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On Solving Fully Fuzzy Multi-criteria De Novo Programming via Fuzzy Goal Programming Approach

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ABSTRACT

In this paper, a Multi-criteria De Novo Linear Programming (F- MDNLP) problem has been developed under consideration of the ambiguity of parameters. These fuzzy parameters are characterized by fuzzy numbers. A fuzzy goal programming approach is applied for the corresponding multi-criteria De Novo linear programming ($\alpha-$MDNLP) problem by defining suitable membership functions and aspiration levels. The advantage of the approach is that the decision maker's role is only the specification of the level α and hence evaluate the $\alpha-$ efficient solution for limitation of his/ her incomplete knowledge about the problem domain. A numerical example is given for illustration.

1. Introduction

In general, there is no single optimal solution in multi-criteria problems, but rather a set of non-inferior (Pareto optimal) solutions from which the Decision Maker (DM) must select the most preferred or best compromise solution as the one to implement. One of the difficulties which occur in the application of mathematical programming is that the coefficients in the formulation are not constants but fluctuating and uncertain.

Fuzzy set theory introduced by Zadeh [32] makes a model has to be set up using data which is approximately known. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers [6]. Fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade [7]. A fuzzy linear programming problem has been formulated by Tanaka and Asai [28] to obtain a reasonable solution under consideration of the ambiguity of parameters. Rommelfanger et al. [23] solved the multi-objective linear optimization problem using the interactive method, where the coefficients of the objectives and/or the constraints are known exactly but imprecisely. Zimmerman [39] introduced fuzzy linear programming with multiple objective functions. Sakawa and Yano [24] introduced the concept of $\alpha-$Pareto optimality of fuzzy parametric program. Bellman and Zadeh [1] introduced the concept of a maximizing decision making problem. Zhao et al. [37] introduced the

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complete solution set for the fuzzy linear programming problems using linear and nonlinear membership functions. Tang and Chang [29] developed a decision-making model and helped the DMs and researchers to understand the effect of multiple criteria decision-making on a capital budgeting investment, coefficients, and resources. They solved the problem based on the proposed method introduced by Gasimov and Yenimez [10]. Kiruthiga and Loganathan [16] reduced the Fuzzy MOLP problem to the corresponding ordinary one using the ranking function and hence solved it using the fuzzy programming technique. Hamadameen [11] proposed a technique for solving fuzzy MOLP problem in which the objective functions coefficients are triangular fuzzy numbers. Under uncertainty, vague, and imprecise of data, Garg [9] suggested an alternative approach for solving multi-objective reliability optimization problem. Chen et al. [5] applied fuzzy goal programming approach to treat equipment's purchasing problem for a flexible manufacturing cell.

De Novo programming is used as a methodology of optimal system design for reshaping feasible sets in linear systems. De Novo programming was proposed by Zeleny [33] ensures creation of an optimal-level model by economizing the constraints resources within the frame of a given budget. The major characteristic of De Novo hypothesis is to realize optimal system design instead of optimizing a given system [35]. Fiala [8] proposed approaches for solving multi-objective De Novo linear programming, also introduced possible extensions, methodological, and real applications. Umarusman and Turkmen [30] have been built the optimum production setting through the de novo programming with the global criterion method. Tabucanon [27] has been shown that the de novo programming formulation deal with the best mixture of input specified as well as the best mixture of the output. De novo programming problem was studied and developed by several authors [3, 4, 12, 36].

De Novo programming model with fuzzy coefficients was proposed by Li and Lee [18]. Also, Li and Lee [19] developed fuzzy goals and fuzzy coefficients for the De Novo programming model simultaneously. Fuzzy programming approach was firstly used by Zimmermann [38] to solve the multi-objective problems. Narasimhan [22] and Ignizio [13] used the fuzzy set theory for solving the problems having multiple goals. Ramadan Hamed Mohamed [20] introduced some new forms of fuzzy programs using the deviational variables concepts of fuzzy goal programming to transform the fuzzy programming into the corresponding crisp. Also, he applied the min-operator to convert the fuzzy programming into the corresponding crisp program. Charnes and Cooper [2] were the first ones presented the goal programming and hence this approach developed by Ijiri [14] and Lee [17]. Veeramani et al. [31] studied fuzzy MOLP problem with fuzzy technological.

