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Optimization of Order Quantity for Multi-Product from Multi-Supplier with Discounted Prices

Ali Izadi[1], Seyed Mahdi Homayouni[2*], Mohammadreza Vasili[2]
[1] Department of Industrial Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran
[2] Department of Industrial Engineering, Lenjan Branch, Islamic Azad University, Isfahan, Iran

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A B S T R A C T

Many researches in inventory control area of knowledge have been focused on single objective and multi-objective problem of determining the economic quantity of order. In single objective problems, costs were considered as the objective. However, multi-objective problems have not been well investigated. For instance, there are no hint to transportation cost, budget, or holding costs, or only capacity and demand constraints have been considered in these researches. This study focuses on developing a model accompanied by costs, quality and the time of delivery.The economic order quantity of multi-product from multi-supplier in multi-period under uncertainty in demand and discounted prices are considered in this paper. In first step, a mathematical model is developed for this problem. This mathematical model is solved by using multi-objective optimization method *i.e.* goal programming. Then, a meta-heuristic method based on multi-objective particle swarm optimization is proposed. Results of the small size numerical examples show that solutions found by using the proposed meta-heuristic method are in average, 5% worse than solutions found by using the mathematical methods; however, it needs much lower computational time.

1. Introduction

Most of companies provide their raw materials and parts from suppliers and sellers (Karpak et al, 2001). Therefore,companies should be aware of this fact that for their own profit, they should concentrate close and long-term cooperation with their suppliers.Decision making on procurement of raw materials has a great effect on performance of companies.In this case,

*Corresponding author name: Seyed Mahdi HomayouniE-mail address:homayouni@iauln.ac.ir

economic order quantity has a direct effect on the company's profit; and if there is non-optimal decision on buying the products, company may face into a great financial losses (Mendoza and Ventura, 2012).

In most of industries, the cost of raw materials and parts cover a major part of rounded cost of products, in some cases it is up to 70% final product costs (Weele,2014). Thus, providing raw material and its inventory control have an important role on efficiency and effectiveness of companies. Some major factors must be kept in mind while determining order quantity include but not limited to delivery time, quality of product, suppliers capacity, geographical position of suppliers, and so on (Ghodsypour and O'Brien,2001).

Although many theoretical optimization models have been developed for this problem in recent years; these models are not applicable in real cases due to some unrealistic assumptions, and undefined parameters. According to Davis's(1993) classification, three types of uncertainties for demand, process and prices can be defined in supply chain management. Uncertainties in demand is the most important among others which is investigated in this research. This would be happen because of incorrect prediction of customers' demands in selling centers.

In this paper, a three-objective mathematical model is developed which is minimizing costs, defective products, and delivery times. The costs include procurement, ordering, holding and transportation costs. The model is developed for multiple products, multiple suppliers and multiple periods. Moreover, demand is stochastic and all-unit quantity discounted prices are considered in this model. In remainder of the paper, section 2 review of the main researches in this area of knowledge is presented. Section 3 include the modelling of the problem. In section 4, a meta-heuristic method based on particle swarm optimization (PSO) method for this problem is described. Numerical results of the small size test cases for both solving the mathematical model of the problem, and the meta-heuristic optimization method is presented. Finally, in section 6 conclusion remarks of the research is presented.

2. Literature Review

Most of the researches for order allocation considered single-objective related to the cost objective function. In this section, some of the related researches is reviewed. Ventura (2008) proposed a two-step method for selection and order allocation of suppliers. In the first step, suppliers are ranked using the analytical hierarchy process (AHP), and in the second step using a non-linear mathematical model, the order quantity is determined. Che and Wang (2008) investigated on the relations between assembly and order quantity using bill of material. The purpose of this research is to minimize the assembly time in addition to the raw material costs. They proposed a solving method based on genetic algorithm with multiple objectives, claiming that their method is qualitative, trustable and strong enough to solve applied real cases. Mendoza and Ventura (2012) have developed a nonlinear model for choosing suppliers and determining order allocation to each of them based on their capacity constraints.

