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Neutrosophic Perspective on DEA

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ABSTRACT

Several attempts have been made to deal with uncertain input and output data in Data Envelopment Analysis (DEA). However, due to the limitation of these methods, they cannot be applied for solving DEA with indeterminacy, impreciseness, vagueness, inconsistent and incompleteness information. So this paper for the first time, deals with the Neutrosophic Data Envelopment Analysis and present a new model to solve it.

1. Introduction

With the advent of technology and the complexity and volume of information, senior executives have required themselves to apply scientific methods to determine and increase the productivity of the organization under their jurisdiction.

Data Envelopment Analysis (DEA) is a linear programming for measuring the relative efficiencies of homogeneous decision making units (DMUs) without knowing production functions, just by utilizing input and output information [1,2]. DEA technique has just been effectively connected in various cases such as financial services, agricultural, health care services, education, manufacturing, telecommunication, and supply chain management. Classical DEA models, for example, CCR and BCC models [1, 2] require crisp inputs and outputs, which may not generally be accessible in real world applications.

However, the watched values of the input and output data in genuine issues are infrequently uncertain [3-12], loose assessments might be the consequence of unquantifiable, inadequate and non- obtainable information. It is useful to consider the knowledge of experts about the parameters as fuzzy data. The concept of fuzzy set was introduced in [13]. After this, many researchers have been applied this theory for different problems; see [14-22] and references there in. There are also many studies reported utilizing fuzzy set theory in DEA; see [23-33] and references there in.

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Although, fuzzy set theory has been introduced as a powerful tool to quantify vague data and several authors have suggested various fuzzy methods in DEA, However, there is a key shortcoming in previous approaches.

Smarandache [34-36] initiated the notion of Neutrosophic Set (NS) which is generalization of classical set, fuzzy set, intuitionistic fuzzy set, and so on. A neutrosophic set is a part of neutrosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic sets are relatively new extensions of intuitionistic fuzzy sets. Neutrosophic set is a generalization of fuzzy set and intuitionistic fuzzy set. In the neutrosophic set, the degree of membership-truth (T), the degree of indeterminacy (I), and the degree of non-membership-falsehood (F) are independent, therefore their sum (as single-valued numbers) can be up to 3.

Neutrosophic set has been used in solving problems that involve indeterminacy, uncertainty, impreciseness, vagueness, inconsistent, incompleteness, etc. [36].

The neutrosophic logic has been approved by many researchers in a short time. Especially, a significant acceleration in the number of publications on neutrosophic sets is observed after 2015 [37-46]. So, when the model is uncertainty with DEA then the concepts of imprecise DEA emerge that mentioned in [22-33]. But these logics does not have the term indeterminacy. To handle such situations, this paper aims to propose a new model of DEA with neutrosophic input and output.

2. Neutrosophic Set

Definition 1: [34]. Let X be a space point or objects, with a genetic element in X denoted by *x*. A single-valued NS, V in X is characterised by three independent parts, namely truth-Membership Function T_V , indeterminacy- Membership Function I_V and falsity- Membership Function F_V , such that:

$$T_V : X \to [0,1], I_V : X \to [0,1], \text{ and } F_V : X \to [0,1].$$

Now, V is denoted as $V = \{< x, (T_V(x), I_V(x), F_V(x)) > \ | x \in X\}$, satisfying

$$0 \le T_V(x) + I_V(x) + F_V(x) \le 3.$$

Definition 2: [40]. (single-valued triangular neutrosophic number (SNTNN)): Let $\hat{r}^N = \left\langle \left(\overset{\frown}{r_{ij,l}}, \overset{\frown}{r_{ij,m}}, \overset{\frown}{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ is a special NS on the real number set R, whose truth-MF $\Delta_{\tilde{r}}(x)$, indeterminacy-MF $\nabla_{\tilde{r}}(x)$, and falsity-MF $\mho_{\tilde{r}}(x)$ are given as follows:

