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Residual Lifetime Prediction for Multi-State System Using Control Charts to Monitor Affecting Noise Factor on Deterioration Process

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ABSTRACT

In this research, multi-state complex systems are analyzed in order to measure reliability and predict residual of systems' lifetime under the effect of an out of control noise factor. Hence, the analytic method helps us to estimate multi-state system reliability, and then means residual lifetime that is calculated under normal conditions. Finally, the calculation is updated for out of control noisy condition using the accelerated method. To reveal the applied results, the proposed policy is implemented in a case study in a molding machine on SNJ Co. at Isfahan.

1. Introduction

As a system operates in its normal, standard and controlled condition offers a level of quality. However, when it is in the middle of its operation age factor, surpasses a predefined L level. This system stops working, failure happens, and this malfunction causes too much reduction in system's performance. Under the current condition, even when it is still working, the system is considered as a failure. The L threshold indicates the minimum level of performance of a system, by which manufacturing a product with the required quality is still possible. Determining L is based on a level of expertise and information pertaining to asymmetry. In addition, it is assumed that a failure does not happen deliberately or by design. Threshold L can be assumed as a level of failure that must not be surpassed by considering economic or safety aspects. If the level of failure is maintained more than L, an inspection is required [2, 7].

Reliability in binary systems is defined as a system and its parts that work together and may fail. In multi-state reliability states, systems and its parts are considered to have more than two performance states. These states define engineering systems in a more realistic, precise and complex manner and evaluate major problems and performance of these systems. Reliability analysis is a critical task conducted in all states of the system in order to maintain and keep them. There have been numerous efforts to combine multistate systems and system analysis [5, 6, 8]. A production system may be under

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exterior shocks and fail. After failure, it is replaced by a new system. In each failure and replacement, a significant cost is incurred. Therefore, there is a need to replace the system before failure [4, 9].

High level of stress is very detrimental to the processes of machinery and equipment. Exhaustion causes fractures and when there is a load beyond a threshold and during loading and unloading, failure happens. Accidental failure in structures happens when a fracture reaches its threshold. Effects of every stress can be understood in the cumulative damage. Consecutive load and unload will result in exhaustion for metal parts and in turn it leads to their failure. Exhaustion starts with one or two fractures and it grows within the structure. It eventually leads to the loss of service or demolition of the structure. Start of an early fracture, which can expand and lead to exhaustion, is a vague phenomenon. Therefore, it must be defined within a suitable factor [11]. With age, most of the manufacturing system and structured systems suffer from erosion and are exposed to failure as a result of damage. However, most of them are maintained using maintenance systems. A maintenance policy should observe two measurable variables: damage and stress. The latter is based on stress. Such policy is a necessity when detection reaches the first critical threshold and system fails [3].

Wu and Wang using a static and adoptive control curve created a formula for expected long term cost per time for a monitoring system [12]. In the adoptive control curve, the sample size is constant, but sampling intervals vary. There are three states for the system: normal, pending failure, and total failure. Two levels of maintenance are used for maintaining the system: Minor maintenance is used when a malfunction is discovered by inspection, and it brings back the system to work. Major maintenance, however, is only used when a failure happens. In this paper, expected cost per time is calculated in these situations. The purpose of this analysis is to find the optimized policy for the process of sampling inspection. Predicting system residual lifetime is an important feature of reliability which is highly useful in the analysis of dynamic behavior of systems. Predicting the residual lifetime of systems is attended in several fields such as reliability analysis of survival function, quality assurance, etc. By predicting the residual of the lifetime of the equipment, behaviors of systems can be predicted and quality of products can be controlled [4, 1, 10].

In this paper, reliability is shed light upon in the presence of a stress factor and under several damage mechanisms. Failure indicators have different aspects. In the presented paper, by using analytical methods and by predicting mean of the residual lifetime, the system is controlled and monitored in the presence of stress factor. This paper is organized as follows: In the second Section, the process of failure and relevant variables are described. In the third Section, maintenance policies are described. Section 4 analyzes the validity of the proposed maintenance policy and a molding machine, "Hang9", in SNJ Co using a numerical case study. Results of the research are presented in Section 5.

Nomenclature

(X_m) $m \in N$: Discrete time process describing the deterioration at time t_m .

$(Y_m)m \in N$: Discrete time process describing a stress at time t_m .

Z_m : discrete time process describing the system state at time t_m

l_i : standard quality threshold of the working state i , i=0,1,2,…, k-1

Δt :unit time length.

t_m : discrete time.

a : scale parameter.

b : sensitivity to stress.

AF : failure acceleration factor due to stress.

t_s : discrete time due to stress

t_0 : lifetime in normal state

σ : standard deviation of standard deviation of Y_t

λ : stress threshold.