On multi-objective problems, Karpak et al. (2001) investigated a goal programming model minimizing costs, defects, and delivery time. Susuz (2009) utilized a combination of AHP and nonlinear multi-objective programming model considering discounted prices and capacity and budget constraints to optimize order quantity. Rezaei and Davoodi (2011)proposed a multi-objective nonlinear integer model for multiple products and periods. This model optimize three objective functions include costs, quality and the service level.

In fuzzy models, Amid et al. (2006) developed a fuzzy multi-objective optimization model for choosing suppliers and determining optimized order quantity. Amid et al. (2009)formulated a multi-objective model considering discounted prices. Amid et al. (2011) proposed a fuzzy Max-min model. They used AHP to determine supplier selection criteria based on cost, quality and service, and the order quantity with discounted prices have been calculated. Razmi and Maghool (2009) developed a fuzzy model for multi-product, multi-period considering various kinds of discount policies. Mohammad Ebrahim et al. (2009) constructed a mathematical model includes all units discount, incremental discount, and total business volume discount policies, which demonstrated on multi- objective formulation for single buying examples.

Based on reviewed researches, lack of trust in input data is the most important uncertainty in this kind of problems. Yang et al. (2011) developed a model for multi-product order quantity allocation under stochastic demand, considering service level and budget constraints. Li and Zabinsky (2011)have developed various decision making models for supplier section under various probability distributions.

3. Problem Description and Formulation

The mathematical model for the problem is developed, in this section. For a buyer and multiple suppliers environment, a model aiming to minimize procurement, holding, order and transportation costs for the buyer, in presence of uncertainty in demand (considered as a uniform random variable) and discounted prices. In this problem, buyer needs to select one or more of the suppliers, and determines the order quantity of each product in each period. Suppliers can propose all-unit quality discount with determined levels. The capacity of suppliers for each product in each period is pre-determined and constrained. Based on Banerjee's (1986) suggestions, buyer follow batch ordering policy. In this policy, economic production quantity of the producers is equal to the economic order quantity of the buyer and is fixed in all the periods. Three objectives for this problem is formulized in the proposed mathematical model.

- Minimizing the total annual cost of the supply chain
- Minimizing the total number of defective products produced in the supply chain
- Minimizing the total number of late delivered products produced by the vendors

3.1. Model assumptions and notations

The following assumptions for the problem are considered:

- The buyer can purchase the required quantity from multiple suppliers.
- The buyer needs to buy multiple products from the suppliers.
- The suppliers offer all-unit quantity discount on the periodic order.
- Stock out is not allowed for both the buyer and the suppliers.
- Inventory can be transferred from a period to the other periods.
- Holding costs is calculated for the end of period inventories.
- Suppliers have limited capacity to provide products

Furthermore, the following parameters and decision variables are used in modeling the problem:

Index:

i Index for products, $i=1,2,3,\dots,m$

j Index for suppliers, $j=1,2,3,\dots,n$

t	Index for periods, *t*=1, 2, 3,…, *T*
k	Index of discount intervals, *k*=1, 2, 3,…, *K*

Model parameters:

P_{ij}^{k}	Discounted unit price of the discount interval *k* offered by supplier *j* for product *i*
a_{it}	Ordering costs for product *i* in period *t*
h_{it}	Holding costs of product *i* in period *t*
C_{ij}	Capacity of supplier *j* in production of product *i* per period
f_{ij}	Defective rate of the product *i* for supplier *j*
L_{ij}	Lateness rate of the product *i* for supplier *j*
A_{jt}	Transportation costs for supplier *j* per vehicle in period *t*
V_j	Vehicle capacity for supplier *j*
K_i	Space required to store product *i* in warehouse or vehicle
B_t	Budget constraint allocated in period *t*
$Dmin_{it}$	Lower bound of the demand of product *i* in period *t*
$Dmax_{it}$	Upper bound of the demand of product *i* in period *t*
$X_{max\,ijt}$	Upper bound of the order quantity
$X_{min\,ijt}$	Lower bound of the order quantity
$I_{max\,i}$	Total storage capacity for product *i*
u_{ijk}	Upper bound of the discount interval *k* offered by supplier *j* for product *i*

