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A New Technique for Investigating the Dynamic Response of a Beam Subjected to a Load-Moaving System

Ibrahim Mousa Abu-Alshaikh *

Department of Mechanical Engineering, University of Jordan, Amman-11942, Jordan.

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ABSTRACT

The dynamic response of a homogeneous elastic simply-supported beam subjected to a load system moving with a uniform velocity is studied in detail in this paper. Analytical expressions for the dynamic responses of the beam and the load-moving system are obtained by means of a new technique using decomposition method whereby the generalized displacement of the beam is written as an infinite series. The method is versatile and simple so that its application to other related problems is possible. Comparisons between different cases of load-moving systems are made clear. Interaction, load, mass, velocity effects on the beam as well as on the load-moving system are investigated. It is concluded that the inertia effect of the load-moving system cannot be neglected when the traveling velocity and its mass ratio to that of the beam are large.

1. Introduction

The general problem of predicting the transverse vibrations of continuous media resulting from the passage of moving loads is of considerable practical interest in the dynamics of structures. This problem has many applications in engineering systems. Some of the applications are vibrations occur in bridges and railroad tracks due to moving vehicles, vibrations occur in piping systems due to fluid flow, machining operations, machine chains and belt drives, thermal processing subjected to moving heat sources, computer disk drives, and video cassette recorders. In general, three types of fundamental problems are usually considered in this type of work: (a) the moving force problem, (b) the moving mass problem, and (c) the moving oscillator problem. The moving force is the simplest model. In this model, the basic dynamic characteristics of the beam are obtained. The drawback here is the interaction between the beam and the moving subsystem is ignored. This model is widely used when the mass of the subsystem in comparison to the mass of the beam is sufficiently small. The moving mass problem is considered to be an improvement to the moving force problem. The inertia of the subsystem is accounted for in this model. The moving oscillator problem is the most general of these where the coupling stiffness between the moving subsystem and the beam is finite. Over the past few decades, different aspects of this problem have been dealt with. However, almost all the literature in this field can be easily categorized in either one of the aforementioned three types. A complete review of literature in this field is certainly beyond the scope of this paper. In here, some of the work most pertinent to the

* Corresponding author
E-mail: i.abualshaikh@ju.edu.jo
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subject matter of this paper will be discussed. The text by Fryba [1] contained a variety of problems related to the moving loads on structures and a large number of references. With regard to the first type, different problems have been solved [2-7]. Perhaps the most important problem is the one that deals with inherent randomness due to material properties, force, support, and speed. Extensions to rotating beams and Timoshenko beams have been discussed in [8-10]. The vibration response of beams on elastic foundation due to moving force have been discussed in [11, 12]. The response of a simply supported beam subjected to a moving mass using a dynamic Green's function approach is treated in [13]. The same problem, using an approximation based on modal analysis is presented in [14]. The problem of an elastic system subjected to a moving oscillator is studied in [15]. In this paper, the problem is formulated using the relative displacement model in which the authors showed that in the limiting case the moving mass problem is recovered. A similar problem using an improved series expansion in terms of the eigenfunctions of the distributed system is presented in [16].

This paper presents an extension to the aforementioned work. A homogeneous elastic simply supported beam subjected to a load-moving system modeled as a sprung mass as shown in Figure 1 is investigated. Furthermore, due to the spring attachment the dynamic interaction effect between the moving system and the beam is studied. The solution is obtained by means of the decomposition method. The basic idea of this method is to decompose the dependent variable, which is the generalized displacement in this case into an infinite series. The components of the series are then found recursively. Similar ideas of decomposing dependent variables are implemented in solving nonlinear functional equations of various kinds [17, 18].

2. Formulation

The problem to be considered is that of transverse vibrations of a simply supported uniform homogeneous elastic Bernoulli-Euler beam of finite length originally at rest. The beam is acted upon by a load moving system which travels from left to right with a uniform velocity v, as shown in Fig 1. The governing equations of the beam and the moving system are:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \mu\frac{\partial^2 w(x,t)}{\partial t^2} = P(t)\delta(x-vt) \tag{1}$$

$$m\ddot{z}(t) + kz(t) = kw(vt,t). \tag{2}$$

Where E is the modulus of elasticity, I is the moment of inertia of the cross-section, μ is the mass per unit length, $w(x,t)$ is the transverse dynamic displacement of the beam at point x and time t, $P(t)$ is the load of the moving system, v is the traveling speed of the load, and g is the gravitational acceleration. The $\delta(.)$ is the familiar delta function, m is the mass of the moving system, k is the stiffness of the suspension of the system, and z is the vertical displacement of the moving system. The load of the moving system $P(t)$ is written for the case of the moving oscillator problem as:

$$P(t) = mg - k\left(z(t) - w(vt,t)\right) \tag{3}$$

For the case of the moving mass and moving force problems, this load is written respectively as:

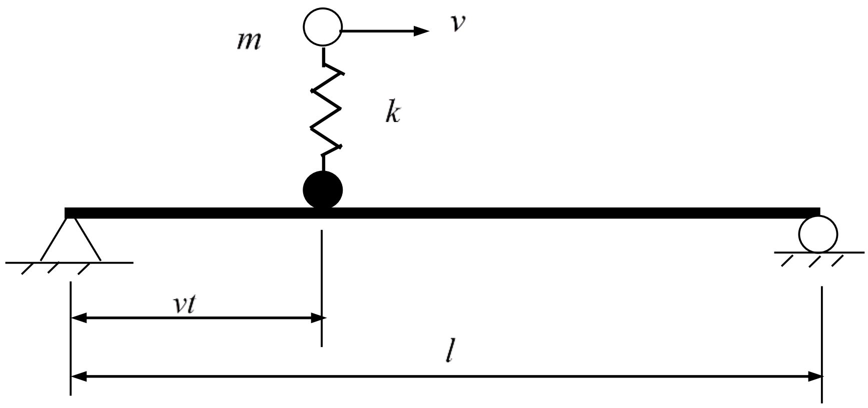

Fig 1. Load moving system model over a simply supported beam.

$$P(t) = mg - m\frac{\partial^2 w(vt,t)}{\partial t^2} \tag{4}$$

$$P(t) = mg \tag{5}$$

It should be noted that Eqs. (4-5) can be obtained as special cases from Eq. (3). In modal form, the transverse displacement of the beam for the case of the moving mass problem is written as:

$$w(x,t) = \sum_{n=1}^{\infty} Y_n(t)\sin\frac{n\pi x}{l}. \tag{6}$$

Where $Y_n(t)$ is the generalized displacement or the modal response of the beam. Substituting Eq. (6) into Eq. (1) to obtain:

$$EI\sum_{n=1}^{\infty} Y_n(t)\left(\frac{n\pi}{l}\right)^4 \sin\frac{n\pi x}{l} + \mu\sum_{n=1}^{\infty}\ddot{Y}_n(t)\sin\frac{n\pi x}{l} = mg\delta(x-vt)$$
$$-m\left[\sum_{n=1}^{\infty}\sin\frac{n\pi x}{l}\ddot{Y}_n(t)\right]\delta(x-vt) \tag{7}$$

Multiplying both sides of Eq. (7) by $\sin\frac{k\pi x}{l}$, integrating over the domain, and considering the orthogonality conditions, the differential equation of the nth mode of the generalized displacement is written as:

$$\ddot{Y}_n(t) + \omega_n^2 Y_n(t) = \Psi g\sin(\Omega_n t) - \Psi\sin(\Omega_n t)\sum_{k=1}^{\infty}\sin(\Omega_k t)\,\ddot{Y}_k(t). \tag{8}$$

Where ω_n is the natural frequency of the beam defined as

$$\omega_n^2 = \left(\frac{n\pi}{l}\right)^4 \frac{EI}{\mu}. \tag{9}$$

In addition, the parameters Ψ and Ω_n are defined as

$$\Psi = 2m/\mu l\,,\ \Omega_n = n\pi v/l \tag{10}$$

It should be noted that Eq. (8) satisfies the initial conditions $Y_n(0) = 0$, $\dot{Y}_n(0) = 0$. Now a solution of Eq. (8) is sought. Various methods have been presented in the literature to solve Eq. (8) [13, 14]. In here, the generalized displacement of the beam subjected to a moving mass is presented by decomposing the solution of Eq. (8) into an infinite series of the form below:

$$Y_n(t) = \sum_{j=0}^{\infty} Y_n^j(t). \tag{11}$$

Where $Y_n^j(t);\ j = 0, 1, 2, ...$ are the components of $Y_n(t)$ to be determined recursively. The recursive relations for the first three components of Eq. (11) can be found by substituting Eq. (11) into Eq. (8) to obtain:

$$\ddot{Y}_n^{\mathbf{0}}(t) + \omega_n^{\mathbf{2}} Y_n^{\mathbf{0}}(t) = \Psi g \sin(\Omega_n t) \tag{12.a}$$

$$\ddot{Y}_n^1(t) + \omega_n^2 Y_n^1(t) = -\Psi \sin(\Omega_n t) \sum_{k=1}^{\infty} \sin(\Omega_k t)\, \ddot{Y}_k^0(t), \tag{12.b}$$

$$\ddot{Y}_n^{\mathbf{2}}(t) + \omega_n^{\mathbf{2}} Y_n^{\mathbf{2}}(t) = -\Psi \sin(\Omega_n t) \sum_{k=\mathbf{1}}^{\infty} \sin(\Omega_k t)\, \ddot{Y}_k^{\mathbf{1}}(t), \tag{12.c}$$