λ_i:stress threshold at state i , , i=0,1,2,…, k-1

$f_{\lambda_i}^{(k)}$: probability density function of the deterioration increment after m periods of time of a non-failed at kth state of system.

e(t) : function of Mean Residual Lifetime.

μ : mean of random sample of density function.

τ : X-inspection period.

ε : preventive replacement threshold

UCL upper control limit.

2. Description of the Combined Stress-Degradation Failure Process

In this section, a multi-state system, its failure process, the relationship of stress, and the behavior of the system are described under the condition of damage. Afterwards, a mathematical model that is derived from these relationships is presented.

2.1. Description of the System Failure Process

Take a multi-state system with two failure mechanisms. First pertains to too much damage and Second pertains to fatal shock which can simultaneously happen on the surface and in stressful environment. Take cumulative failure damage of a k-state system. When this system is exposed to a progressive damage bigger than a predetermined threshold, its normal state is over and it enters into the next state. The duration of which system is in this state, equals the time interval between previous state threshold and the current state threshold. As the system reaches the ending threshold of the current state, the system enters into the next performance state and damage rate increases. This process continues up to k-1 threshold when system enters into (k) *th* state. The system stops working in this state and a failure is realized. Damage threshold in each state is the lease performance level required to produce quality products in that level. Variable $(X_m)\ m \in N$ is the persistent increase of detection process in discrete time of $t_m = m\Delta t$ in which $\Delta t = 1$. Variable $(X_m)\ m \in N$ has a one fuzzy exponential distribution with a parameter of λ and density function of $f(x) = \lambda e^{-\lambda x}$. Cumulative exhaustion shows the growth of a fracture. It is assumed that in a k-state system density function of damage process, a parametric λ is exponential function.

The first state, which begins from the start of the system, is the normal lifetime of the system. The duration of this state is determined based on standard quality threshold l_0 of the working condition. In this state, exponential density function has a parameter of λ_0. The second state starts when the system reaches l_0 lifetime threshold and its performance decreases in comparison with the previous state. The residual productivity of the system in this state is from l_0 lifetime threshold to $l_1$threshold. The $l_1$threshold is determined on the basis of quality and productivity of the 2nd state. In this state, the system has an exponential density function with λ_1 damage rate. As time progresses and threshold l_{k-2} is surpassed, the system enters into k-1 state and its performance decreases in comparison with the previous states. In this state, erosion is too much. In state k-1, damage rate is λ_{k-2}. When productivity of state k-1 decreases and reaches threshold k-1, the system enters into k state. In this state, products do not have the least acceptable quality and total failure happens. In state k, the system has an exponential density function with damage parameter of λ_{k-1} . The relationship between density function parameters are independent and $\lambda_0 < \lambda_1 < \lambda_1 < \cdots < \lambda_{k-1}$.

2.2. Failure Caused by Stress

The second damage mechanism of failure is caused by stress. Y_m $(m \in N)$ is the stress trend variable. $Y_m(m \in N)$ is the indicator of stress strength in t_m. Y_m is a random variable with exponential density function for stress threshold γ:

- If $Y_m < \gamma$ system is in its nominal state and it is not significantly affected by stress.
- If $Y_m \geq \gamma$ system is in the stress state and there is an increased chance of failure because of stress.

Proportion coefficient AF is the momentum factor, which stems from stress. In parametric condition, the relationship between the lifetime in normal state t_0 and in stress state t_s is proportionate and proportion coefficient, i.e. momentum factor AF can be calculated using expression 1:

$$t_0 = AFt_s. \tag{1}$$

In practice, proportion coefficient can be calculated by dividing estimated centrality in the normal state by its value in a stress state with specific lifetime density function. Knowing the relationship between lifetime centrality and level of stress, this can be calculated through empirical data and regression. AF coefficient is estimated by Arrhenius method. In this method, using Expression 2 and having temperature in the Kelvin degrees, lifetime is calculated as follows:

$$t_0 = ke^{\frac{c}{T}}. \tag{2}$$

kth State
density function parameter : λ_{k-1} — $Threshold\ l_{k-2}$
lifespan $[l_{k-1}, \infty)$ minutes ←
k-1th State
density function parameter : λ_{k-2} — $Threshold l_{k-3}$
lifespan $[l_{k-2}, l_{k-1})$ minute ←
.
.
.
Second State
density function parameter : λ_1 — $Threshold\ l_1$
Lifespan $[l_0, l_1)$ minute ←
first State
density function parameter : λ_0 — $Threshold\ l_0$
Lifespan $(0, l_0)$ minute ←

Fig. 1. Multi- state system.