Decision variables:

Q_{ijt}^{k}	Order quantity per period *t* for product *i* from supplier *j* in discount interval *k*
Y_{ijt}^{k}	If the order quantity per period *t* from supplier *j* for product *i* falls on the interval corresponding to this variable = 1, otherwise =0
I_{it}	Inventory of the product *i* carried over from period *t* to period *t*+1 (I_{i0}=0).

3.2. Objective functions

The first objective function considered in this paper is the total supply chain annual cost. The sum of the procurement, ordering, holding and transportation costs in all periods should be minimized. Most of existing studies only include the first three types of costs and ignore transportation costs. It is noted that the total procurement costs are the sum of the procurement costs of all products from all selected suppliers in all periods. This objective function is presented in equation (1).

$$\text{Min } Z_c = \sum_i \sum_j \sum_k \sum_t P_{ij}^{k}.\,Q_{ijt}^{k} + \sum_i \sum_j \sum_k \sum_t a_{it}.\,Q_{ijt}^{k} + \sum_i \sum_t I_{it}\,.\,h_{it} + \sum_j \sum_t A_{jt} \left\lceil \frac{\sum_i \sum_k K_i.\,Q_{ijt}^{k}}{V_j} \right\rceil \quad (1)$$

The second objective function (shown in equation 2) minimizes the annual defective products purchased in the supply chain. This minimization leads to improving product quality and consequently more efficiency in the supply chain. The third objective function stated as the minimization of the products delivered lately to the buyer, in a year. Equation (3) represent the third objective.

$$\text{Min } Z_s = \sum_i \sum_j \sum_k \sum_t Q_{ijt}^{k}.\,f_{ij} \quad (2)$$

$$\text{Min } Z_l = \sum_i \sum_j \sum_k \sum_t Q_{ijt}^k . L_{ij} \quad (3)$$

3.3. Model constraints

Constraints (4) restrict the maximum order quantity of the products up to the capacity of product i produced by supplier j. Constraints (5) calculate the amount of inventory at the end of period t. Constraints (6) ensures that demand for product i must be met in period t. Constraints (7) restricts all the costs for the buyer up to budget in period t. Constraints (8) show that the buyer has a limited storage capacity in each period should be considered. Constraints (9) sets the minimum and maximum for order quantities. Constraints (10, 11, and 12) describe how the quantity ordered to each supplier falls into one of the intervals offered by the supplier. Constraints (13) state that decision variables are non-negative.

$$\sum_k Q_{ijt}^k \leq C_{ij} \qquad \forall i,j,k,t \quad (4)$$

$$I_{it} = \sum_j \sum_k Q_{ijt}^k \left(1 - f_{ij}\right) - \overline{D_{it}} + I_{i(t-1)} \qquad \forall i,t \quad (5)$$

$$I_{i(t-1)} + \sum_j \sum_k Q_{ijt}^k . (1 - f_{ij}) \geq \overline{D_{it}} \qquad \forall i,t \quad (6)$$

$$\sum_i \sum_j \sum_k P_{ijk} . Q_{ijt}^k \leq B_t \qquad \forall t \quad (7)$$

$$\sum_i \sum_j \sum_k Q_{ijt}^k . (1 - f_{ij}) + I_{i(t-1)} \leq I_{max\, i} \qquad \forall i \quad (8)$$

$$X_{min_{ijkt}} \leq Q_{ijt}^k \leq X_{max_{ijkt}} \qquad \forall i,j,k,t \quad (9)$$

$$u_{ij(k-1)} . Y_{ijt}^k \leq Q_{ijt}^k < u_{ijk} . Y_{ijt}^k \qquad \forall i,j,k,t \quad (10)$$