The remainder of the paper is as: In Section 2, some preliminaries are introduced. In Section 3, a problem formulation and solution concepts is introduced. In Section 4, a fuzzy goal programming approach is applied. A solution procedure for solving the problem is suggested in Section 5. A numerical example is given in Section 6. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

Here, definition of fuzzy numbers, triangular fuzzy numbers, interval confidence, and some of arithmetic operations needed in order to discuss our problem conveniently are recalled [15, 21].

Let $I(R)=\left\{[a^-, a^+]: a^-, a^+ \in R=(-\infty,\infty), a^- \leq a^+\right\}$ denote the set of all closed interval numbers on R.

Definition 1. Assume that: $[a^-, a^+],\ [b^-, b^+] \in I(R)$, we define:

$$[a^-, a^+](+)[b^-, b^+] = [a^- + b^-, a^+ + b^+] \tag{1}$$

$$[a^-, a^+](-)[b^-, b^+] = [a^- - b^+, a^+ - b^-] \tag{2}$$

$$[a^-, a^+](\cdot)[b^-, b^+] = [\min (a^-.b^-, a^-.b^+, a^+.b^-, a^+.b^+), \max (a^-.b^-, a^-.b^+, a^+.b^-, a^+.b^+)] \tag{3}$$

The order relation $"\leq"$ in $I(R)$ is defined by:

$$[a^-, a^+](\leq)[b^-, b^+], \text{ if a only if } a^- \leq b^-, a^+ \leq b^+. \tag{4}$$

Definition 2. A fuzzy number $\tilde{a}$ is a mapping:

$\mu_{\tilde{a}} : R \to [0, 1],$ with the following properties:

- $\mu_{\tilde{a}}(x)$ is an upper semi- continuous membership function;
- $\tilde{a}$ is a convex fuzzy set, i. e., $\mu_{\tilde{a}}(\lambda x^1 + (1-\lambda)x^2) \geq \min\{\mu_{\tilde{a}}(x^1), \mu_{\tilde{a}}(x^2)\}$, for all $x^1, x^2 \in R, 0 \leq \lambda \leq 1.$
- $\tilde{a}$ is normal, i. e., $\exists x_0 \in R$ for which $\mu_{\tilde{a}}(x_0) = 1.$
- Supp $(\tilde{a}) = \{x \in R : \mu_{\tilde{a}}(x) > 0\}$ is the support of the $\tilde{a}$, and its closure cl (supp $(\tilde{a})$) is compact set.

Let $F(R)$ denotes the set of all compact fuzzy numbers on R that is for any $g \in F(R), g$ satisfies the following:

- $\exists x \in R : g(x) = 1.$
- For any $0 < \alpha \leq 1, g_\alpha = [g_{\bar{\alpha}}, g_{\bar{\alpha}}]$ is a closed interval numbers on $R.$

Definition 3. The $\alpha-$cut set of $\tilde{a} \in F(R), 0 \leq \alpha \leq 1$, denoted by $(\tilde{a})_\alpha$ and is defined as the ordinary set:

$$(\tilde{a})_\alpha = \begin{cases} \{x \in R : \mu_{\tilde{a}}(x) \geq \alpha, & 0 < \alpha \leq 1 \\ cl(\sup p(\tilde{a})), & \alpha = 0 \end{cases}$$

In the F- MDNP problem, the following notions are used:

$\tilde{p}_i$, $\tilde{B}$, $\tilde{c}_j^k$, $\tilde{d}_j^s$, and $\tilde{a}_{ij} \in F(R), i = 1, 2, ..., m, \quad j = 1, 2, ..., n, \quad k = 1, 2, ..., K, \quad s = 1, 2, ..., S$, and their $\alpha-$ level sets are:

$$\left(\tilde{C}\right)_\alpha = \left(\tilde{c}_j^k\right)_\alpha = \left\{c \in R^{K(1\times n)} : \mu_{\tilde{c}_j^k}(c_j^k) \ge \alpha; \forall j; k\right\},$$

$$\left(\tilde{D}\right)_\alpha = \left(\tilde{d}_j^s\right)_\alpha = \left\{ d \in R^{S(1\times n)} : \mu_{\tilde{d}_j^s}(d_j^s) \ge \alpha; \forall j, s\right\},$$