In this expression, C and K are constants, which are estimated using empirical data and sampling using regression method. The linear formula is $\mathrm{Ln}(t_0) = \mathrm{Ln}(k) + c \times \frac{1}{T}$. Thus, the following relations hold true:

$$\mathrm{Ln}(t_0) - \mathrm{Ln}(k) = c \times \frac{1}{T}$$
$$\mathrm{Ln}\left(\frac{t_0}{k}\right) = \frac{c}{T} \tag{3}$$
$$t_0 = ke^{\frac{c}{T}}.$$

Since the system is sensitive to stress, when time progresses, different states are realized. Chance of failure due to stress is a function of current damage X_k and time t_k. Z_m is the indicator of system state. $Z_m = 0$ denotes the normal state, and $Z_m = 1$ is the indicator of failure. Conditional possibilities of damage are shown, based on the two damage mechanisms and considering $X_m, Y_m, t_m \geq 1$ and λ_i^1 where i=0, 1, 2,..., k-2 and failing progress in stress condition and in state i.

i*th* state

$$P(Z_m = 0 | X = x < L_0, \ Y_m < \gamma) = \lambda_i e^{-\lambda_i t_m}$$

$$P(Z_m = 1 | X = x < L_0, \ Y_m \geq \gamma) = F_A \ \lambda_i^1 e^{-\lambda_i^1 t_m}$$

kth state

$$P(Z_m = 1 | L_1 \leq X = x, \ Y_m < \gamma) = \lambda_{k-1} e^{-\lambda_{k-1} t_m}$$

2.3 Reliability of the System

Reliability of the system is the summation of all the favorable states of system. Density function of reliability of damage increases after level k, since period of no failure for each state of a failure and stressed system is $f_{\lambda}^{(k)}$.

$$f_{\lambda_i}^{(k)}(x) = ((1-p)AF + p)^k \frac{\lambda_i^k}{\lambda_i^{k-1}\,(k-1)!} \times (1-e^{-\lambda_i x})^{k-1} e^{-\lambda_i x} \prod_{j=1}^{k} e^{-bj}\,, \tag{4}$$

where $p = p(Y_m < \gamma) = \phi\left(\left(\frac{(\gamma-m)}{\sigma}\right)\right)$ and $\phi(0)$ are normal cumulative density function and the mean and the standard deviation of stress factor Y_m are respectively m and σ. Variable b is the indicator of system sensitivity to stress factor. Reliability of each system state in $t_k = m\Delta t$ can be calculated through integral of $f_{\lambda}^{(k)}$ from 0 to L_i (equation 5):

$$\int_0^{L_i} f_{\lambda_i}^{(k)}(x)dx = ((1-p)AF + p)^k \int_0^{L_i} \frac{\lambda_i^k}{\lambda_i^{k-1}\,(k-1)!} \times (1-e^{-\lambda_i x})^{k-1} e^{-\lambda_i x} \prod_{j=1}^{k} e^{-bj}\,dx\,. \tag{5}$$

3. Definition of the Maintenance Policy

In this section, a framework of decision making for maintenance is presented. First, this framework is defined. Then, system inspection, system replacement, and predicting mean of system lifetime are defined. Finally, the process of predicting mean residual lifetime of multi-state system, which is required to make decisions about residual lifetime of the system that, is delineated.

3.1. Structure of the Maintenance Policy

Failure state was described in the previous section. It is assumed that failure is not obvious, thus an inspection is required to know this system state. In this regard, two types of inspection can be used: superficial inspection (x-inspection) and state inspection (z-inspection). Maintenance policy provides the opportunity to do Preventive or Corrective replacements. Finally, a typical maintenance can be used to decrease the effects of the stressful environment.

3.2. X-Inspection and Z-Inspection

Inspection of damage level (X-inspection) is defined as accumulation level of damage X_m, which can only be observed via costly inspection with the cost of X. The first inspection is conducted after replacing failed system and after τ time unit since start of the programmed system. After that, a system is inspected in equal intervals (τ, 2τ, …). In this regard, it is assumed that inspection is through and the precise reported level of damage is X_k. Throughout these inspections of system state (z-inspection), stress variable Y_m is monitored against time t_m in a classic control chart. This chart is used to determine failure through observing changes in Y_m . If Y_m is more than UCL then the least inspection of Z is conducted to determine system state ($Z_m = 0 or 1$).

3.3. Preventive and Corrective Replacement

Maintenance policy gives us the liberty to replace as an act of Preventive or Corrective. When system has failed through X-inspection or a Z-inspection failure, replacement is Corrective. Preventive replacement is conducted to prevent system failure. If e is the threshold of Preventive replacement, and damage level is between (ε, L), the replacement is conducted during an X-inspection.