$$\sum_k Y_{ijt}^k = 1 \qquad \forall i,j \quad (11)$$

$$u_{ij0} = 0 \qquad \forall i,j \quad (12)$$

$$I_{i0} = 0 \; I_{it} \geq 0 \qquad \forall i \quad (13)$$

To solve the mathematical model more efficiently, the model is converted into an unconstrained model. In this paper, using the concept of violation (Smith and Coit, 1996), the constraints are converted into an objective function. In this method, first the violation rate for each constraint is calculated. In second step, the mean of violation rates as the new objective function for all the constraints is calculated. Equation (14) shows the forth objective function of the model. If the forth objective equals to zero, it means that the solution found for the problem is feasible. In last step of modelling for this problem, using the method proposed by Koziel and Michalewicz (1997), a four-objective model (shown in equation 15) which should be optimized simultaneously, is provided.

$$\begin{aligned} &\overline{\mathrm{V_T}} = (\mathrm{V_c} + \mathrm{V_d} + \mathrm{V_b} + \mathrm{V_l})/4 \\ &\text{Min } \overline{\mathrm{V_T}} \end{aligned} \quad (14)$$

$$\begin{aligned} &\text{Min } \mathrm{Z_c} \\ &\text{Min } \mathrm{Z_s} \\ &\text{Min } \mathrm{Z_l} \end{aligned} \quad (15)$$

$\text{Min } V_T$

4. The Proposed MOPSO

Knowles and Corne (1999) initially proposed the swarm strategy for optimization. Particle swarm optimization is a stochastic optimization technique that draws inspiration from the behavior of a flock of birds or the collective intelligence of a group of social insects with limited individual capabilities. In PSO, individuals, referred to as particles, are "flown" through hyper dimensional search space. Changes to the position of the particles within the search space are based on the social psychological tendency of individuals to emulate the success of other individuals. A swarm consists of a set of particles, where each particle represents a potential solution. The position of each particle is changed according to its own experience and that of its neighbors.

PSO has been found to be successful in a wide variety of optimization tasks but until recently it had not been extended to deal with multiple objectives. Multi objective particle swarm optimization (MOPSO) is an approach in which Pareto dominance is incorporated into PSO. This method allows the application of PSO for the problems with multiple objectives (Coello et al, 2004). It uses a secondary (i.e., external) repository of particles that is later used by other particles to guide their own flight. MOPSO seems particularly suitable for multi-objective optimization mainly because of the high speed of convergence that the algorithm presents for single objective optimization (Knowles and Corne, 1999).

4.1. Description of the Algorithm

PSO using a Pareto ranking scheme (Knowles and Corne, 1999) could be the straightforward way to extend the approach to handle multi-objective optimization problems. The historical record of best solutions found by a particle (i.e., an individual) could be used to store non-dominated solutions generated in the past. The use of global attraction mechanisms combined with a historical archive of previously found non-dominated vectors would motivate convergence toward globally non-dominated solutions. Flow chart of the MOPSO is shown in fig 1.

