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Combinatorial Optimization of Permutation-Based Quadratic Assignment Problem Using Optics Inspired Optimization

Soheila Badrloo[1,*]***, Ali Husseinzadeh Kashan***[2]

[1]*Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.*

[2]*Department of Industerial Engineering, Tarbiat Modares University, Tehran, Iran.*

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ABSTRACT

A lot of real-world problems such as assignment of special rooms in hospitals, operating room layout, image processing, etc., could be formulated in terms of Quadratic assignment problem. Different exact methods are suggested to solve these problems, but because of the special structure of these problems, by increasing the size of the problem, finding an exact solution become more complicated and even impossible. So, employing meta-heuristic algorithms is inevitable, due to this problem we use Optics Inspired Optimization (OIO) in this paper. The obtained results and its comparison with the solutions of the central library of Quadratic Assignment Problem (QAPLIB) show that the proposed algorithm can exactly solve small-sized problems with 100% efficiency while the efficiency of medium-to-large size instances is 96%. Accordingly, one can conclude that the proposed OIO has generally high efficiency for solving permutation-based problems.

1. Introduction

In mathematics and engineering, meta-heuristic algorithms are getting to be the most vital part in tackling the optimization problems. Wide utilization ranges of meta-heuristic calculations have come up over the final three decades, and numerous meta-heuristic algorithms such as genetic algorithm, differential evolution algorithm, particle swarm optimization algorithm, artificial bee colony algorithm, ant colony algorithm, etc., have gotten to be well known for real-world optimization issues. In common, the meta-heuristic algorithms show a few instruments from nature as creature nourishing propensities, mating inspiration or chasing method [20].

Decision making and optimization problems are divided into two categories from solution approach viewpoint; tractable and intractable or hard. For some of these problems (P category), an exact solution can be obtained in polynomial time, while for NP category, one cannot solve them in polynomial time. Only for some of the NP-hard problems which are the hardest problems of NP category, one can solve them inexactly in polynomial time. An important category of combinatorial problems is those in which the problem solution can be represented in terms of a permutation of elements (objects, items, members, actions, etc.). In this category of problems, if there are n objects, the goal is to determine a sequence of them, where a given criteria is optimized concerning the problem nature. In this regard, one can mention

Corresponding author
E-mail address: Soheylabaderloo87@yahoo.com
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to Traveling Salesman Problem (TSP), Flow Shop Scheduling Problem (FSSP) and Quadratic Assignment Problem (QAP) as the problems belonging to the permutation-based optimization problems.

One of the other most important permutation problems with special importance in facility layout planning and other places is QAP whose objective in these problems is assigning some facilities into some locations, where total cost is minimized. The permutation-based combinatorial optimization problems often belong to NP-hard category of problems and finding the optimal solution within reasonable computation time is currently impossible for any arbitrary number of elements. Accordingly, meta-heuristic algorithms inspired from nature have been much-paid attention by the researchers in recent years which have similarities with social/nature systems and have yielded acceptable results [27].

Meta-heuristic algorithms with specialized knowledge have the best performance. These algorithms are used widely for both discrete and continuous variables. The main goal of population- and routing-based meta-heuristic algorithms is to find the global optimum infeasible region through random movements. The key difference among different algorithms is about route of the algorithm, whose objective is the next movement in solution space. In this research, Optics Inspired Optimization (OIO) algorithm as an innovative method extracted from meta-heuristics design algorithms is used. Despite OIO algorithm is intrinsically designed for continuous space, however, its application in discrete optimization problems including permutation-based combinatorial optimization ones has not been investigated so far. This is happened because its usage necessitates introducing appropriate methods and tools to transform the search process of optimal solution from continuous space into discrete one [28].