3.4. Mean Residual Lifetime Prediction

If $X_n, \dots, X_2, X_1$ have a density function of $f(x) \geq 0$ and mean $\mu = E(x)$ and variance of $\sigma^2 \leq \infty$, function of Mean Residual Lifetime (MRL) Prediction in time t is expressed in Eq. (6).

$$\mathrm{e(t) = E\{T - t|T > t\}} = \int_t^{\infty} \frac{\mathrm{R(x)dx}}{\mathrm{R(t)}} = \frac{\int_t^{\infty} \mathrm{xf(x)dx}}{\mathrm{R(t)}} - \mathrm{t}\,. \quad (6)$$

MRL function for the random sample with an exponential density function and μ parameter as its mean is the inverse of mean density function:

$$\mathrm{e(t)} = \frac{1}{\mu}, \quad (7)$$

mean of the residual lifetime of a multi-state system with an exponential density function equates the number of states of the system. When a system is working in its normal state, damage rate parameter in each state is shown as λ_i and when the system fails because of stress, damage rate parameter is shown as λ_i^1.

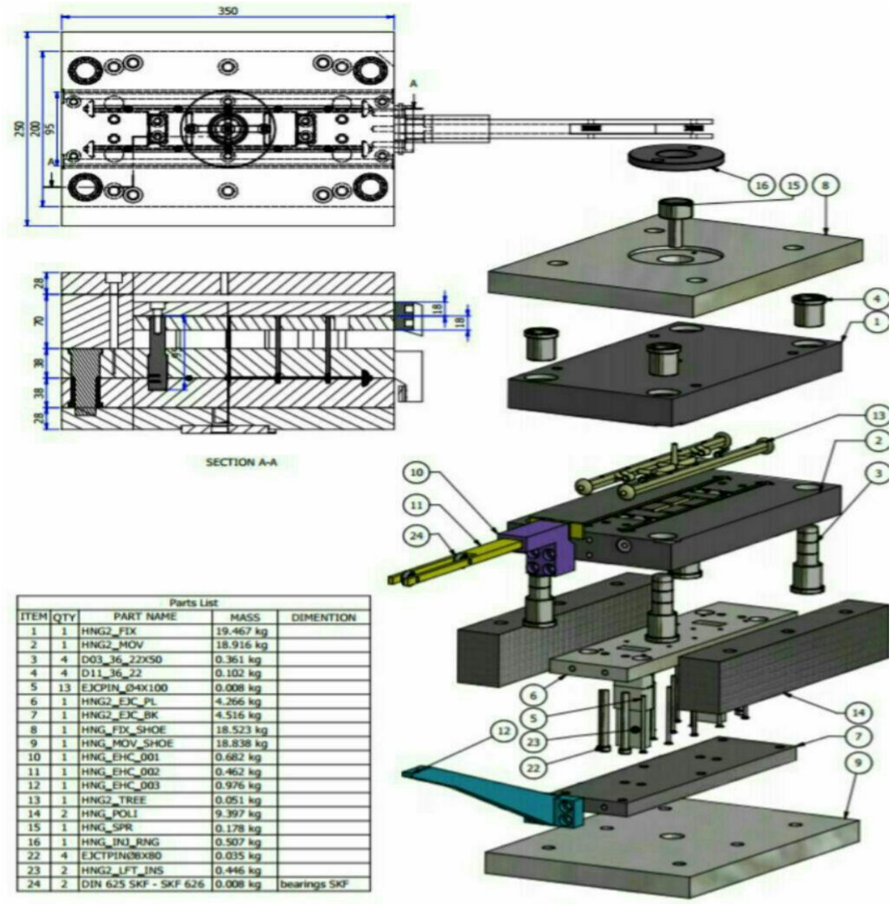
Mean Residual Lifetime in system state i:

$$\mathrm{e(t)} = \begin{cases} \mathrm{i-1} & \textit{operation of system in normal state} \\ {}^{1}_{\mathrm{i-1}} & \textit{operation of system under shock} \end{cases}$$

4. Case Study

In order to analyze the validity of the proposed maintenance policy, a molding machine, "Hang9", in SNJ Co. (located in Isfahan, Iran) is used (Fig. 2). A 6-state system with exponential exhaustion cumulative density function and fracture growth function with the parameter λ is assumed. As it was mentioned in Eq. (3), when damage progress variable of a system is more than a predetermined threshold, the system enters the next performance level. From a level to the next one, performance is decreased and exhaustion is increased. When State 5 of performance is over and the State 6 begins, the system fails completely.