$$\left(\tilde{p}\right)_\alpha = \left(\tilde{p}_i\right)_\alpha = \left\{p \in R^{m\times 1} : \mu_{\tilde{p}_i}(p_i) \ge \alpha; \forall i\right\}, \left(\tilde{B}\right)_\alpha = \left\{B \in R : \mu_{\tilde{B}}(B) \ge \alpha\right\},$$

$$\left(\tilde{A}\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha = \left\{a_{ij} \in R^{m\times n} : \mu_{\tilde{a}_{ij}}(a_{ij}) \ge \alpha; \forall i, j\right\}.$$

Definition 4. (Sakawa [26]). A triangular fuzzy number (T. F. N.) is denoted by $\tilde{A} = (a_1, a_2, a_3)$ and its membership is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3, \\ 0, & x > a_3. \end{cases}$$

Also, T. F. N., parametric form for level α can be characterized as:

$$\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]; \forall 0 < \alpha \le 1.$$

3. Problem Formulation and Solution Concepts

A typical multicriteria De Novo programming problem introduced by Zeleny [34] is presented with fully fuzzy parameters as:

(P_1)

$$\max \tilde{Z}^k(x, \tilde{c}) = \sum_{j=1}^{n} \tilde{c}_j^k x_j$$

$$\min \tilde{W}^s(x, \tilde{d}) = \sum_{j=1}^{n} \tilde{d}_j^s x_j$$

Subject to

$$\sum_{i=1}^{m} \tilde{p}_i b_i = B, \quad x_j \ge 0, j = 1, 2, \ldots, n; i = 1, 2, \ldots, m.$$

Where, x_j and b_i are decision variables for projects and resources. $\tilde{p}_i$, the price of resources i and $\tilde{B}$ is the total available budget, respectively. It is noted that all of $\tilde{c}_j^k$, $\tilde{d}_j^s$, $\tilde{p}_i$ and $\tilde{a}_{ij}$ are fuzzy parameters that assumed to be characterized as the fuzzy numbers [7]. Note that $\sum_{i=1}^{m} \tilde{p}_i \, \tilde{a}_{ij} = \tilde{A}_j$ represents the unit cost of product j. Referring to A_j, problem (P1) may be reformulated as follows:

(P2)

$$\max \tilde{Z}^k = \sum_{j=1}^{n} \tilde{c}_j^k \, x_j$$

$$\min \tilde{W}^s = \sum_{j=1}^{n} \tilde{d}_j^s \, x_j$$

Subject to

$$M(x, \tilde{A}, \tilde{B}) = \left\{ x \in R^n : \sum_{j=1}^{n} \tilde{A}_j \, x_j = \tilde{B}, x_j \geq 0; \forall j \right\}.$$

The feasible region $M(x, \tilde{A}, \tilde{B})$ is assumed to be compact set. In this work, assume that all of fuzzy numbers $\tilde{p}_i$, $\tilde{B}$, $\tilde{c}_j^k$, $\tilde{d}_j^s$ and $\tilde{a}_{ij} \in F(R)$.

Definition 4. (Fuzzy efficient solution). $x^{\wedge}(\tilde{c}, \tilde{d}) \in M(x, \tilde{A}, \tilde{B})$ is said to be fuzzy efficient solution of the problem (P2) if $\tilde{Z}^k(x^{\wedge}, \tilde{c}) \geq \tilde{Z}^k(x, \tilde{c})$, $\tilde{W}^s(x^{\wedge}, \tilde{d}) \leq \tilde{W}^s(x, \tilde{d})$, $\tilde{Z}^k(x^{\wedge}, \tilde{c}) \neq \tilde{Z}^k(x, \tilde{c})$, or $\tilde{W}^s(x^{\wedge}, \tilde{d}) \neq \tilde{W}^s(x, \tilde{d})$.