$$\Delta_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(x-\overset{\frown}{r}_{ij,l}\right)}{\left(\overset{\frown}{r}_{ij,m}-\overset{\frown}{r}_{ij,l}\right)} & \overset{\frown}{r}_{ij,l}\le x<\overset{\frown}{r}_{ij,m}, \\ 1 & x=\overset{\frown}{r}_{ij,m}, \\ \dfrac{\left(\overset{\frown}{r}_{ij,k}-x\right)}{\left(\overset{\frown}{r}_{ij,k}-\overset{\frown}{r}_{ij,m}\right)} & \overset{\frown}{r}_{ij,m}<x\le\overset{\frown}{r}_{ij,k}, \\ 0 & otherwise.\end{cases} \tag{1}$$

$$\nabla_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(r_{ij,m}-x\right)}{\left(r_{ij,m}-r_{ij,l}\right)} & r_{ij,l}\le x<r_{ij,m}, \\ 0 & x=r_{ij,m}, \\ \dfrac{\left(x-r_{ij,m}\right)}{\left(r_{ij,k}-r_{ij,m}\right)} & r_{ij,m}<x\le r_{ij,k}, \\ 1 & otherwise.\end{cases} \tag{2}$$

$$\mho_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(\underline{r}_{ij,m}-x\right)}{\left(\underline{r}_{ij,m}-\underline{r}_{ij,l}\right)} & \underline{r}_{ij,l}\le x<\underline{r}_{ij,m}, \\ 0 & x=\underline{r}_{ij,m}, \\ \dfrac{\left(x-\underline{r}_{ij,m}\right)}{\left(\underline{r}_{ij,k}-\underline{r}_{ij,m}\right)} & \underline{r}_{ij,m}<x\le\underline{r}_{ij,k}, \\ 1 & otherwise.\end{cases} \tag{3}$$

Where $0\le\Delta_{\tilde{r}}(x)+\nabla_{\tilde{r}}(x)+\mho_{\tilde{r}}(x)\le 3,\quad x\in\hat{r}^N$.

Definition 3: [41]. **(**Arithmetic operation). Let $\hat{r}^N=\left\langle\left(\overset{\frown}{r}_{ij,l},\overset{\frown}{r}_{ij,m},\overset{\frown}{r}_{ij,k}\right),\left(r_{ij,l},r_{ij,m},r_{ij,k}\right),\left(\underline{r}_{ij,l},\underline{r}_{ij,m},\underline{r}_{ij,k}\right)\right\rangle$ and $\hat{s}^N=\left\langle\left(\overset{\frown}{s}_{ij,l},\overset{\frown}{s}_{ij,m},\overset{\frown}{s}_{ij,k}\right),\left(s_{ij,l},s_{ij,m},s_{ij,k}\right),\left(\underline{s}_{ij,l},\underline{s}_{ij,m},\underline{s}_{ij,k}\right)\right\rangle$ be two arbitrary SVTNNs, and $\theta\ge 0$; then:

$$\hat{r}^N\oplus\hat{s}^N=\left\langle\left(\overset{\frown}{r}_{ij,l}+\overset{\frown}{s}_{ij,l},\overset{\frown}{r}_{ij,m}+\overset{\frown}{s}_{ij,m},\overset{\frown}{r}_{ij,k}+\overset{\frown}{s}_{ij,k}\right),\left(r_{ij,l}+s_{ij,l},r_{ij,m}+s_{ij,m},r_{ij,k}+s_{ij,k}\right),\right.$$
$$\left.\left(\underline{r}_{ij,l}+\underline{s}_{ij,l},\underline{r}_{ij,m}+\underline{s}_{ij,m},\underline{r}_{ij,k}+\underline{s}_{ij,k}\right)\right\rangle$$

$$\hat{r}^N\otimes\hat{s}^N=\left\langle\left(\overset{\frown}{r}_{ij,l}\cdot\overset{\frown}{s}_{ij,l},\overset{\frown}{r}_{ij,m}\cdot\overset{\frown}{s}_{ij,m},\overset{\frown}{r}_{ij,k}\cdot\overset{\frown}{s}_{ij,k}\right),\left(r_{ij,l}\cdot s_{ij,l},r_{ij,m}\cdot s_{ij,m},r_{ij,k}\cdot s_{ij,k}\right),\right.$$
$$\left.\left(\underline{r}_{ij,l}\cdot\underline{s}_{ij,l},\underline{r}_{ij,m}\cdot\underline{s}_{ij,m},\underline{r}_{ij,k}\cdot\underline{s}_{ij,k}\right)\right\rangle$$