The first State, which is comprised of the start point until its critical threshold $l_0 = 1200$ minutes, with an exhaustion rate of λ=0.000277, is the best working condition of the system. As l_0 lifetime threshold is exceeded, system enters into the second performance state. Productivity of this state is from l_0 threshold to l_1 =2400 minutes. In the second State, system has an exponential density function with a parameter of $\lambda_1 = 0.000416$. In the third performance State, system has a lifespan of (2400, 3600] minutes and exhaustion rate of $\lambda_2 = 0.000833$. The 4th performance State of the system starts from 3600 minutes. Since exhaustion of the system in this state is rather high, damage rate is $\lambda_3 = 0.0015$. As a result, this state is shorter than the previous states. The 5th State of the system is when the system is on the brink of high failure. Damage rate in this period is $\lambda_4 = 0.001666$. Productivity falls to 12.346. The 6th State of the system is when system fails completely and products do not have the least acceptable quality. Parameters of density function are independent and $\lambda_0 < \lambda_1 < \lambda_2 < \lambda_3 < \lambda_4 < \lambda_5$. The stress factor of the research in question, is temperature. Temperature has little effect on system lifetime and damage process in a range of (22, 26, 20). In order to show the effect of temperature in the mentioned range on system lifetime, from 25 different states in system lifetime and 25 sample data that each of which containing 5, were gathered. Control chart of mean-range was illustrated based on the data resulted from Minitab 16.



ITEM	QTY	PART NAME	MASS	DIMENTION
1	1	HNG2_FIX	19.467 kg	
2	1	HNG2_MOV	18.916 kg	
3	4	D03_36_22X50	0.361 kg	
4	4	D11_36_22	0.102 kg	
5	13	EJCPIN_Ø4X100	0.008 kg	
6	1	HNG2_EJC_PL	4.266 kg	
7	1	HNG2_EJC_BK	4.516 kg	
8	1	HNG_FIX_SHOE	18.523 kg	
9	1	HNG_MOV_SHOE	18.838 kg	
10	1	HNG_EHC_001	0.682 kg	
11	1	HNG_EHC_002	0.462 kg	
12	1	HNG_EHC_003	0.976 kg	
13	1	HNG2_TREE	0.051 kg	
14	2	HNG_POLI	9.397 kg	
15	1	HNG_SPR	0.178 kg	
16	1	HNG_INJ_RNG	0.507 kg	
22	4	EJCTPINØ8X80	0.035 kg	
23	2	HNG2_LFT_INS	0.446 kg	
24	2	DIN 625 SKF - SKF 626	0.008 kg	bearings SKF

Fig. 2. Machine, “Hang9” state.

Fig. 3 depicts the preliminary data of the chart of range control R and mean control $\bar{x}$. R and $\bar{x}$ are mapped in the chart for each sample. All samples within control range are depicted and no organized trend was observed among the samples. Thus, it can be concluded that the process has been under control and the resulted range of the control is appropriate for present and future products. All of the depicted data has been gathered using retrospective analysis. The preliminary samples for R chart are depicted in Fig. 3 and there is no indication of the system being out of control. Since R chart shows a controlled situation for process changeability, it is logical. $\bar{x}$ chart with a center line of $\bar{\bar{x}} = 24.051$ and control limits of UCL= 26.205 and LCL = 21.898 is depicted in Fig. 3. In $\bar{x}$ control chart in Fig. 3, there is no condition of an out of control system. Therefore, since both $\bar{x}$ and R charts seem to be controlled, it can be concluded that according to the levels depicted in the charts, the process is under control, and as a result, experimental control limits can be set for controlling present and future products.

Other samples of products, after finalizing control charts were chosen to help to analyze the effect of temperature of the environment on systems. R and $\bar{x}$ of all samples are depicted in the control chart, immediately after choosing each sample. $\bar{x}$ and R charts of the data related to these new samples are shown in Fig. 4. In the R control chart, samples of 27, 28, and 29 are depicted in the upper end of the control chart and it can be concluded that before this time for some reason a choice has been made. Also, the general trend of the points in the $\bar{x}$ control chart, from 25[th] subgroup on, shows a change in mean activity. In $\bar{x}$ chart, samples of 26 to 30 are depicted in the upper end control limits and it can be concluded that before this time, for some reason, a deviation has happened. The general trend of the points in the $\overline{x}$ control chart, from 26 Subgroup, shows a change in the mean of the process.