For a certain degree of α, the problem (P2) can be written as in the following non fuzzy form (Sakawa and Yano [24]):

(P3)

$$\max Z^k = \sum_{j=1}^{n} c_j^k \, x_j$$

$$\min W^s = \sum_{j=1}^{n} d_j^s \, x_j$$

Subject to

$$x \in M(x, A, B) = \left\{ x \in R^n : \sum_{j=1}^{n} A_j \, x_j = B, x_j \geq 0; \forall j \right\},$$

$$c_j^k \in \left(\tilde{c}_j^k\right)_\alpha, d_j^s \in \left(\tilde{d}_j^s\right)_\alpha, A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha \text{ , and } \tilde{B} \in \left(\tilde{B}\right)_\alpha \text{ .}$$

Definition 5. ($\alpha-$efficient solution). $x^*(c,d)\in M(x,A,B)$ (M is the feasible region) is said to be $\alpha-$efficient solution of the problem (P3) if and only if there does not exist another x, $c_j^k \in \left(\tilde{c}_j^k\right)_\alpha$, $d_j^s \in \left(\tilde{d}_j^s\right)_\alpha, A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha, B \in \left(\tilde{B}\right)_\alpha$, such that $Z^k(x^*,c) \le Z^k(x,c), W^s(x^*,d) \ge W^s(x,d)$, and $Z^k(\hat{x},c) \ne Z^k(x,c)$, or $W^s(\hat{x},d) \ne W^s(x,d)$, where the corresponding values of parameters c^{*k}, d^{*s}, A^*, B^* are called $\alpha-$level optimal parameters.

Definition 6. ($\alpha-$ optimal compromise solution). A feasible solution $x^\wedge \in M(x,A,B)$ is said to be $\alpha-$optimal compromise solution of problem (P3) if and only if $Z(x^\wedge,c) \ge \underset{x\in E}{\wedge} Z(x,c)$, $c_j^k \in \left(\tilde{c}_j^k\right)_\alpha$, $d_j^s \in \left(\tilde{d}_j^s\right)_\alpha$, $A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha$, and $\tilde{B} \in \left(\tilde{B}\right)_\alpha$, where $\wedge$ referees to "minimum" and E is the set of $\alpha-$efficient solutions.

4. Fuzzy Goal Programming for Solving Problem (P3)

4.1 Fuzzy Programming Approach

Bellman and Zadeh [1] introduced three basic concepts: Fuzzy goal (G), fuzzy constraints (C), fuzzy decision (D), and handling the application of these concepts under fuzziness for the decision-making processes. Depending on the fuzzy goal and the fuzzy constraints, the fuzzy decision (D) can be defined as:

$$D = C \cap G \tag{5}$$

Whose membership function is characterized as:

$$\mu_D(x) = \min\left(\mu_C(x),\ \mu_G(x)\right) \tag{6}$$

To describe the fuzzy goals for the problem (P3), let us introduce the linear membership functions

$$\mu_k(Z^k(x)) = \begin{cases} 0, & Z^k(x) \le L^k, \\ \dfrac{Z^k(x,c)-L^k}{U^k-L^k}, & L^k \le Z^k(x) \le U^k, \\ 1, & Z^k(x) \ge L^k, \textit{ and} \end{cases} \tag{7}$$

$$\mu_s(W^s(x)) = \begin{cases} 1, & W^s(x) \le L^s, \\ \dfrac{U^s-W^s(x,d)}{U^s-L^s}, & L^s \le W^s(x) \le U^s, \\ 0, & W^s(x) \ge U^s. \end{cases} \tag{8}$$

Where, μ_k and μ_s are membership functions of the objective functions k and s, respectively. All of

U^k, U^s, L^k, and L^s are the upper and lower bounds of $Z^k(x,c)$ and $W^s(x,d)$. They are estimated at $\alpha = 0,$ as follows:

$$U^k = \max_{x\in M} (Z^k(x,c)),\quad U^s = \max_{x\in M} (W^s(x,d)),\ L^k = \min_{x\in M} (Z^k(x,c)),\ \text{and}\ L^s = \min_{x\in M} (W^s(x,d)),$$

$k = 1,2,\ldots,K,\ s = 1,2,\ldots,S.$ It is assumed that: $U^k \neq L^k$ and $U^s \neq L^s$.