Various methods have been proposed to solve this kind of problems in a precise and innovative way. But due to the specific structure of such combinatorial problems have such as the size of the problem increases, the time required to obtain the optimal solution also increases and sometimes it may take years. The aim of population-based and routing metaheuristic approaches is to optimize the overall spatialization by random motion. The key difference between the algorithms lies in the path of the algorithm, all of which aim at the next move in the answer space. OIO algorithm, unlike other algorithms, equip a mechanism to handle constraints and to avoid the drawbacks of typical techniques, a feasibility measure is used beside the objective function value to bias the search towards feasible regions. Consequently, this algorithm is capable to find the global optimum of many investigated problems in in a reasonable and fast time.

The main target of this paper is to apply the proposed OIO algorithm to the QAP problem. The proposed OIO algorithm's performance on QAP instance was compared with the well-known meta-heuristic algorithms such as Genetic Algorithm. Finally, we present the solution quality of the proposed OIO algorithm for solving QAP using some instances in QAPLIB. The rest of the paper is organized as follows:

In Section 2, the basic information about quadric assignment problem will be offered, literature review and fundamental information about the Quadratic Assignment Problem (QAP) problem will be offered in Section 3. Section 4 the basic information about the original OIO algorithm is given briefly. QAP tests and the performance of the OIO algorithm is discussed in Section 5. In the last section, the conclusion and future works are presented.

2. Quadratic Assignment Problem (QAP)

The QAP was first introduced by Koopmans and Beckman in 1957 as a mathematical model of inseparable economics activities establishment problem [19]. This problem can generally be defined as follows:

A set of facilities are assigned to a set of locations, where total cost as a function of distance and flow between facilities is minimized. Also, the assignment cost of a given facility which is located in a specific position should be minimized, as well. In other words, the goal is to establish a set of machines, facilities, warehouses, etc., which are connected within a specific environment such as shop floor [15].

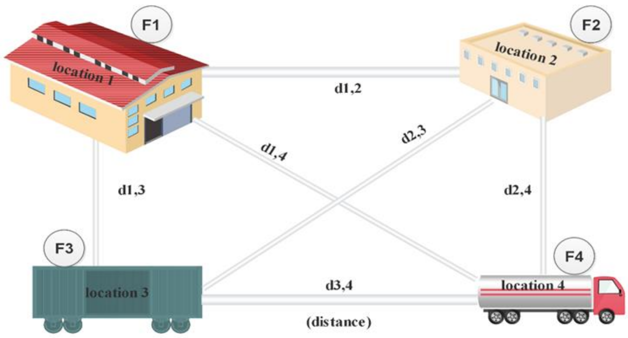


Fig. 1. *Quadratic assignment problem (QAP).*

QAP can be formulated as a combinatorial optimization problem in the design of buildings layout and facility layout planning of industrial units and even lots of other cases, Fig.1 show one example of Quadratic Assignment Problem. Production companies spend a huge time and cost to design or redesign of their facilities. These designs have a significant impact on the system's performance since poor layouts lead to increasing the costs and decreasing the efficiency of the system compared to what is desired for the customers [4]. The QAP could be a challenging issue with a wide extent of applications in assorted areas counting backboard wiring issue. Since it belongs to NP-hard category, exact (deterministic) algorithms can only solve small-size due to this problem, developing stochastic and meta-heuristic algorithms could be paid more attention. These algorithms can obtain near-optimal solutions within reasonable computational time.

In QAP, there is a set named $N=\{1,2,\ldots,n\}$ and some $n \times n$ matrices called $F=\{f_{ij}\}$, $D=\{d_{ij}\}$ and $C=\{c_{ij}\}$ with the goal of finding a permutation of set N, which can minimize the following function in terms of Eq. (1) [19]:

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} d_{\phi(i)\phi(j)} + \sum_{i=1}^{n} C_{i\phi(i)}. \quad (1)$$

One can mention to designing the keyboard of a computer [7], which designing electronic kits and finding the facilities positions and telecommunication posts to minimize total used wires, as other applications of QAP. Also, locating a university campus or small town which was first introduced by [10], could be another application. In the latter problem, n buildings are to be constructed, where d_{kl} and f_{ij} are the distance and traffic volume between building k and l and building i and j, respectively. The goal is to minimize the total traversed distance between all buildings. The other applications of QAP are planet layout problem in a hospital [9], design of control panel and typewriter keyword [8], chemical reactions analysis in organic compounds [29], ranking the archeology data [9], etc.