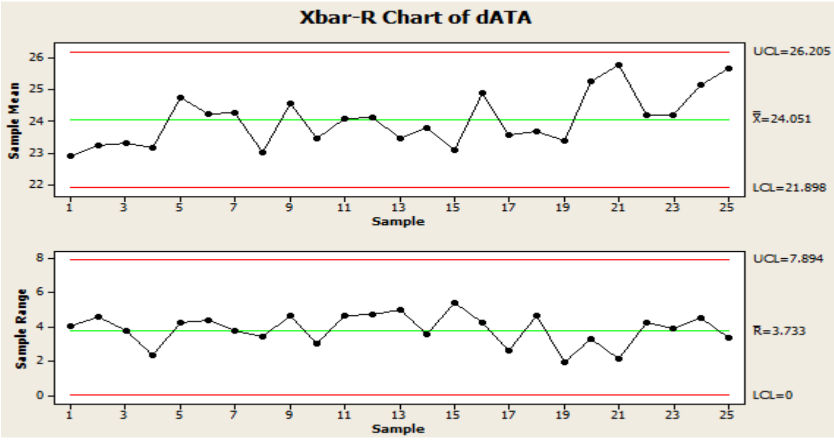


Fig. 3. Control chart $\overline{x}$ in controlled condition.

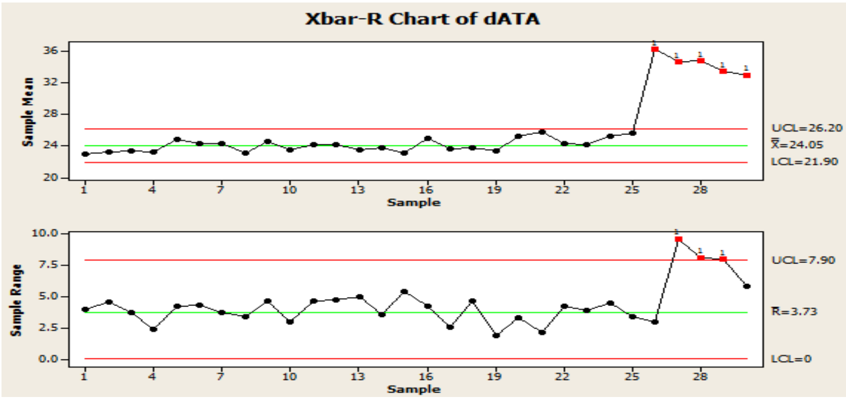


Fig. 4. Control chart $\overline{x}$ with out of control range points.

Y_m $(m \in N)$ is the variable of stress in time t_m. Random variable Y_m has an exponential density function. if $\gamma = 26.2$ system is in the stress threshold; therefore if $Y_m < 26.2$ system is in its nominal state and stress factor has little effect on failure probability, and if $Y_m \geq 26.2$ system is stressed and the probability of failure due to the stress rises. The increase of system temperature more than the upper mean control limit can precipitate damage trend of the system and decrease the system lifetime. In such condition, proportion coefficient AF indicates the momentum of stress process in damage process of the system. It is necessary to monitor and estimate AF in all system states and analyze its effect on each state. Using Minitab 16, data of damage rates for different time periods for Hang9 are collected, so that the stress factor can be analyzed for different system states. Based on 100-item samples of system lifetime in three conditions (three different temperature of 33.5, 40, 46.5 Celsius degrees), exponential density function for lifetime of each system state are determined, and using Minitab, damage rate data for each state of the chart in 3 different temperatures (under the effect of stress factor) are generated. Estimation of the factor of stress momentum for failure process and in each system state is calculated using Expression 3 and the gathered data for AF coefficient. Estimation of the factor of acceleration stress of the failure process AF coefficient in each state of system by the data that obtained H9 device for each state of system uses Eq. (3) based on Ernius method unto Kelvin's degree that is shown in Table 1. The output of software for the first to fifth states is shown respectively in Figures 5-9.

Table 1. Estimation of AF coefficient.

state	C	K	Life time in 24.05 temperature	$t_{median}(33.5)$	$t_{median}(40)$	$t_{median}(46.5)$	AF
1	4935.8	0.00021178	1269.5701	2033.987571	1645.9242	1417.4677	0.6241779
2	5731.7	0.000007397	1774.3168	1312.3978	1052.304561	946.513953	1.351965
3	3519.9	0.00591656	1779.11	655.0644	547.328	472.689	2.716
4	6624.2	1.7473	845.55	461.995	303.947	262.494	1.83
5	2415.7	0.01021	34.74426	32.75	27.466	23.72	1.061

NonLinear Regression: $Ln(t_0)$= Ln k + C×1/T
Method
Algorithm Gauss- Newton
Max iterations 200
Tolerance 0.00001
Starting Values for Parameters
Parameter value
Lnk 1
C 1
Equation
$Ln(t_0)$= -8.45931 + 4935.88 × 1/T
Parameter Estimate
Parameter Estimate SE Estimate
Lnk -8.46 4.67
C 4935.88 1460.30
$Ln(t_0)$ = Lnk + C× 1/T

Fig. 5. Minitab output, in the 1th system state.