In compact form, these relations are written for $j \ge 0$ as

$$\ddot{Y}_n^{j+1}(t) + \omega_n^2 Y_n^{j+1}(t) = -\Psi \sin(\Omega_n t) \sum_{k=1}^{\infty} \sin(\Omega_k t)\, \ddot{Y}_k^j(t). \tag{13}$$

Where Eq. (12.a) is still valid for $Y_n^{\mathbf{0}}(t)$. The exact solution of Eq. (12 a) can be obtained using Duhamel integral as

$$Y_n^{\mathbf{0}}(t) = \frac{\Psi g}{\omega_n^{\mathbf{2}} - \Omega_n^{\mathbf{2}}} \left(\sin(\Omega_n t) - \frac{\Omega_n}{\omega_n} \sin(\omega_n t) \right) \tag{14}$$

Substituting Eq. (14) into the right hand side of Eq. (12 b) to obtain

$$\ddot{Y}_n^{\mathbf{1}}(t) + \omega_n^{\mathbf{2}} Y_n^{\mathbf{1}}(t) = g\Psi \left\{ -\mathbf{2}\sin(\Omega_n t) \sum_{k=\mathbf{1}}^{\infty} A_k + \sum_{k=\mathbf{1}}^{\infty} \left\{ \begin{array}{l} A_k \sin((\Omega_n + \mathbf{2}\Omega_k)t) + \\ A_k \sin((\Omega_n - \mathbf{2}\Omega_k)t) + \\ B_k \sin((\omega_k + \Omega_n - \Omega_k)t) - \\ B_k \sin((\omega_k + \Omega_n + \Omega_k)t) - \\ B_k \sin((-\omega_n + \Omega_n - \Omega_n)t) + \\ B_k \sin((-\omega_n + \Omega_n + \Omega_n)t) \end{array} \right\} \right\}, \tag{15}$$

In which,

$$A_k = \frac{\Psi\, \Omega_k^2}{4\left(\Omega_k^2 - \omega_k^2\right)},\ B_k = \frac{\Psi\, \Omega_k \omega_k}{4\left(\Omega_N^2 - \omega_k^2\right)}. \tag{16}$$

Where $\ddot{Y}_k^0(t)$ in Eq. (12. b) is found easily from Eq. (14). It follows that the solution for the second component of the generalized displacement, $Y_n^{\mathbf{1}}(t)$, is obtained as

$$Y_n^1(t,\beta) = g\Psi\left\{\begin{array}{l} \dfrac{-2}{\omega_n^2-\beta^2}\left(\sin(\beta t)-\dfrac{\beta}{\omega_n}\sin(\omega_n t)\right)\sum_{k=1}^{\infty}A_k + \\ \sum_{k=1}^{\infty}\dfrac{A_k\left(\sin((\beta+2\Omega_k)t)-\dfrac{(\beta+2\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(\beta+2\Omega_k)^2} + \\ \sum_{k=1}^{\infty}\dfrac{A_k\left(\sin((\beta-2\Omega_k)t)-\dfrac{(\beta-2\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(\beta-2\Omega_k)^2} - \\ \sum_{k=1}^{\infty}\dfrac{B_k\left(\sin((\omega_k+\beta+\Omega_k)t)-\dfrac{(\omega_k+\beta+\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(\omega_k+\beta+\Omega_k)^2} + \\ \sum_{k=1}^{\infty}\dfrac{B_k\left(\sin((-\omega_k+\beta+\Omega_k)t)-\dfrac{(-\omega_k+\beta+\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(-\omega_k+\beta+\Omega_k)^2} + \\ \sum_{k=1}^{\infty}\dfrac{B_k\left(\sin((\omega_k+\beta-\Omega_k)t)-\dfrac{(\omega_k+\beta-\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(\omega_k+\beta-\Omega_k)^2} - \\ \sum_{k=1}^{\infty}\dfrac{B_k\left(\sin((-\omega_k+\beta-\Omega_k)t)-\dfrac{(-\omega_k+\beta-\Omega_k)}{\omega_n}\sin(\omega_n t)\right)}{\omega_n^2-(-\omega_k+\beta-\Omega_k)^2} \end{array}\right\} \tag{17}$$