3. Literature Review

Optimization methods are classified into two categories: exact and approximation methods. The exact techniques find the optimal solution of a problem and ensure the optimality conditions, while approximation (heuristic) methods generate high-quality solutions within reasonable computational time, however, do not guarantee to reach to global optimum. Exact methods endure from high computational complexity such as dynamic programming, branch-and-bound, and cutting planes or combinations of among these methods. There is currently no solution for this problem that runs in polynomial time unless P=NP. The heuristic algorithms are problem dependent and require correct strategies to endure from tall computational complexity such as energetic programming, branch-and-bound, and cutting planes or combinations of among these strategies. Now there is no arrangement for this issue that runs in polynomial time unless P=NP. The heuristic calculations are issue subordinate and require modifications when applying to diverse problems [3].

QAP is a problem with exponential complexity. Generally speaking, problems with a size of larger than 20 cannot be solved optimally by exact methods, or need a huge computational time which is not reasonable at all. In these cases, meta-heuristics as approximation methods are employed, equipped with mechanisms to get out of local optima and can be applied to a large variety of optimization problems. Different classes of these algorithms are developed in recent decades, some of the most popular algorithms are Genetic Algorithm (GA) [13], Particle Swarm Optimization (PSO) algorithm [23], Artificial Bee Colony (ABC) algorithm [17], Differential Evolution (DE) algorithm [22], Ant Colony (ACO) algorithm [11], etc. OIO algorithm is one of the newest meta-heuristic algorithms which was first introduced by [14] and is employed in this paper.

Dokeroglu et al. [30] used Artificial Bee Colony (ABC) optimization for the Quadratic Assignment Problems. The ABC has been detailed to be a proficient meta-heuristic for the arrangement of numerous intractable issues. It has promising results making it a great candidate to get (near)-optimal solutions for well-known NP-Hard issues. In this paper 125 of 134 benchmark problem instances have been solved optimally from the QAPLIB library and the result showed 0.27% deviation for 9 large problem instances that could not be solved optimally. The efficiency of the ABC optimization algorithms is competitive with state-of-the meta-heuristic algorithms.

Kilic and Yuzgec [31] solved the quadratic assignment problem with tournament selection-based antlion optimization algorithm. In this paper, they compare the QAP results with several other meta-heuristic algorithms. The results provide the proposed TALO algorithm had the best efficiency in comparison with those of the other meta-heuristic algorithms. Due to the accomplishment, of the TALO algorithm, this paper suggested that this algorithm can be applied for other optimization problems.

Abdel-Baset et al. [3] Integrated the Whale Algorithm with Tabu Search for solving quadratic assignment Problem. This work used Tabu Search to increase the quality of the solution obtained by WA for QAP problem as a local search algorithm. This paper included fourteen different case studies, that included 122 test problems which have been employed for analyzing the efficiency of the proposed WAITS. The results appeared that this new algorithm finds near-optimal solutions with an acceptable computational time.

Tosun [25] used a Parallel Hybrid Algorithm (PHA) for the solution of the quadratic assignment problem. In this paper, a PHA with three stages was proposed. At first, a genetic algorithm was utilized to get a high-quality seed. Afterwar, an enhancement stage was run on the initial seed. At last, a robust Tabu search was done on the intermediate solution to discover a near-optimal result. A parallel hybrid algorithm was able to solve big problems instance size of 256 within11h, and with a higher precision than the well-known solutions.