NonLinear Regression: $Ln(t_0)$= Ln k + C×1/T
Method
Algorithm Gauss- Newton
Max iterations 200
Tolerance 0.00001
Starting Values for Parameters
Parameter value
Lnk 1
C 1
Equation
$Ln(t_0)$= -11.9075 +5731.76 × 1/T

Fig. 6. Minitab output, in the 2th system state.

NonLinear Regression: $Ln(t_0)$= Ln k + C×1/T
Method
Algorithm Gauss- Newton
Max iterations 200
Tolerance 0.00001
Starting Values for Parameters
Parameter value
Lnk 1
C 1
Equation
$Ln(t_0)$= -5.1339 + 3519.9 × 1/T
Parameter Estimate
Parameter Estimate SE Estimate
Lnk -5.13 4.35
C 3519.90 1361.40
$Ln(t_0)$ = Lnk + C× 1/T

Fig. 7. Minitab output, in the 3th system state.

NonLinear Regression: $Ln(t_0)$= Ln k + C×1/T
Method
Algorithm Gauss- Newton
Max iterations 200
Tolerance 0.00001
Starting Values for Parameters
Parameter value
Lnk 1
C 1
Equation
$Ln(t_0)$= -15.5639 + 6624.22 × 1/T
Parameter Estimate
Parameter Estimate SE Estimate
Lnk -15.56 4.22
C 6624.22 1319.66
$Ln(t_0)$ = Lnk + C× 1/T

Fig. 8. Minitab output, in the 4th system state.

NonLinear Regression: $Ln(t_0)$= Ln k + C×1/T
Method
Algorithm Gauss- Newton
Max iterations 200
Tolerance 0.00001
Starting Values for Parameters

Parameter	value
Lnk	1
C	1

Equation
$Ln(t_0)$= -4.58502 + 2415.73 × 1/T
Parameter Estimate

Parameter	Estimate	SE Estimate
Lnk	-4.59	4.22
C	2415.73	1321.15

$Ln(t_0)$ = Lnk + C× 1/T

Fig. 9. Minitab output, in the 5th system state.

Due to stress and system fails, the density function of the system is $f_0(t_0) = \frac{1}{AF} f_s(t_0)$. $f_0(t)$ is the system density function in normal state. $f_s(t)$ is the system density function when it is under the effect of stress. Because of the sensitivity of the system to stress, and the fact that stress is a function of current damage level X_m in time t_m, the probability of failure increases with time. Variable Z_m indicates system state.

- If $Z_m = 0$ system is in good condition.
- If $Z_m = 1$ system is in failure state.

Conditional probability of failure, based on two damage mechanisms and considering values of X_m, Y_m, $t_m \geq 1$ is depicted as follow:

$$P(Z_m = 1|X = x \geq 1200)=1$$

1th state

$$P(Z_m = 0|X = x < 1200, Ym < \gamma) = 1 - e^{-0.000277x-bt_m}$$

$$P(Z_m = 1|X = x < 1200, Ym \geq \gamma) = 1 - 0.6241779 * e^{-0.000277x-bt_m}$$

2th state

$$P(Z_m = 1|X = x \geq 2400) = 1$$

$$P(Z_m = 1|1200 \leq X = x < 2400, Ym < \gamma) = 1 - e^{-0.000416x-bt}$$

$$P(Z_m = 1|1200 \leq X = x < 2400, Ym \geq \gamma) = 1 - 1.351965 * e^{-0.000416x-bt}$$

3th state

$$P(Z_m = 1|X = x \geq 3600) = 1$$

$$P(Z_m = 1|2400 \leq X = x < 3600, Ym < \gamma) = 1 - 0.000833 * e^{-0.000833x-bt}$$

$$P(Z_m = 1|2400 \leq X = x < 3600, Ym \geq \gamma) = 1 - 2.716 * e^{-0.000833x-bt}$$

4th state

$$P(Z_m = 1|X = x \geq 4200) = 1$$

$$P(Z_m = 1|3600 \leq X = x < 4200, Ym < \gamma) = 1 - e^{-0.0015x-bt}$$

$$P(Z_m = 1|3600 \leq X = x < 4200, Ym \geq \gamma) = 1 - 1.83 * e^{-0.0015x-bt}$$