Where $\beta = \Omega_n$ in Eq. (17). Similarly, the solution for the third component of the generalized displacement, $Y_n^2(t)$, is written as:

$$Y_n^2(t) = g\Psi\left\{\begin{array}{l} -2\,Y_n^1(t,\Omega_n)\sum_{k=1}^{\infty}A_k + \sum_{k=1}^{\infty}A_k\;Y_n^1(t,\Omega_n+2\Omega_k) + \\ \sum_{k=1}^{\infty}A_k Y_n^1(t,\Omega_n-2\Omega_k) - \sum_{k=1}^{\infty}B_k Y_n^1(t,\omega_k+\Omega_n+\Omega_k) + \\ \sum_{k=1}^{\infty}B_k Y_n^1(t,-\omega_k+\Omega_n+\Omega_k) + \sum_{k=1}^{\infty}B_k Y_n^1(t,\omega_k+\Omega_n-\Omega_k) - \\ \sum_{k=1}^{\infty}B_k Y_n^1(t,-\omega_k+\Omega_n-\Omega_k) \end{array}\right\} \tag{18}$$

The analytical results presented in Eqs. (14, 17, 18) and the numerical solutions obtained suggest that one can neglect higher order terms in the decomposition and consider only the first three terms with sufficient degree of accuracy. Thus, the generalized displacement of the beam is obtained by substituting Eqs. (14, 17, 18) into Eq. (11). It then follows that the transverse dynamic displacement of the beam is obtained directly from Eq. (6). It should be mentioned that the solution to the moving force problem is obtained only from the first term of Eq. (11). The velocity and acceleration responses of the beam are obtained by differentiating Eq. (11) once and twice with respect to time.

The dynamic response of the load-moving system is obtained from Eq. (2) which is rewritten as

$$\ddot{z}(t)+\omega_L^2 z(t)=\omega_L^2\sum_{n=1}^{\infty}\sin\left(\frac{n\pi v t}{l}\right)Y_n(t). \tag{19}$$

Where ω_L is the natural frequency of the load moving system defined as

$$\omega_L^2=\frac{k}{m} \tag{20}$$

Using the Duhamel integral, the displacement response of the moving system can be obtained

$$z(t)=\omega_L\int_0^t\sum_{n=1}^{\infty}\sin\left(\frac{n\pi v\tau}{l}\right)Y_n(\tau)\sin\omega_L(t-\tau)d\tau \tag{21}$$

Where $Y_n(t)$ is substituted in Eq. (21) from the results obtained in Eq. (11). The velocity and acceleration responses of the load-moving system are obtained by differentiating Eq. (21) once and twice with respect to time.

3. Numerical Results and Discussion

To clarify the analysis and results arrived at in this paper several graphs representing different cases are shown. The following parameters are adopted for the beam under investigation. The moment of inertia, modulus of elasticity $E=3.3\times10^{10}\,N/m^2$, and the mass per unit length of the beam $\mu=5000\,kg/m$. The loading system is assumed to have three masses *m* = 25000kg, 75000kg, or 100000 kg; and the spring constant of the loading system $k=20\times10^6\,N/m$. Three cases of traveling speeds are considered; that is, *v*= 25*m/s, v*= 75*m/s, and v*= 100*m/s.* The traveling speed may be defined in terms of a dimensionless speed parameter α, which is written as

$$\alpha=\frac{v}{v_{cr}} \tag{22}$$

Where v_{cr} is the critical speed, defined as

$$v_{cr}=\frac{\omega_1 l}{\pi} \tag{23}$$

Thus the traveling speeds considered correspond to speed parameters $\alpha=0.13$, $\alpha=0.38$, $\alpha=0.5$ respectively. In Figures 2 (a-c) and 3(a-c) are shown the displacement and acceleration responses for the beam versus the normalized time parameter $s=\frac{tl}{v}$. Here the figures corresponding to the velocity response are not shown for brevity. When *s* =0, the load moving system is at the left-hand side of the beam $x=0$, and when *s* =1 the load moving system is at the right-hand side of the beam $x=l$. Figures 4 (a-c) and 5(a-c) are similar to Figures 2(a-c) and 3(a-c) but for the load moving system. In Fig 2(a) is shown the dynamic mid-span displacement response of the beam for the three cases of traveling speeds where the mass of the loading system *m*=25000kg. In here, the mass ratio of the loading system to the mass of the beam, $m/\mu l$, is equal to 0.1. It is shown for this case and for both the force and mass moving systems (including inertia effect) that the mid-span displacements of the beam overlap for the case of small traveling speed with slight difference when the speed increases. In other words and for this case, the inertia effect or the interaction effect of the moving system has no effect or a limited effect on the dynamic displacement of the beam. Similar to Figure 2(a), Figures 2(b, c) show the mid-span displacement response of the beam for both the force and mass moving systems but for *m*=75000kg and *m*=100000kg, respectively. These two masses correspond to mass ratios of 0.3 and 0.4, respectively. In these figures, it is shown that the dynamic displacement response increases as the mass ratio of the loading system to the mass of the beam and the traveling speed increase. It is also clear that the inertia effect of the mass of the moving system or the interaction effect becomes more pronounced as the mass of the loading system as well as the traveling speed increase.