Husseinzadeh Kashan [28] Introduced optimization inspired by Optics as a new algorithm for constraining. The aim of this paper was as below:

- Optimization inspired from Optics Introducing as a new algorithm.
- The instrument of algorithm is basic which lets its implementation easily.
- Exploring the use of this algorithm on designing in mechanical engineering.
- Capacity of this algorithm to find the optimum solution of numerous investigated problems.
- The algorithm carries on constantly and conducts so reliable than other algorithms.

There are several nature-inspired algorithms which adapt their source of inspiration from Physics, e.g., Ray optimization [18], Spiral Dynamics inspired optimization [24], Central force optimization [12], etc. It has been observed that concave surfaces/mirrors reflect the light rays toward the principal axis. Furthermore, many new meta-heuristic algorithms are launched to mimic the behavior of many species as insects, flowers, water, whales which leads to a new tsunami of meta-heuristic algorithms. Some of the recent algorithms use metaphors to give a new name to an improvement to an existing algorithm with little novelty.

The motive of this paper is to propose a method for tackling QAP and find near-optimal solutions to alleviate high computational cost as the main drawback of exact methods. The meta-heuristic algorithms can't guarantee to obtain the optimal solutions but they have an additional advantage over the exact methods which is obtaining acceptable results within a reasonable time. As such, an integrated version of the meta-heuristic Optic Inspired Optimization algorithm is proposed.

Table 1. *Algorithms for solving QAP.*

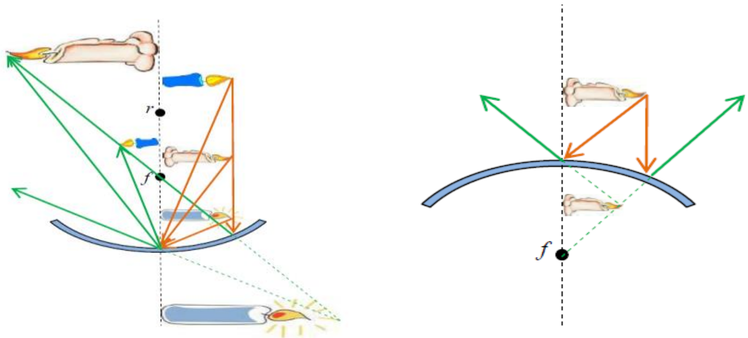
Author	Year	Solution algorithm	Result
Pradeepmon et al.	*2018*	*modified discrete particle swarm optimization algorithm*	*The algorithm is used to solve some benchmark instances of QAP taken from QAP Library and the results showed that minute deviations from best-known solutions.*
Xia and Zhou	*2018*	*Ant Colony Optimization (ACO)*	*This algorithm shows that ACO can outperform the 2-exchange local search algorithm on an instance of the QAP problem.*
Ahmed	*2018*	*A hybrid algorithm combining Lexi search and genetic algorithms*	*a comparative think about has been carried out between LSGA and bound together molecule swarm optimization (UPSO) for the same occasions. In terms of arrangement quality, LSGA beat UPSO for all category of occurrences. Moreover, in terms of computational time, but for seven occasions, LSGA outflanked UPSO.*
Abdel-Baset et al.	*2018*	*Elite Opposition-Flower Pollination Algorithm (EOFPA)*	*This algorithm compares with the best proposals from the related literature and showed that the proposed algorithm is predominant to some other algorithms*

Author	Year	Solution algorithm	Result
Shylo	*2017*	*Repeated Iterated Tabu Search Method*	*compare this algorithm with the best existing algorithms for solving this problem had shown its competitiveness with respect to both its performance and quality of solutions.*
Pradeepmon et al.	*2016*	*discrete version of Particle Swarm Optimization (DPSO)*	*Taguchi's robust plan method has been designed in this paper for discrete molecule swarm optimization calculation for tackling quadratic task issues.*
Lim et al.	*2016*	*Biogeography-Based Optimization (BBO)*	*Out of 61 data from QAPLIB tested, the hybrid method could obtain the well-known solutions for 57 of them.*
Ahmed	*2015*	*Improved Genetic Algorithm (IGA)*	*The efficiency of some benchmark QAPLIB instances have been tested by the proposed GA using SCX and then compared with GAs using other existing crossover operators.*
Azarbonyad and Babazadeh	*2014*	*Proposed genetic algorithm*	*The gotten comes about in sensible time appear the effectiveness of proposed GA*