5th state

$$P(Z_m = 1|X = x \geq 4800) = 1$$

$$P(Z_m = 1|4200 \leq X = x < 4800, Ym < \gamma) = 1 - e^{-0.01666x-bt}$$

$$P(Z_m = 1|4200 \leq X = x < 4800, Ym \geq \gamma) = 1 - 1.061 * e^{-0.01666x-bt}$$

6th state

$$P(Z_m = 1|X = x \geq 4800) = 1 .$$

4.1. Evaluation of the Proposed Maintenance Policy

In this section, the proposed maintenance policy that was mentioned in Section 3 is evaluated using a case study of Hang9 molding machine. As it was mentioned earlier, failure cannot be revealed. Therefore, an inspection is required to know system condition. The first inspection is conducted after replacement of a failed system and τ =600 minutes after reboot of the programmed system. After that, system is inspected in equal intervals ($\tau, 2\tau, 3\tau$, etc). It is assumed that the inspections are complete and the accurate level of damage X_m is perceived. Stress variable Y_m per time t_m is constantly monitored using a mean control chart $\bar{x}$. This control chart is employed to determine failure using observations of Y_m. If Y_m is more than UCL then at least inspection of Z is conducted to understand system state (Z_m=0 or 1). The threshold of precautious replacement is $\varepsilon = 1000$ minutes. If the level of damage is within (ε, L), a precautious replacement is implemented in x-inspection. Since in 1st, 2nd, 3rd, 4th, and 5th states, stress factor can cause progress of damage, considering normal or stressed state of the system, the mean residual lifetime is estimated using Expressions 6 and 7.

Prediction of mean of residual lifetime of the system in the 1st state:

$$e(t) = \begin{cases} \lambda_0 = 0.000277 & \text{operation of system in normal state} \\ \lambda_0^1 = 0.0004889 & \text{operation of system under shock} \end{cases}$$

Prediction of mean of residual lifetime of the system in the 2st state:

$$e(t) = \begin{cases} \lambda_1 = 0.000416 & \text{operation of system in normal state} \\ \lambda_1^1 = 0.00073216 & \text{operation of system under shock} \end{cases}$$

Prediction of mean of residual lifetime of the system in the 3st state:

$$e(t) = \begin{cases} \lambda_2 = 0.000833 & \text{operation of system in normal state} \\ \lambda_2^1 = 0.001466 & \text{operation of system under shock} \end{cases}$$

Prediction of mean of residual lifetime of the system in the 4st state:

$$e(t) = \begin{cases} \lambda_3 = 0.0015 & \text{operation of system in normal state} \\ \lambda_3^1 = 0.00264 & \text{operation of system under shock} \end{cases}$$

Prediction of mean of residual lifetime of the system in the 5st state:

$$e(t) = \begin{cases} \lambda_4 = 0{,}01666 & \text{operation of system in normal state} \\ \lambda_4^1 = 0{,}02921 & \text{operation of system under shock} \end{cases}$$

According to the definition of system inspection, stress factor is monitored and evaluated using mean control chart. Inspecting damage level of the system is a complete inspection. During inspection of exhaustion level, if the machine has reached the precautious replacement threshold or is within(ε, L), precautious replacement is implemented. Afterward, based on the stress condition, the mean residual lifetime is predicted:

- If the mean residual lifetime of the system state is predicted to be higher than the length of the next inspection period: Precautious replacement has not been conducted.
- If mean residual lifetime of the system state is predicted to be lower than the length of the next inspection period: Precautious replacement has been conducted.

5. Conclusion

In the present study, the accelerated stress factor was considered outside the range of environment temperature under the operating conditions. Range of environment temperature of the system was obtained in operating conditions by the domain-average control graph. The rate of failure progression in each state of the system degradation was achieved in the presence of accelerated stress factor. With the development of the failure process, the failure rate increased in each state of system. The rate of failure progression in each state of the system was compared to the operating condition in the presence of the accelerated stress factor. By summing up all the probabilities of the development of the failure process of system states in the process of deterioration, the reliability of a multi-state system was calculated. For each state of system, remainder of the lifetime prediction of the multi-state system has calculated in development of the failure process. Proposed repairs and maintenance were presented according to two types of inspection: Inspection of failure level (X-Inspection) and inspection of system state (Inspection-Z). When there was inspection of the erosion system, if the device was in preventive replacement threshold distance, the threshold of failure in each state, and variation state of system in period of time, preventive replacement was done. Then, based on the stress state, remainder of average lifetime was predicted. According to remainder of average lifetime of state of the system, decision is made for corrective replacement to prevent system failure. Validation of the proposed method has been implemented by machine modeling that has six state and it is under accelerated stress factor at SNJ Company in Isfahan. As future studies, researchers are encouraged to analyze prediction of residual lifetime of a system, comprising different multi state machines and under stress factors.

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