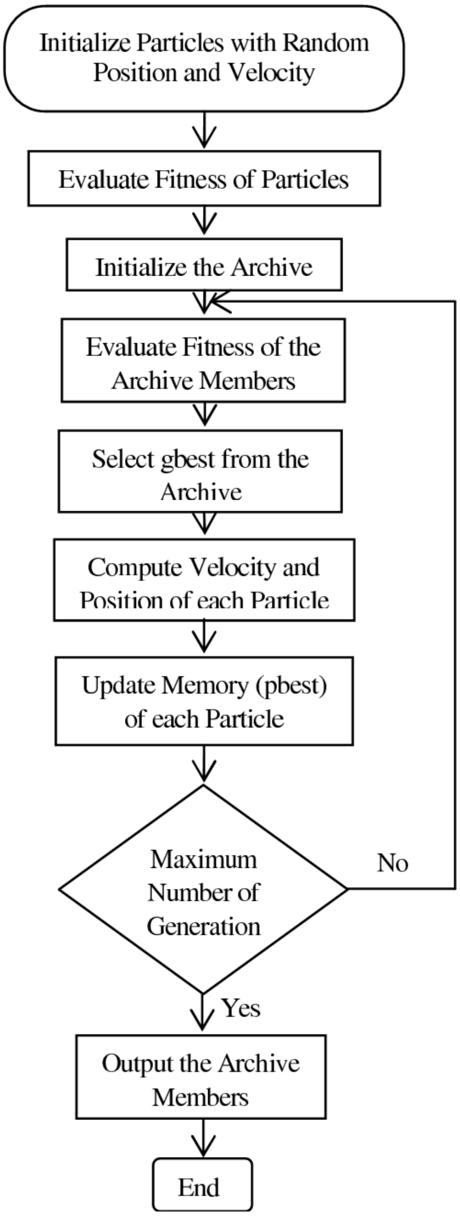

Fig 1 MOPSO Flow Chart

4.2. MOPSO Parameter Tuning

In the proposed MOPSO algorithm there are nine parameters that should be tuned. These parameters are as followings. By performing various preliminary tests the best value for each parameter can be determined as shown in table (1). For example, for results of the test performed to find the suitable maximum number of iterations is shown in fig 2.

- Maximum number of iterations (M)
- Population size (P)
- Repository size (R)
- Inertia weight (w)
- Personal learning coefficient (c1)
- Global learning coefficient (c2)
- Leader selection pressure (beta)

- Deletion selection pressure (gamma)
- Mutation rate (r)