The popularity of heuristics and meta-heuristics in solving QAP problems have shown above table (Table 1). By the way, we want to know why we need new or improve method for this problem. The answer is that as an optimization problem, QAP can be better solved by a new memetic or improved algorithm when considering a specific dataset. This motivated our try to improve a solution for QAP using the recently proposed Optics Inspired Optimization (OIO). The goal of this paper is to require a procedure for solving QAP and find the best optimal solutions. By the way, the meta-heuristic algorithms can't ensure to get the ideal solutions but they have an additional advantage over the exact methods which are obtaining acceptable results within a reasonable time. As such, an integrated version of the meta-heuristic whale optimization algorithm with optics search (OIO) is proposed.

4. Optics Inspired Optimization (OIO)

The content of this section is mainly focused on optic theory which constitutes the structure of the optics algorithm so-called OIO algorithm. Optic is a branch of physics containing behavior and properties of light which comprises its interactions with matter and the construction of tools that use or detect it. One can find the practical applications of optics in a vast variety of technologies and everyday objects such as telescopes, mirrors, microscopes, lenses, etc. A curved or spherical mirror is a mirror with a curved reflective surface which may be either concave (bulging inward) or convex (bulging outward). Most of curved mirrors have surfaces that are shaped like part of a sphere. The behavior of light reflected by a curved mirror is subject to the same laws as that of plane mirrors, known as the “laws of reflection”. The first law says that the incident ray, the reflected ray, and the normal ray all lie on the same plane, and the second law says that the angle between the incident and the normal rays equals to the angle between the reflected ray and the normal one.



(a). *Concave mirror*. (b). *convex mirror*.

Fig. 2. *Location effect of an object toward focal distance on the object's image.*

Due to the law of reflection, the converging and diverging behavior of concave and convex mirrors cause that curved mirrors show different image types. The OIO is a recently proposed algorithm for unconstrained optimization which treats the surface of the function to be optimized as a wavy mirror in which each peak is assumed to reflect as a convex mirror and each valley to reflect as a concave one. Each individual is treated as an artificial light point that its glittered ray is reflected back by the function surface, given that the surface is convex or concave, and the artificial image (a new solution) is formed based on mirror equations adopted from Optics. There are several constraint handling techniques which have been proposed for handling infeasible solutions. However, these techniques may suffer from problem dependency, no unique way for designing their operators, no unique way for updating their internal parameters, increasing the computational complexity, etc. To equip OIO with a mechanism to handle constraints and to avoid the drawbacks of typical techniques, a feasibility measure is used beside the objective function value to bias the search towards feasible regions. Such a consideration requires to modify several modules in the basic OIO algorithm.

A concave mirror has a reflecting surface that bulges inward (away from the incident light). Such mirrors are used to focus light. Concave mirrors show different image types depending on the distance between the object and the mirror.

These kinds of mirrors are called "converging", because they tend to collect light that falls on them, refocusing parallel incoming rays toward a focus. In concave mirrors, if an object lies between focal point and mirror, its image is virtual, upright and larger than the object, while if the object lies behind of focal point (i.e., the distance of object to mirror is more than focal distance), the image is always real and inverted, however the image size depends on the object's position.