By using the linear membership functions (7) and (8), and the fuzzy decision defined by (6), the problem (P_3) can be rewritten as:

(P_4)

$$\max \min_{\substack{k=1,2,\ldots,K\\ s=1,2,\ldots,S}} x \ \left\{\mu_k(Z^k(x,c)),\ \mu_s(W^s(x,d)\right\}$$

Subject to

$$x \in M(x,A,B) = \left\{ x \in R^n : \sum_{j=1}^{n} A_j x_j = B, x_j \geq 0, j = 1,2,\ldots,n \right\},$$

$$c_j^k \in \left(\tilde{c}_j^k\right)_\alpha, d_j^s \in \left(\tilde{d}_j^s\right)_\alpha, A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha, \text{ and } \tilde{B} \in \left(\tilde{B}\right)_\alpha.$$

By introducing the auxiliary variable ζ, the problem (P_4) can be reduced to the following conventional linear programming problem:

(P_5)

$$\max\ \zeta$$

Subject to

$$\zeta \leq \frac{Z^k(x,c) - L^k}{U^k - L^k}, k = 1,2,\ldots,K,$$
$$\zeta \leq \frac{U^s - W^s(x,d)}{U^s - L^s}, s = 1,2,\ldots,S,$$

$$x \in M(x,A,B) = \left\{ x \in R^n : \sum_{j=1}^{n} A_j x_j = B, x_j \geq 0, j = 1,2,\ldots,n \right\},$$

$$c_j^k \in \left(\tilde{c}_j^k\right)_\alpha, d_j^s \in \left(\tilde{d}_j^s\right)_\alpha, A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha \text{ and } \tilde{B} \in \left(\tilde{B}\right)_\alpha.$$

4.2 Goal Programming Approach

Fuzzy goal programming is a well-known approach in multi-criteria decision-making and its applications. To formulate the problem (P_5) as a goal programming problem (Sakawa [26]), let us introduce the negative and positive deviational variables as

$$Z^k(x,c^-) - d_k^- + d_k^+ = \hat{Z}_k, k = 1,2,...,K, \text{ and } W^s(x,d^+) - q_s^+ + q_s^- = \hat{W}_s, s = 1,2,...,S.$$

Where, $\hat{Z}_k$ and $\hat{W}_s$ are the aspiration levels of the objective functions k and s , respectively. With these goals, problem (P_5) can be rewritten as follows:

(P_6)

$$\max \ \zeta$$

Subject to

$$\zeta \le \frac{Z^k(x,c) - L^k}{U^k - L^k}, k = 1,2,...,K,$$

$$\zeta \le \frac{U^s - W^s(x,d)}{U^s - L^s}, s = 1,2,...,S,$$

$$Z^k(x,c) - d_k^- + d_k^+ = \hat{Z}_k, k = 1,2,...,K,$$

$$W^s(x,d) - q_s^+ + q_s^- = \hat{W}^s, s = 1,2,...,S,$$

$$d_k^+, d_k^-, q_s^-, q_s^+ \ge 0, \quad k = 1,2,...,K, \quad s = 1,2,...,S,$$

$$0 \le \zeta \le 1,$$

$$c_j^k \in \left(\tilde{c}_j^k\right)_\alpha, d_j^s \in \left(\tilde{d}_j^s\right)_\alpha, A_j \in \left(\tilde{A}_j\right)_\alpha = \left(\tilde{a}_{ij}\right)_\alpha \text{ , and } \tilde{B} \in \left(\tilde{B}\right)_\alpha,$$

$$x \in M(x,A,B) = \left\{ x \in R^n : \sum_{j=1}^{n} A_j x_j = B, x_j \ge 0, j = 1,2,...,n \right\}.$$

It is clear that problem (P_6) is not linear programming, even all the membership functions defined in Eqs. (7) and (8) are linear. To solve this problem using the linear programming technique, let us introduce the following two set-valued functions:

$$U_j(c_j^k, c_j^s) = \left\{ \begin{array}{l} (x,\zeta) \in R^{n+1} : \zeta \le \mu_k(Z^k(x,c), k = 1,2,...,K; \ \zeta \le \mu_s(W^{s_s}(x,d) \\ s = 1,2,...,S, \ j = 1,2,...,n \end{array} \right\}, \tag{9}$$

$$V_j(A_j, B) = \left\{ x \in R^n : \sum_{j=1}^{n} A_j x_j = B, j = 1, 2, \ldots, n \right\} \tag{10}$$