Table 1 MOPSO parameter tuning

M	P	R	W	c(1)	c(2)	beta	gamma	r
1000	15	100	2	2	1	1	.2	.5

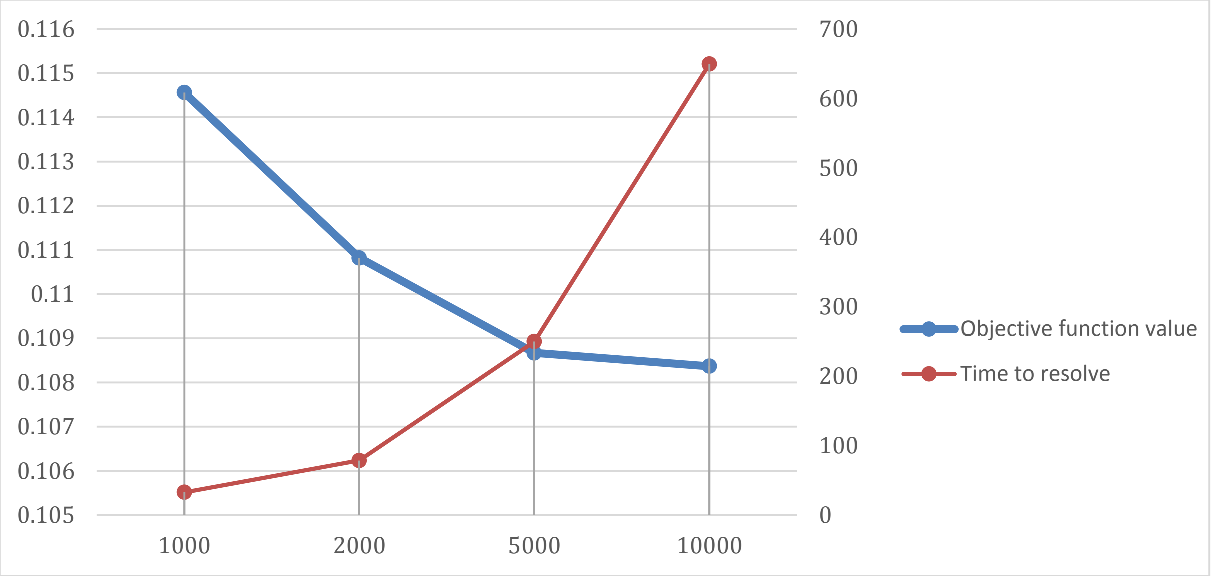

Fig 2 Number of Iterations versus Computational Time

5. Numerical example

Here, the efficiency and applicability of the MOPSO is illustrated by a set of numerical examples. 10 examples, based on the information presented in table (2) has been created. For comparison purposes, all 10 examples is solved by using “goal programming” method. The relative percentage deviation (PD) as a common performance measure to evaluate the algorithms is used. PD shows that how much an algorithm is different from the best obtained solution on average and it is calculated according to the equation (16).

Table 2 Input data for construction of numerical examples

Parameter	**f**	**l**	**a**	**c**	**h**	**A**	***k***	**V**	**B**	**D**	**Imax**	**p**
Select a range of initial values	[0. 3]	[.001-.3]	[.03-6]	[10-100] *10^3	[.03-6]	[20-600]	[1-10]	[200-500] *10^3	[1-10] *10^6	[0 80] *10^3	[2 9] *10^3	According discounts
Percent increase over the period	--	--	20%	--	20%	20%	--	--	20%	--	--	--

$$PD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} * 100 \qquad (16)$$

Obviously from the table (3), as the complexity of the examples increases (the number of variables more than 72), classic method "goal programming” cannot find a feasible solution. While in MOPSO, output solutions will improve by incensement of variables numbers. Based on percentage deviation of the first examples, it is concluded that far initiative algorithm have pleasant results. Because in small size examples results obtained by two methods are closed to each other, the quality of slotions found by the MOSO to large size examples. In the table, negative percentage deviation stated a better function of MOPSO algorithm in comparison to goal programming. Objective function is normalized and illustrated in equation (17).

$$\textbf{Objective function} = \alpha * {Z_c}/{Z_c^{max}} + \beta * {Z_s}/{Z_s^{max}} + \delta * {Z_l}/{Z_l^{max}} + \gamma * {V_T}/{V_T^{max}} + \alpha + \beta + \delta + \gamma = 1 \quad (17)$$

Table 3 The numerical results of small size examples solved by goal programming & MOPSO methods

Problem	Number of products	Number of supplier	Period	Discount intervals	Variable	Goal Programming		MOPSO			PD%
						Objective function	CPU time(s)	Objective function	Number of Iterations	CPU time(s)	
1	1	3	1	4	12	0.0098	6	0.0107	500	13	*9.18%*
2	2	1	2	4	16	0.0205	8.5	0.0221	500	16	*7.80%*
3	2	2	1	4	16	0.0251	8	0.0261	500	16	*3.98%*
4	2	3	1	4	24	0.0259	42	0.0266	500	25	*2.70%*
5	2	2	2	4	32	0.0567	383	0.056	500	25	*-1.23%*
6	2	3	2	4	48	0.0605	450	0.06	500	30	*-0.83%*
7	3	2	2	4	48	0.0082	892	0.008	500	30	*-2.44%*
8	3	2	3	4	72	0.0152	1170	0.0135	500	66	*-11.18%*
9	3	3	3	4	108	No	-	0.0279	1000	120	..
10	3	1	10	4	120	No	-	0.052	1000	72	..

6. Conclusion

In this article, a multi-objective functions mathematical model has been developed considering uniform random demand and all unit discounted prices. The purpose of this model is to minimize delivery time of products; also some constraints considered such as production capacity of suppliers, buying budget, buyer's storage capacity, minimum and maximum of order quantity. Major improvements of the proposed model compared to the existing models include planning for multiple periods, stochastic demand for products, discounted prices of the products and considering transportation's cost in the objective function. Ten numerical small size examples have been created using random data. These examples have been solved by goal programming and MOPSO methods. The results of both methods for these examples are similar. However, in some cases MOPSO gained better solutions than goal programming method due to the complexity of the examples.

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