$$\theta\hat{r}^N=\left\langle\left(\theta\overset{\frown}{r}_{ij,l},\theta\overset{\frown}{r}_{ij,m},\theta\overset{\frown}{r}_{ij,k}\right),\left(\theta r_{ij,l},\theta r_{ij,m},\theta r_{ij,k}\right),\left(\theta\underline{r}_{ij,l},\theta\underline{r}_{ij,m},\theta\underline{r}_{ij,k}\right)\right\rangle\ if\ \ (\theta>0),$$

$$-\hat{r}^N = \left\langle \left(-\overline{r_{ij,k}}, -\overline{r_{ij,m}}, -\overline{r_{ij,l}}\right), \left(-r_{ij,k}, -r_{ij,m}, -r_{ij,l}\right), \left(-\underline{r_{ij,k}}, -\underline{r_{ij,m}}, -\underline{r_{ij,l}}\right) \right\rangle .$$

Definition 4: [40]. Let $\hat{r}^N = \left\langle \left(\overline{r_{ij,l}}, \overline{r_{ij,m}}, \overline{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ then the score function and the accuracy function are defined as follows:

$$s(\tilde{r}) = \frac{1}{12}\left[8 + \left(\overline{r_{ij,l}} + 2\cdot\overline{r_{ij,m}} + \overline{r_{ij,k}}\right) - \left(r_{ij,l} + 2\cdot r_{ij,m} + r_{ij,k}\right) - \left(\underline{r_{ij,l}} + 2\cdot\underline{r_{ij,m}} + \underline{r_{ij,k}}\right)\right], \quad (4)$$

$$H(\tilde{r}) = \frac{1}{4}\left[\left(\overline{r_{ij,l}} + 2\cdot\overline{r_{ij,m}} + \overline{r_{ij,k}}\right) - \left(\underline{r_{ij,l}} + 2\cdot\underline{r_{ij,m}} + \underline{r_{ij,k}}\right)\right]. \quad (5)$$

Definition 5: [30]. $\hat{r}^N = \left\langle \left(\overline{r_{ij,l}}, \overline{r_{ij,m}}, \overline{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ and $\hat{s}^N = \left\langle \left(\overline{s_{ij,l}}, \overline{s_{ij,m}}, \overline{s_{ij,k}}\right),\right.$ $\left(s_{ij,l}, s_{ij,m}, s_{ij,k}\right)$, $\left.\left(\underline{s_{ij,l}}, \underline{s_{ij,m}}, \underline{s_{ij,k}}\right)\right\rangle$ be two arbitrary SVTNNs, the ranking of $\tilde{r}$ and $\tilde{s}$ by score function is described as follows:

$$\begin{gathered} \text{if } s(\tilde{r}) \prec s(\tilde{s}) \text{ then } \tilde{r} \prec \tilde{s}, \\ \text{if } s(\tilde{r}) \approx s(\tilde{s}) \text{ and if} \\ H(\tilde{r}) \prec H(\tilde{s}) \text{ then } \tilde{r} \prec \tilde{s} \\ H(\tilde{r}) \succ H(\tilde{s}) \text{ then } \tilde{r} \succ \tilde{s} \\ H(\tilde{r}) \approx H(\tilde{s}) \text{ then } \tilde{r} \approx \tilde{s}. \end{gathered}$$

3. Data Envelopment Analysis

The efficiency of a DMU is established as the ratio of sum weighted output to sum weighted input, subjected to happen between one and zero. Let a set of $n\left[0,1\right]$ DMUs, with each DMUj $(j = 1,2,...,n)$ by using m inputs x_{ij} $(i = 1,2,...,m)$ and producing s outputs y_{rj} $(r = 1,2,...,s)$. If DMUp is under consideration, the CCR model for the relative efficiency is the following model [1]:

$$\begin{aligned} &\theta_p^* = \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \\ &s.t. \\ &\qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad \forall j \\ &\qquad u_r, v_i \ge 0 \qquad\qquad \forall r, i. \end{aligned} \quad (6)$$