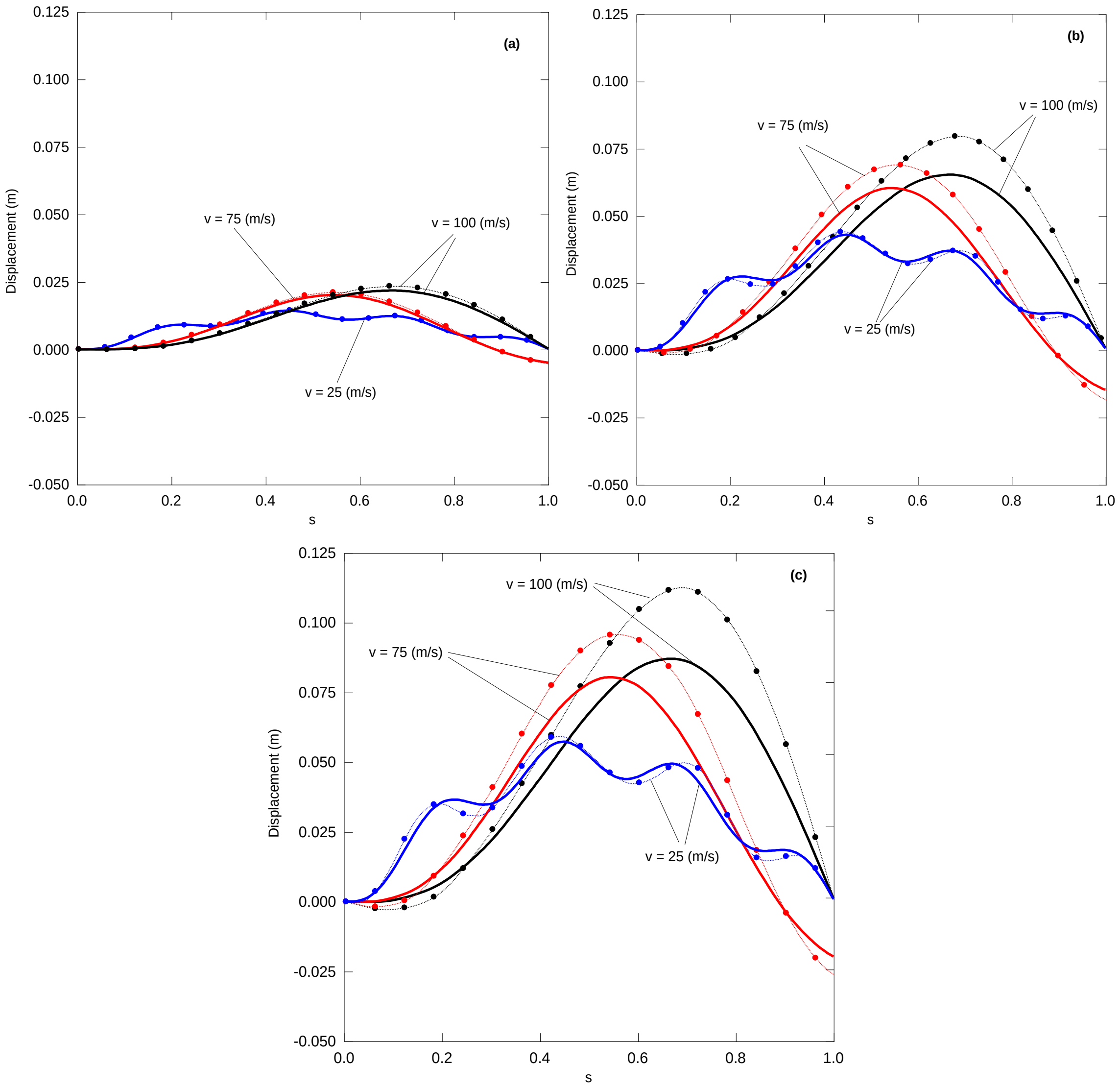

Fig 2. Beam mid-span displacement response versus the dimensionless time.
(a) *m*=25000 *kg*, (b) *m*= 75000 *kg*, (c) *m*= 100000 *kg*; *v*= 25 *m/s*, *v*= 75 *m/s*, *and v*= 100 *m/s.*
(⋯●⋯) moving mass problem; (———) moving force problem.