Proposition 1. (Sakawa [25]). (1) If $\left(c_j^k\right)^1 \leq \left(c_j^k\right)^2$, then $U_j\left(\left(c_j^k\right)^1, .\right) \supseteq U_j\left(\left(c_j^k\right)^2, .\right)$, and if $\left(d_j^s\right)^1 \leq \left(d_j^s\right)^2$, then $U_j\left(. , \left(d_j^s\right)^1\right) \supseteq U_j\left(. , \left(d_j^s\right)^2\right)$; (2) If $A_j^1 \leq A_j^2$, then $V_j\left(A_j^1, .\right) \supseteq V_j\left(A_j^2, .\right)$, and if $B^1 \leq B^2$, then $V_j\left(., B^1\right) \supseteq V_j\left(., B^2\right)$.

From the properties of the $\alpha -$cut sets of fuzzy $\tilde{c}_j^k, \tilde{d}_j^s, \tilde{B}$, and $\tilde{A}_j$, it should be noted that c_j^k, d_j^s, $\tilde{B}$, and $A_j, j = 1, 2, \ldots n, k = 1, 2, \ldots, K,$ $s = 1, 2, \ldots, S$, can be denoted by the intervals of confidence as $\left[\left(c_j^k\right)^-, \left(c_j^k\right)^+\right], j = 1, 2, \ldots, n$; $\left[\left(d_j^s\right)^-, \left(d_j^s\right)^+\right], s = 1, 2, \ldots, S$; $\left[B^-, B^+\right]$, and $\left[A_j^-, A_j^+\right], j = 1, 2, \ldots, n.$

Through the use of proposition 1 and the arithmetic operations of intervals of confidence, the problem (P_6) can be rewritten as follows:

(P_7)

$$\max \ \zeta$$

Subject to

$$\zeta \leq \frac{Z^k(x, c^-) - L^k}{U^k - L^k}, k = 1, 2, \ldots, K,$$

$$\zeta \leq \frac{U^s - W^s(x, d^+)}{U^s - L^s}, s = 1, 2, \ldots, S,$$

$$Z^k(x, c^-) - d_k^- + d_k^+ = \hat{Z}_k, k = 1, 2, \ldots, K,$$

$$W^s(x, d^+) - q_s^+ + q_s^- = \hat{W}_s, s = 1, 2, \ldots, S,$$

$$d_k^+, d_k^-, q_s^-, q_s^+ \geq 0, k = 1, 2, \ldots, K; s = 1, 2, \ldots, S,$$

$$0 \leq \zeta \leq 1.$$

$$x \in M(x, A, B) = \left\{ x \in R^n : \sum_{j=1}^{n} \left[\left(A_j\right)^-, \left(A_j\right)^+\right] x_j \leq \left[B^-, B^+\right], x_j \geq 0, j = 1, 2, \ldots, n \right\}.$$

5. Solution Procedure

In this section, a solution procedure to solve problem (P_7) can be summarized as:

Step 1. Calculate the individual minimum (lower bound) and the individual maximum (upper bound) of each objective function under the given constraints at $\alpha = 0$.

Step 2. Ask the decision maker to specify the initial value of $\alpha\,(0 < \alpha < 1)$ and the initial aspiration level.

Step 3. Define the membership functions (7) and (8) and the initial aspiration level.

Step 4. Solve (P_6) through (P_7).

Step 5. Introduce the solution to the DM. If the DM satisfied, go to step6. Otherwise, go to step 2.

Step 7. Stop.