A convex mirror is a curved mirror in which the reflective surface bulges toward the light source. Convex mirrors reflect light outwards and always form a virtual image, since the focus (f) and the center of curvature (r) are both imaginary points positioned inside the mirror, which are not accessible. A collimated beam of light diverges after reflection from a convex mirror, since the normal to the surface differs with each spot on the mirror. The image on a convex mirror is always virtual, upright and smaller than the object. The ray diagram in Fig. 2(b) shows how an image is formed by a spherical convex mirror. Only two rays are used in Fig. 2, however it should be pointed out that there are an infinite number of rays, obviously. Actually, suffice it to draw only two rays so as to locate the image of a point on an object. Accordingly, only two rays are considered coming from the top of object (candles in our case) and then depicted as shown in Fig. 2. When these two rays are extended backward (or forward in case of a concave mirror), their intersection locates the image point position. Similarly, one can depict the two rays coming from each point of the object and then obtain the corresponding image point. Accordingly, one can see the whole image of the object.

The spherical mirror model can be used in order to develop a simple equation for such a kind of mirrors. Using triangle relationship and the laws of reflection, it is also possible to develop a quantitative relationship between the object and image distances. Suppose that f be the focal length, r be the radius of curvature (r = 2f), p be the object position, and q be the image position. The mirror equation can be determined approximately as follows in terms of Eq. (2):

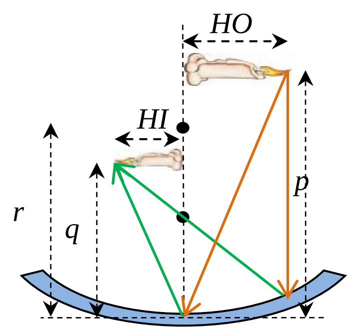


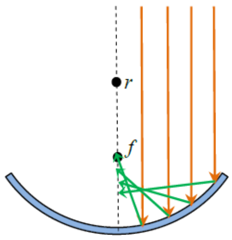
Fig. 3. *Triangular relationships between object and its images.*

$$\frac{2}{r}=\frac{1}{p}+\frac{1}{q}\Rightarrow q=\frac{rp}{2p-r}. \tag{2}$$

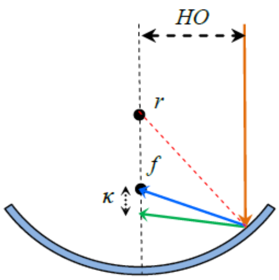
All distances are measured from the vertex, i.e., the point where the axis meets the mirror. Generally, distances are considered positive, if they lie on the same side of the mirror as the light rays themselves. However, if they lie behind the mirror, the distances are taken negative into account. In other words, both of r (or f) and q are negative for a convex mirror and only q is negative for a concave mirror, only when the object lies between the vertex and the focal point.
Magnification (m) is another property of a spherical mirror, which determines how much larger or smaller the image is relative to the object. In practice, m is a simple ratio of the image height (HI) to the object height (HO). Using triangle geometry, this ratio can be written in terms of image position and object position as follows (Eq. 3):

$$m=-\frac{q}{p}=\frac{HI}{HO}\Rightarrow HI=-HO\frac{q}{p}. \tag{3}$$

Virtual images are always behind the mirror, which means that the image position is negative. When the absolute value of a magnification is less than 1, the image is smaller than the object and when the absolute value of the magnification is greater than 1, the image is larger than the object. A negative magnification means that the image is inverted relative to the object.



(a). Spherical mirror.



(b). Aberration amount of ray from focal point f.

Fig. 4. *Spherical mirrors and aberration amount of object from mirror.*

In order to form an image, this equation uses only rays that are close to and almost parallel with the main axis. Such a situation is physically imposed through assuming that the angle the incident beam makes with the axis is small enough to make the approximations $\sin(\theta) \approx \theta$ for rays coming from the axis. In reality, the ray coming from an object toward a mirror is diverging, so not the entire ray is close to or parallel to the axis. Those rays which are far from the main axis do not converge to a single point. The fact is that a spherical mirror does not bring all parallel rays to a single point known as "spherical aberration".

This defect is most noticeable for light rays striking the outer edges of the mirror. Rays that's trike the outer edges of the mirror fail to focus in the same precise location as light rays that strike the inner portions of the mirror. This causes that the images of objects as seen in spherical mirrors are often blurry.