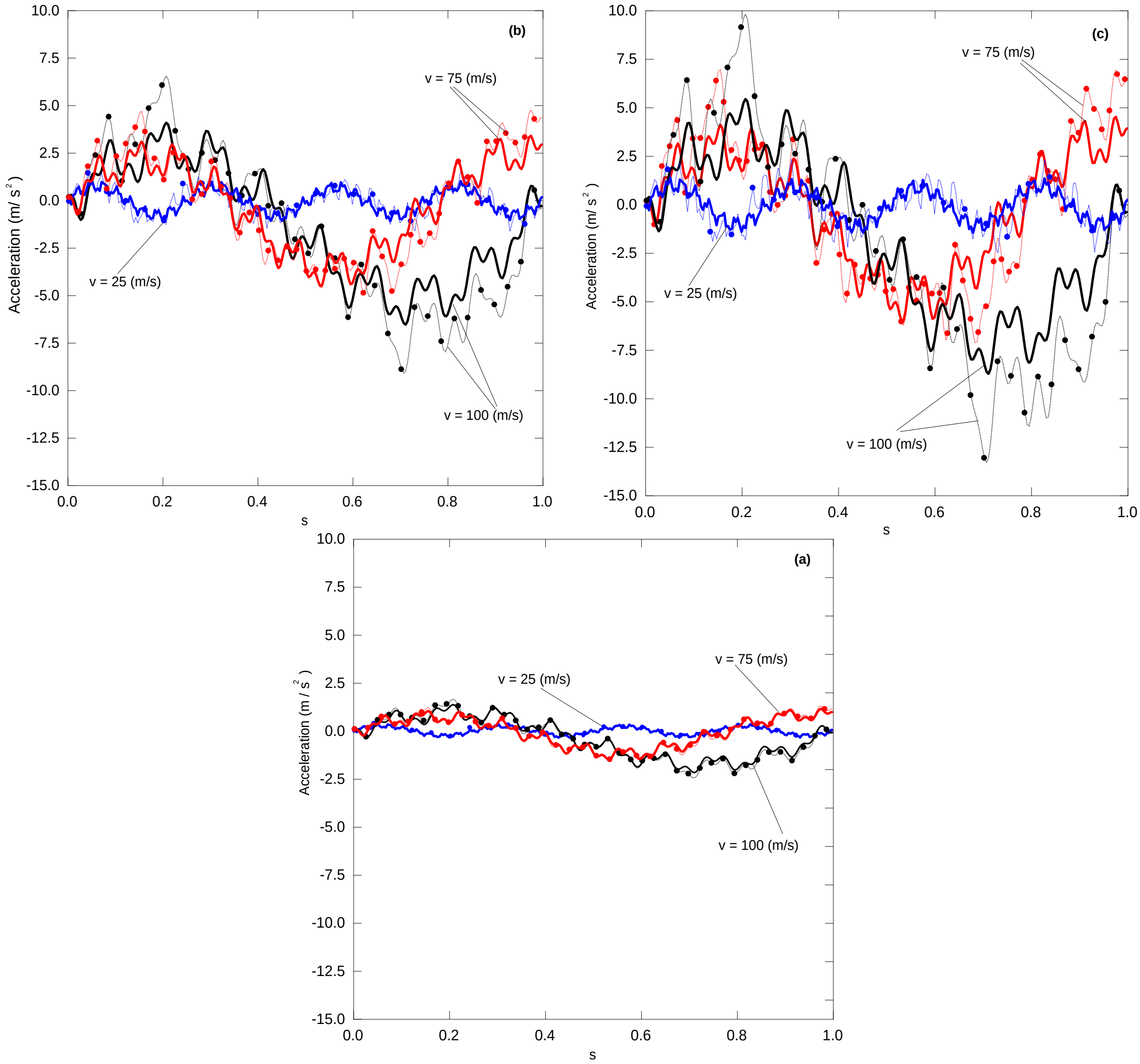

Fig 3. Beam mid-span acceleration response versus the dimensionless time.
(a) m=25000 *kg*, (b) m= 75000 *kg*, (c) m= 100000 *kg*; v= 25 *m/s*, v= 75 *m/s, and* v= 100 *m/s* .
(┈┈●┈┈) moving mass problem; (▬▬) moving force problem.

Figures 2(a-c), show also that the amplitude of the displacement response for the force and mass moving systems increases as speed increases. This follows from the fact that the energy content of the beam increases when speed increases. This observation becomes more evident for the case of mass moving system and this is obviously due to the inertia or the interaction effect. As can be seen from Figures 3(a-c), the acceleration response of the beam is more sensitive to the inertia effect or interaction effect. This becomes quite clear for larger speeds. One can conclude that adopting the moving force model for the moving system is sufficient as long as the mass ratio of the moving system and the traveling speed are small enough. Once this ratio and the traveling speed become large, the interaction effect or the inertia effect can not be neglected, and hence a more advanced model for the moving system is to be considered. The acceleration as well as the velocity analysis can further be utilized to estimate the service life of the beam. The displacement, and acceleration responses of the moving system are shown in Figures 4(a-c) and 5(a-c). It is worth mentioning here that the results presented for the moving system are approximate since the beam generalized displacement is obtained first and then substituted in Eq. (21) to obtain the displacement response of the moving system. The velocity and acceleration responses are obtained by taking the time derivatives of Eq. (21). Nevertheless, this type of study helps in understanding the interaction phenomenon between the beam and the moving system and the various parameters affecting it. Comparing Figures 2(a-c) and 4(a-c), the resemblance between the beam displacement and that of the moving system is clear. One can obtain a good idea about the beam response or the beam frequency from that of the moving system. In fact, one can use the velocity or the acceleration responses to obtain this type of information. In this case further analysis should be performed where frequencies are clearly identified.