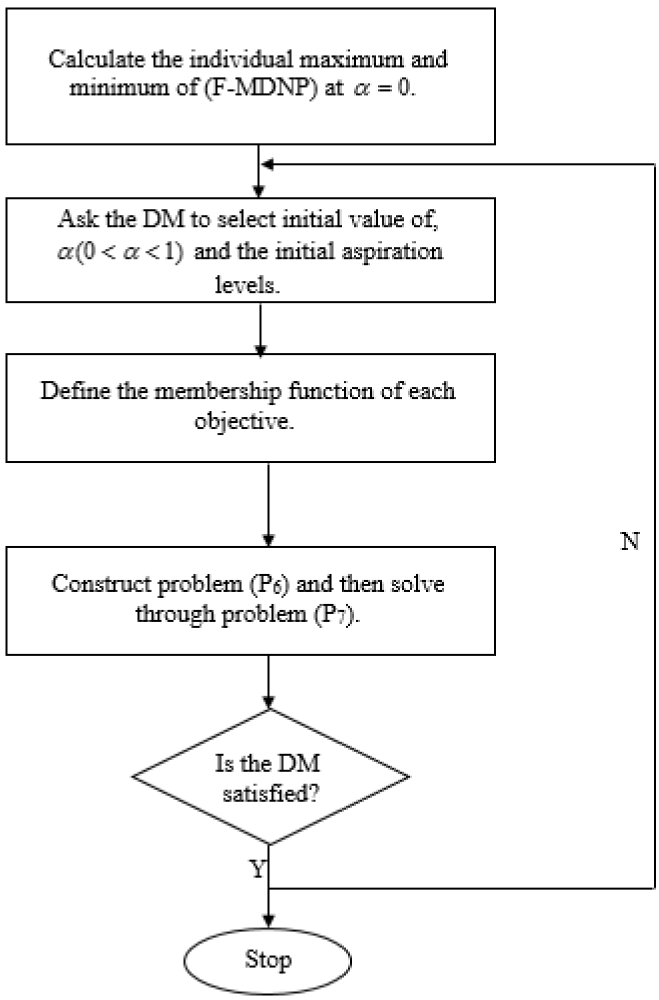

Fig. 1. *Flowchart of the solution procedure of F-MDNP.*

6. Numerical Example

Consider the following F-MDNLP

$$\max \tilde{Z}^1 = (1,2,3)\,x_1 + (3,5,6)x_2 + (6,7,8)x_3 + (0,1,2)x_4\,,$$

$$\tilde{Z}^2 = (2,4,5)\,x_1 + (0,1,2)\,x_2 + (2,3,4)\,x_3 + (10,11,13)x_4,$$

$$\tilde{Z}^3 = (7,9,10)\,x_1 + (2,3,4)x_2 + (0,1,2)x_3 + (1,2,3)\,x_4,$$

$$\min\ \tilde{W}^1 = (0,1,2)\,x_1 + (0,2,3)x_2 + (0,1,2)\,x_2 + (2,3,4)x_4,$$

$$\tilde{W}^2 = (0,1,2)x_1 + (0,1,2)\,x_2 + (0,1,2)\,x_3 + (1,2,3)\,x_4,$$

Subject to

$$(2,3,4)\,x_1 + (3,5,5)\,x_2 + (1,2,3)\,x_3 + (6,8,9)x_4 = (145,150,155),$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Step 1. At $\alpha = 0$, the individual maximum and minimum of each objective function with respect to the given constraints is:

$$\left(Z^1\right)^{\max} = 150,\ \left(Z_2\right)^{\max} = 225, \left(Z^3\right)^{\max} = 450, \left(W^1\right)^{\max} = 75,\ \left(W^2\right)^{\max} = 75;$$
$$\left(Z^1\right)^{\min} = 18.75,\ \left(Z^2\right)^{\min} = 28.4, \left(Z^3\right)^{\min} = 37.5, \left(W^1\right)^{\min} = 50,\ \left(W^2\right)^{\min} = 30.$$

Step 2. Assume the DM selects $\alpha = 0.6$, and the aspiration levels:
$\hat{Z}_1 = 150$, $\hat{Z}_2 = 225$, $\hat{Z}_3 = 450$, $\hat{W}_1 = 75$, $\hat{W}_2 = 75.$

At $\alpha = 0.6$, the problem corresponding to the F-MDNLP is as:

$$\max Z^1 = [1.6, 2.4]x_1 + [4.6, 6.4]x_2 + [6.6, 7.4]x_3 + [0.6, 1.4]x_4,$$
$$Z^2 = [3.6, 4.4]\ x_1 + [0.6, 1.4]\ x_2 + [2.6, 4.6]x_3 + [10.6, 11.4]x_4,$$
$$Z^3 = [8.6, 9.4]x_1 + [2.6, 4.4]x_2 + [0.6, 1.4]x_3 + [1.6, 2.4]x_4,$$
$$\min\ W^1 = [0.6, 1.4]\ x_1 + [1.6, 2.4]x_2 + [0.6, 1.4]x_3 + [2.6, 3.4]x_4,$$
$$W^2 = [0.6, 1.4]\ x_1 + [0.6, 1.4]\ x_2 + [0.6, 1.4]x_3 + [1.6, 2.4]\ x_4,$$