The extent of the ray divergence (k) from the focus called the lateral aberration, can be quantized in terms of the distance HO of the light ray from the principal axis of a concave mirror with the radius of curvature r. The lateral aberration k is determined by Eq. (4) [31].

$$\kappa = \frac{r^2}{2\sqrt{r^2 - HO^2}} - \frac{r}{2}. \tag{4}$$

Eq (4) predicts that when HO is kept constant and the radius of curvature gets larger, the lateral aberration *k* decreases. Such a mechanism is used for correcting artificial spherical aberration in OIO.

First of all, one should define the objects (i.e., population). In this section, the mechanism to model the above-mentioned physical concept is shown artificially with the goal of developing an efficient optimization algorithm named OIO. The OIO algorithm is a population-based evolutionary algorithm inspired from optics science. In this algorithm, it is assumed that some artificial light points (points in R^{n+1} whose mapping in R^n are potential solutions to the problem) are sitting in front of an artificial wavy mirror (function surface) which reflects their images. OIO treats the surface of the function as the reflecting mirror composed of peaks and valleys in order to be optimized. In the following, two terms "mirror" and "function surface" are equivalently will be used. Each peak and valley are treated respectively as a convex reflective surface and concave reflecting surface.

Accordingly, the artificial reflected ray from an artificial light point is reflected back artificially by the function surface, depending on the reflecting surface is partially a part of a peak or a part of a valley. Also, the artificial image point (a new point in R^{n+1} mapped in R^n as a new solution in the search domain) is formed upright (toward the light point position in the search space) or inverted (outward the light point position in the search space) [14]. Fig. 4 shows the main idea for generation of new solutions in a single-dimension search space. In Fig. 4, it is assumed that an artificial light point (i.e. object) is positioned in the joint search-and-objective space in front of the function surface (i.e. mirror) in a particular distance from the vertex (values on the X-axis constitutes the search/solution space and values on the f(X)-axis constitutes the objective space. The set of all points in X−f(X) coordinate system constitutes the joint search-and-objective space). The artificial image is formed in the joint search-and-objective space and its position and height are determined through mirror equation (Eq. (1)) and magnification equation (Eq. (2)). Eventually, mapping the artificial image position in the search space, depending on the type of the reflecting part of the function surface (convex or concave) and depending on the position of the artificial light point (i.e. object) in the joint search-and-objective space, there are 4 different situations under which new solutions are generated (Fig. 5). Assuming that solution O in the population, a different solution F (vertex point) is randomly peaked form the population. If the corresponding objective function value of F is worse that the corresponding objective function value of O, it is consequently assumed that the surface is convex and a new solution is generated upright somewhere toward O, on the line connecting O and F (Fig. 5(a)). While, if the corresponding objective function value of F is better that the corresponding objective function value of O, it is consequently assumed that the surface is concave and a new solution is generated upright somewhere toward O (Fig. 5(b)), or inverted outward O (Fig. 5(c)) on the line connecting O and F in the search space [14].