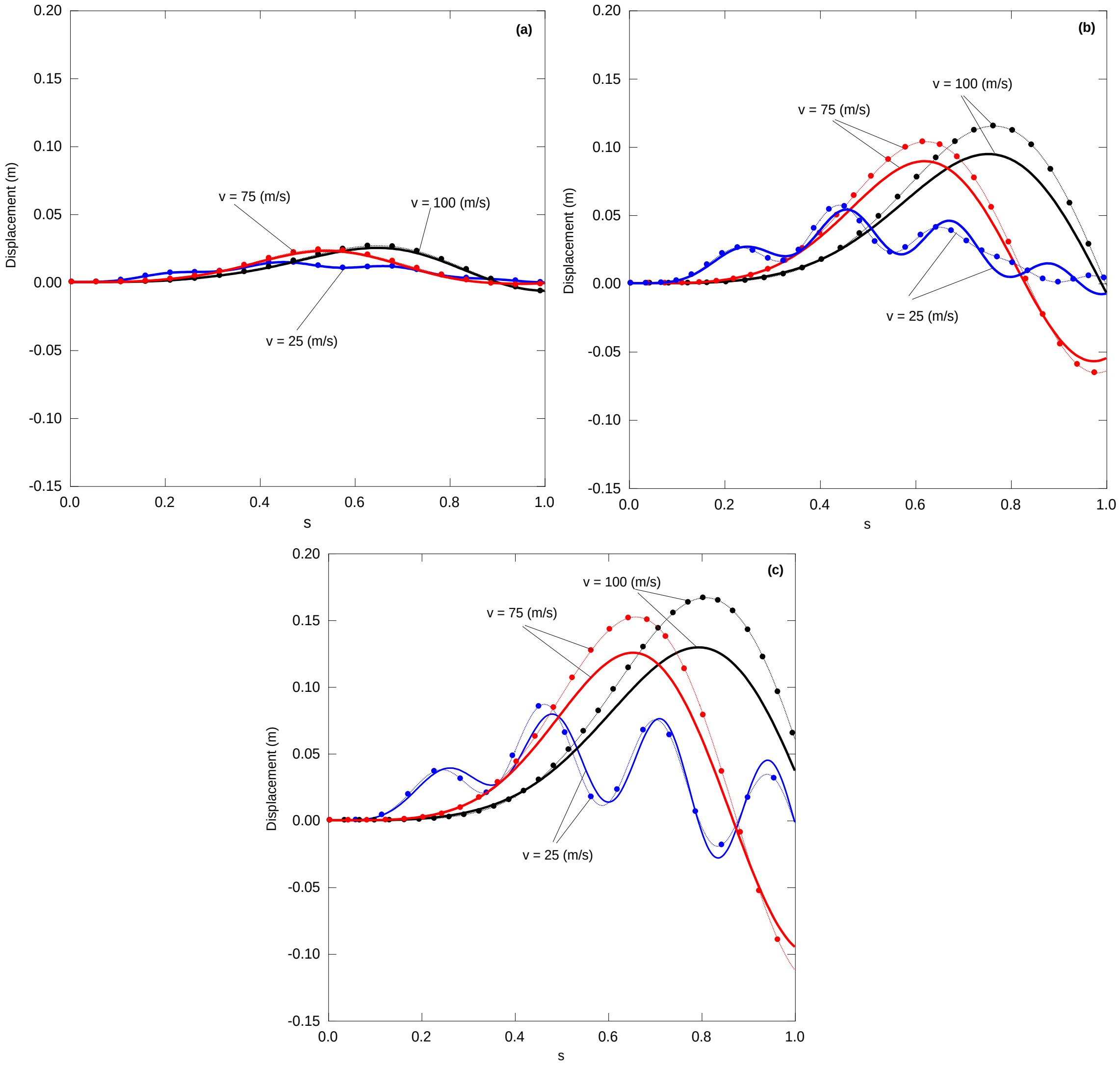

Fig 4. Moving system mid-span displacement response versus the dimensionless time.
(a) *m*= 25000 *kg*, (b) *m*= 75000 *kg*, (c) *m*= 100000 *kg*; *v*= 25 *m/s*, *v*= 75 *m/s, and v*=100 *m/s.*
(·····●·····) moving mass problem; (———) moving force problem.