Subject to

$$[2.6, 3.4]x_1 + [4.6, 5.4]\ x_2 + [1.6, 2.4]\ x_3 + [7.6, 8.4]x_4 = [149.6, 150.4],$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Step 4. Solve problem (P_6) through (P_7) as

$$\max \zeta$$

Subject to

$$\begin{aligned}
&0.0853x_1 + 0.2453x_2 + 0.352x_3 + 0.032x_4 - 7\zeta \ge 1,\\
&0.12676x_1 + 0.021127x_2 + 0.09155x_3 + 0.37324x_4 - 6.9225\zeta \ge 1,\\
&0.22933x_1 + 0.06933x_2 + 0.016x_3 + 0.04267x_4 - 11\zeta \ge 1,\\
&0.018667x_1 + 0.032x_2 + 0.01867x_3 + 0.04533x_4 + 0.3333\zeta \le 1,\\
&0.018667x_1 + 0.018667x_2 + 0.018667x_3 + 0.032x_4 + 0.6\zeta \le 1.
\end{aligned}$$

$$\begin{aligned}
&2.6x_1 + 4.6x_2 + 1.6x_3 + 7.6x_4 \le 149.6,\\
&3.4x_1 + 5.4x_2 + 2.4x_3 + 8.4x_4 \le 150.4,\\
&1.6x_1 + 4.6x_2 + 6.6x_3 + 0.6x_4 - d_1^+ + d_1^- = 150,\\
&3.6x_1 + 0.6x_2 + 2.6x_3 + 10.6x_4 - d_2^+ + d_2^- = 225,\\
&8.6x_1 + 2.6x_2 + 0.6x_3 + 1.6x_4 - d_3^+ + d_3^- = 450,\\
&1.4x_1 + 2.4x_2 + 1.4x_3 + 3.4x_4 - q_1^- + q_1^+ = 75,\\
&1.4x_1 + 1.4x_2 + 1.4x_3 + 2.4x_4 - q_2^- + q_2^+ = 75,\\
&0 < \zeta \le 1,\\
&x_1, x_2, x_3, x_4, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, q_1^-, q_1^+, q_2^-, q_2^+ \ge 0.
\end{aligned}$$

The problem is solved using any software (say, Lingo) and the results are:

$$\begin{aligned}
&x_1 = 31.94712,\ x_2 = 0,\ x_3 = 5.237233,\ x_4 = 0,\ d_1^+ = 0,\ d_1^- = 64{,}31887,\\
&d_2^+ = 0,\ d_2^- = 96.37355,\ d_3^+ = 0,\ d_3^- = 172.1124,\ q_1^- = 0,\ q_1^+ = 22.94190\ ,\\
&q_2^- = 0,\ q_2^+ = 22.94190, \zeta = 0.5037334,\ Z_1 \in [\,85.6811, 115.4286],\\
&Z^1 \in [122.2370, 164.6586],\ Z^2 \in [\,277.8876\ ,\ 307.63505],\ W^1 = W^2 \in [122.3106,\ 52.0581].
\end{aligned}$$

Step 5. Assume that the DM has been satisfied with the solution, go to Step 6.

Table 1. *The fuzzy efficient solution*.

Variables	Value	Objective values
x_1	31. 94712	$\zeta = 0.5037334$
	0	$Z^1 = (63.370518, 100.554871, 137.739224)$
x_2		$Z^2 = (74.368706, 143.500179, 180.684532)$
		$Z^3 = (223.62984, 292.761313, 329.945666)$
x_3	5.237233	$W^1 = (0, 37.184353, 74.368706) = W^2$
x_4	0	

7. Conclusion

In this paper, the fuzzy multi-criteria de novo programming problems was studied. Fuzzy goal programming approach was used to obtain $\alpha-$optimal compromise solution and to achieve satisfactory results for the DM. The decision maker's role only was the evaluation of the $\alpha-$ efficient solutions to limit the influences of his/ her incomplete knowledge about the problem domain.

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