Where $u_r (r = 1,2,....,s)$ and $v_i (i = 1,2,....,m)$ are the weights of the i th input and r th output. This fractional program is calculated for each DMU to find out its best input and output weights. To simplify the computation, the nonlinear program shown as (6) can be converted to a linear programming (LP) and the model was called CCR model.

$$\theta_p^{*} = \max \sum_{r=1}^{s} u_r y_{rp}$$
$$s.t:$$
$$\sum_{i=1}^{m} v_i x_{ip} = 1 \qquad (7)$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad \forall j$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

We run program (7) n -times to work out the efficiency of n DMUs. The DMUp is efficient if $\theta^{*} = 1$, otherwise, is inefficient.

4. Proposed Model

The Neutrosophic CCR model with neutrosophic input and output data is given in Model (8).

$$\theta_p^{*} = \max \sum_{r=1}^{s} u_r \tilde{y}_{rp}$$
$$s.t.$$
$$\sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1 \qquad (8)$$
$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0, \qquad \forall j$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

Where $\tilde{x}_{ij}\,(i = 1,2,...,m)$ and $\tilde{y}_{rj}\,(r = 1,2,...,s)$ are neutrosophic input and fuzzy output for the j th DMUj.

Let $\tilde{x}_{ij} = \left\langle \left(\overline{x_{ij,l}}, \overline{x_{ij,m}}, \overline{x_{ij,k}}\right), \left(x_{ij,l}, x_{ij,m}, x_{ij,k}\right), \left(\underline{x_{ij,l}}, \underline{x_{ij,m}}, \underline{x_{ij,k}}\right) \right\rangle$ and $\tilde{y}_{rj} = \left\langle \left(\overline{y_{ij,l}}, \overline{y_{ij,m}}, \overline{y_{ij,k}}\right), \left(y_{ij,l}, y_{ij,m}, y_{ij,k}\right), \left(\underline{y_{ij,l}}, \underline{y_{ij,m}}, \underline{y_{ij,k}}\right) \right\rangle$ be two arbitrary SVTNNs related to input and output. By Definition 2.3:

$$\theta_p^{*} = \max \sum_{r=1}^{s} u_r \left\langle \left(\overline{y_{ip,l}}, \overline{y_{ip,m}}, \overline{y_{ip,k}}\right), \left(y_{ip,l}, y_{ip,m}, y_{ip,k}\right), \left(\underline{y_{ip,l}}, \underline{y_{ip,m}}, \underline{y_{ip,k}}\right) \right\rangle$$
$$s.t.$$
$$\sum_{i=1}^{m} v_i \left\langle \left(\overline{x_{ip,l}}, \overline{x_{ip,m}}, \overline{x_{ip,k}}\right), \left(x_{ip,l}, x_{ip,m}, x_{ip,k}\right), \left(\underline{x_{ip,l}}, \underline{x_{ip,m}}, \underline{x_{ip,k}}\right) \right\rangle = \tilde{1}$$
$$\sum_{r=1}^{s} u_r \left\langle \left(\overline{y_{ij,l}}, \overline{y_{ij,m}}, \overline{y_{ij,k}}\right), \left(y_{ij,l}, y_{ij,m}, y_{ij,k}\right), \left(\underline{y_{ij,l}}, \underline{y_{ij,m}}, \underline{y_{ij,k}}\right) \right\rangle +$$
$$\sum_{i=1}^{m} v_i \left\langle \left(-\overline{x_{ij,k}}, -\overline{x_{ij,m}}, -\overline{x_{ij,l}}\right), \left(-x_{ij,k}, -x_{ij,m}, -x_{ij,l}\right), \left(\underline{-x_{ij,k}}, \underline{-x_{ij,m}}, \underline{-x_{ij,l}}\right) \right\rangle \le \tilde{0},$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

Now, using the score function (4), we can transform above model as a crisp DEA model and therefore, solve it by any conventional method.

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