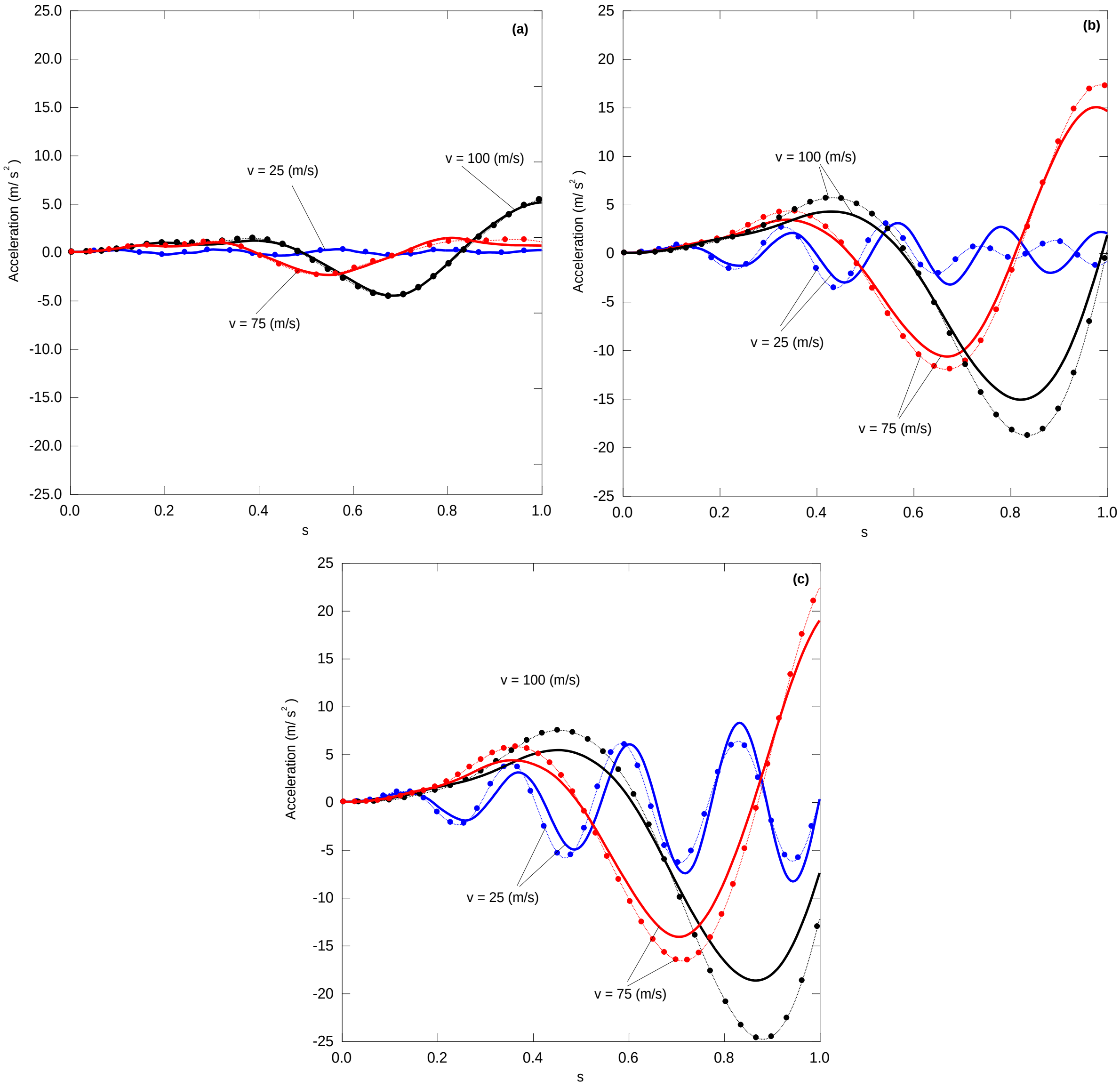

Fig 5. Moving system mid-span acceleration response versus the dimensionless time.
(a) *m*= 25000 *kg*, (b) *m*= 75000 *kg*, (c) *m*= 100000 *kg*; *v*= 25 *m/s, v*= 75 *m/s, and v*= 100 *m/s.*
(⋯●⋯) moving mass problem; (——) moving force problem.

3. Conclusion

In this paper, the dynamic response of a homogeneous elastic simply supported beam subjected to a load system moving with a uniform velocity is investigated. The decomposition method is conveniently implemented to solve for the desired responses. Due to the versatility of the method suggested, other related problems may be solved. Comparison between the moving force and moving mass problems are presented. It is observed that the moving force problem is a special case of the problem considered. Interaction, load, mass, velocity effects on the beam as well as on the load-moving system are presented. It is shown that the inertia effect of the load-moving system can not be neglected when the traveling velocity and the ratio of the mass moving system and that of the beam are large.

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