots,r \\
 & Z_k(x) - (U_k^{\nu} - L_k^{\nu})\gamma \leq L_k^{\nu}, & k=1,\dots,r \\
 & \alpha \geq \beta, \quad \alpha \geq \gamma, \quad \alpha+\beta+\gamma \leq 3 & \\
 & \alpha,\beta,\gamma \in (0,1) & \\
 & \text{all the constraints of (3).} &
\end{array} \tag{13}
$$

Theorem 2. If $(\hat{x},\hat{\alpha},\hat{\beta},\hat{\gamma})$ is a unique optimal solution of problem (13), then $(\hat{x},\hat{\alpha},\hat{\beta},\hat{\gamma})$ is an efficient solution for problem (3).

Proof. On contrary, suppose that $(\hat{x},\hat{\alpha},\hat{\beta},\hat{\gamma})$ is not an efficient solution for problem (3). Then, there exists a feasible solution $x^* \neq \hat{x}$ to problem (3), such that $Z_k(x^*) \leq Z_k(\hat{x})$ for all $k=1,\dots,r$ and $Z_k(x^*) < Z_k(\hat{x})$ for at least one k. Therefore, $\frac{Z_k(x^*)-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}} \leq \frac{Z_k(\hat{x})-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}}$ for all $k=1,\dots,r$ and $\frac{Z_k(x^*)-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}} < \frac{Z_k(\hat{x})-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}}$ for at least one k. Thus, $\max_k\left(\frac{Z_k(x^*)-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}}\right) \leq (<) \max_k\left(\frac{Z_k(\hat{x})-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}}\right)$. Let $\gamma^* = \max_k\left(\frac{Z_k(x^*)-L_k^{\nu}}{U_k^{\nu}-L_k^{\nu}}\right)$ then $\gamma^* \leq (<)\hat{\gamma}$. Similiarly, consider that Let $\beta^* = \max_k\left(\frac{Z_k(x^*)-L_k^{\lambda}}{U_k^{\lambda}-L_k^{\lambda}}\right)$ then $\beta^* \leq (<)\hat{\beta}$.

In the same manner, we have $\frac{U_k^{\mu}-Z_k(x^*)}{U_k^{\mu}-L_k^{\mu}} \geq \frac{U_k^{\mu}-Z_k(\hat{x})}{U_k^{\mu}-L_k^{\mu}}$ for all $k=1,\dots,r$ and $\frac{U_k^{\mu}-Z_k(x^*)}{U_k^{\mu}-L_k^{\mu}} > \frac{U_k^{\mu}-Z_k(\hat{x})}{U_k^{\mu}-L_k^{\mu}}$ for at least one k.

Hence, $\min_k\left(\frac{U_k^{\mu}-Z_k(x^*)}{U_k^{\mu}-L_k^{\mu}}\right) \geq (>) \min_k\left(\frac{U_k^{\mu}-Z_k(\hat{x})}{U_k^{\mu}-L_k^{\mu}}\right)$. Let $\alpha^* = \min_k\left(\frac{U_k^{\mu}-Z_k(x^*)}{U_k^{\mu}-L_k^{\mu}}\right)$ this gives $(\hat{\alpha}-\hat{\beta}-\hat{\gamma}) < (\alpha^*-\beta^*-\gamma^*)$ which means that the solution is not unique optimal. This contradicts the fact that $(\hat{x},\hat{\alpha},\hat{\beta},\hat{\gamma})$ is the unique optimal solution of (13). Hence, it is an efficient solution of (3). □

5 | Compromise Solution algorithm for NFMOLPP

In this section, we summarize the compromise solution procedure developed in Sect. 4 as the following algorithm.

Step 1. Formulate the NFMOLPP as given in problem (1).

Step 2. Transform the NFMOLPP into crisp MOLPP as given in problem (3) by the accuracy function.

Step 3. Find an optimal solution of each single objective LPP and determine the upper and lower bounds by using Eq (4).

Step 4. Using U_k and L_k, obtain the upper and lower bounds for truth, indeterminacy and falsity member-ship function as given in Eqs (5)-(7).

Step 5. Use linear membership functions as given in Eqs (9)-(10) and transform the optimization problem (12) to crisp programming model as in (13).

Step 6. Solve crisp programming problem (13) using suitable techniques or software packages.

6 | Numerical example

To illustrate the application of the proposed approach for a real-life transportation problem, the following numerical example is considered. Since, the parameters of transportation problem vary due to various uncertain situation like weather condition, traffic condition, petroleum price, the crisp value of parameters cannot deal the situation properly. To address this situation, we express parameters by SVTNNs.

Example 1. Consider a transportation problem in which we have two objectives with 2 sources and 3 destinations. The cost of transportation per vehicle is denoted by $\tilde{C}_1^N$ appeared in the first objective and amount of carbon dioxide (CO_2) emission per vehicle $\tilde{C}_2^N$ is appeared in the second objective. The neutrosophic fuzzy parameters related to this Example are summarized in Table 2. The supply of two origins and the demand of three destinations are all SVTN numbers given as follows:

Table 2. Neutrospphic fuzzy parameters for $\tilde{c}_{ij}^{(1)N}$ IF and $\tilde{c}_{ij}^{(2)N}$

$\tilde{c}_{11}^{(1)N} = \langle(20,30,40,50);0.8,0.3,0.6\rangle$ $\tilde{c}_{12}^{(1)N} = \langle(50,60,70,80);0.6,0.4,0.3\rangle$ $\tilde{c}_{13}^{(1)N} = \langle(80,90,110,120);0.7,0.2,0.5\rangle$	$\tilde{c}_{21}^{(1)N} = \langle(45,55,65,75);0.8,0.5,0.3\rangle$ $\tilde{c}_{22}^{(1)N} = \langle(55,65,90,105);0.7,0.4,0.5\rangle$ $\tilde{c}_{23}^{(1)N} = \langle(30,40,60,70);0.9,0.5,0.3\rangle$
$\tilde{c}_{11}^{(2)N} = \langle(8,12,14,18);0.7,0.4,0.3\rangle$ $\tilde{c}_{12}^{(2)N} = \langle(30,40,45,55);0.8,0.2,0.6\rangle$ $\tilde{c}_{13}^{(2)N} = \langle(18,24,30,36);0.6,0.2,0.5\rangle$	$\tilde{c}_{21}^{(2)N} = \langle(25,35,40,50);0.9,0.2,0.4\rangle$ $\tilde{c}_{22}^{(2)N} = \langle(11,16,20,25);0.6,0.3,0.5\rangle$ $\tilde{c}_{23}^{(2)N} = \langle(18,26,32,40);0.7,0.3,0.4\rangle$

$$\tilde{a}_1^N = \langle(60,80,100,120);0.8,0.3,0.4\rangle,\quad \tilde{a}_2^N = \langle(45,65,85,105);0.7,0.3,0.5\rangle$$

$$\tilde{b}_1^N = \langle(35,55,75,95);0.6,0.2,0.5\rangle,\quad b_2^N = \langle(20,30,40,50);0.9,0.4,0.6\rangle$$

$$\tilde{b}_3^N = \langle(50,60,70,80);0.6,0.2,0.7\rangle.$$

Now, the mathematical formulation of the problem can be stated as follows:

$$\begin{aligned} \min \quad & \tilde{Z}_1^N = \sum_{i=1}^{2}\sum_{j=1}^{3} \tilde{c}_{ij}^{(1)N} x_{ij} \\ \min \quad & \tilde{Z}_2^N = \sum_{i=1}^{2}\sum_{j=1}^{3} \tilde{c}_{ij}^{(2)N} x_{ij} \\ \text{s.t} \quad & \sum_{j=1}^{3} x_{ij} \le \tilde{a}_i, \qquad i = 1,2 \end{aligned} \tag{14}$$

$$\sum_{i=1}^{2} x_{ij} \geq \tilde{b}_j, \qquad j = 1,2,3$$
$$x_{ij} \geq 0, \quad \forall\, i = 1,2 \,\&\, j = 1,2,3.$$

Using the notion of accuracy function (4), the crisp version of problem (14) can be stated as follows:

$$\begin{aligned}
\min \quad & Z_1' = 27.125\, x_{11} + 40.625 x_{12} + 75 x_{13} + 45 x_{21} + 55.125 x_{22} + 37.5 x_{23} \\
\min \quad & Z_2' = 9.1\, x_{11} + 34\, x_{12} + 19.575\, x_{13} + 31.875\, x_{21} + 12.66\, x_{22} + 21.75\, x_{23} \\
s.t \quad & x_{11} + x_{12} + x_{13} \leq 65.25 \\
& x_{21} + x_{22} + x_{23} \leq 54.375, \\
& x_{11} + x_{21} \geq 47.125, \\
& x_{12} + x_{22} \geq 27.125 \\
& x_{13} + x_{23} \geq 50.375, \\
& x_{ij} \geq 0, \quad \forall\, i = 1,2 \,\&\, j = 1,2,3.
\end{aligned} \qquad (15)$$

The above problem is solved by taking only one objective function and neglecting the others. The solution sets are obtained as follows:

$z_1 = 4212.281,$

$x_{11} = 47.125, \quad x_{12} = 18.125, \quad x_{13} = 0, \quad x_{21} = 0, \quad x_{22} = 9, \quad x_{23} = 45.375.$

$z_2 = 1718.097,$

$x_{11} = 47.125, \quad x_{12} = 0, \quad x_{13} = 18.125, \quad x_{21} = 0, \quad x_{22} = 27.125, \quad x_{23} = 27.25.$

For each objective, the best and worst values are given as:

$U_1 = 5154.781, \quad L_1 = 4212.281, \; and \; U_2 = 2145.394, \quad L_2 = 1718.097.$

After constructing problem (13) using linear membership functions defined in relations (8-10) and considering $w_1 = w_2 = w_3 = \frac{1}{3}$, we solved it by Lingo software, the following solution is obtained:

$$\alpha = 0.541, \quad \beta = 0.495, \quad \gamma = 0.499,$$

$$x_{11} = 47.125, \quad x_{12} = 7.172, \quad x_{13} = 10.843,$$

$$x_{21} = 0, \quad x_{22} = 14.843, \quad x_{23} = 39.532.$$

7 | Conclusion

In this paper an effective modeling and optimization framework for the NFMOLPP is presented, where the coefficients of the objective functions, constraints and right-hand side parameters are single-valued trapezoidal neutrosophic numbers. In the proposed method, a ranking function of NNs is used to convert the NFMOLPP into an equivalent crisp MOLPP. Then, using the best and worst values of objectives, an appropriate membership function for each objective function is defined to avoid decision deadlock situation in hierarchical structure. In this regard, according to the concept of fuzzy decision set, an optimal compromise solution is selected as the design which it maximizes the degree of acceptance and minimizes the degree of rejection upto some extent and rejection degree by taking all objectives simultaneously. The proposed approach can be used to solve real-world problems arising in industries and business organizations with imprecise and contradictory information. Finally, a transportation problem has been discussed to show the applicability of proposed approach.

As future works, it will be meaningful to investigate the problem in some extended fuzzy environments, such as interval neutrosophic, neutrosophic hesitant, and type-2 neutrosophic environments. It is also interesting to apply the proposed approach for other optimization problems, such as shortest path problems, minimum cost flow problems.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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