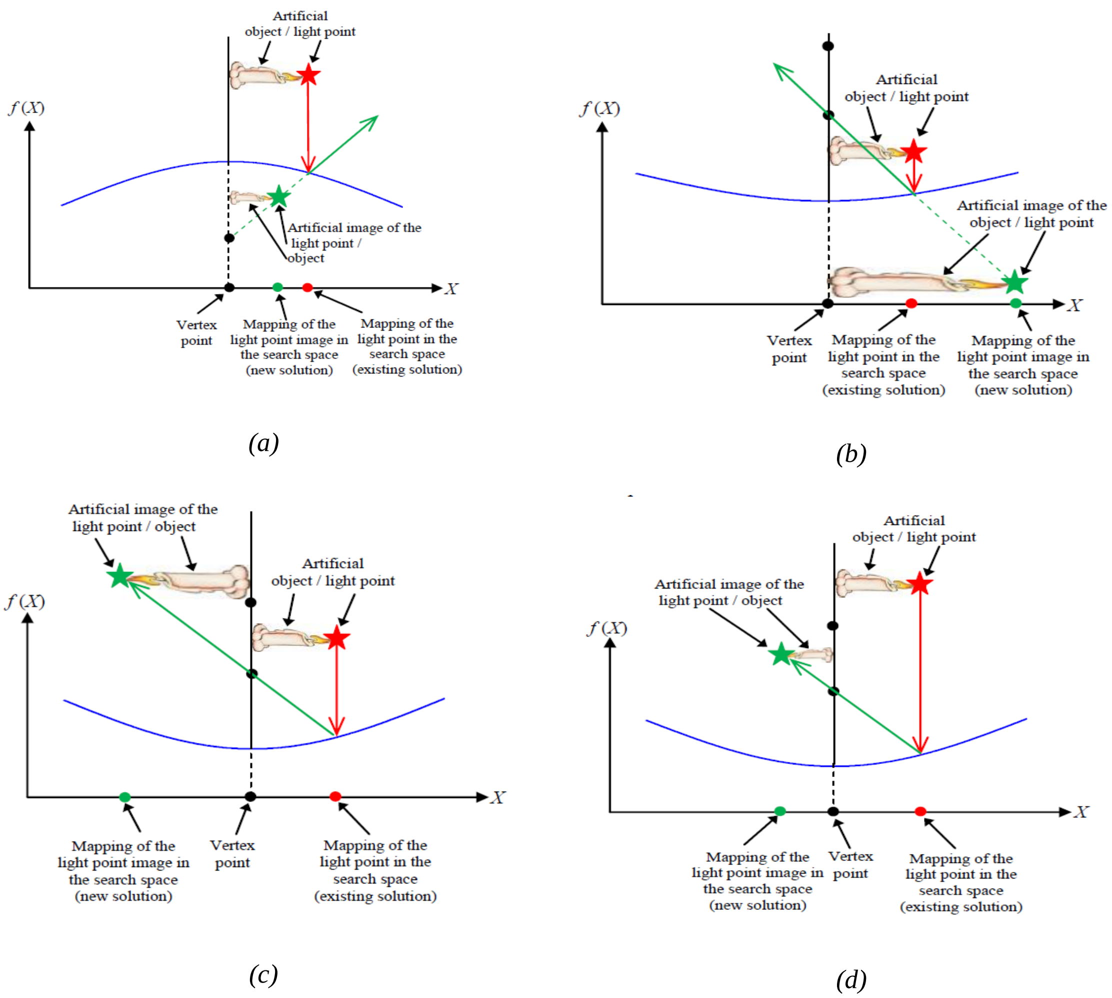


Fig. 5. *Generation of new solutions in single-dimension search space [14].*

As it could be observed from the conceptual model depicted in Fig. 5, the OIO logic for generating new solutions can explore and exploit the optimality during the search. Exploration can be relatively controlled through a big mutation in solution space, while exploitation can be carried out by a relatively smaller mutation during the initial solutions. In the following, the used notations for developing OIO algorithm are first presented and then the steps of the proposed algorithm are explained. Owing to the importance of this matter, these four depictions are separately drawn and their corresponding explanations are distinctly presented, as well.
In fact, the general mechanism is as follows: first of all, the number of NO solutions as initial solutions are randomly generated to form initial positions of the artificial light points in search space. Then, in iteration t, each artificial light point j with location (position) $\vec{O}_j^t = [o_{j1}^t\ o_{j2}^t\ \dots\ o_{jn}^t]$ (j=1,…, NO) in search space (in the joint search-and-objective space in position $[o_{j1}^t\ o_{j2}^t\ \dots\ o_{jn}^t\ s_{j,i_k}^t]$) in front of the artificial mirror (function surface) is positioned in distance p_{j,i_k}^t from the vertex of mirror and its artificial image in joint search-and-objective space is formed in distance of q_{j,i_k}^t (on objective/function axis) from the vertex of the object.

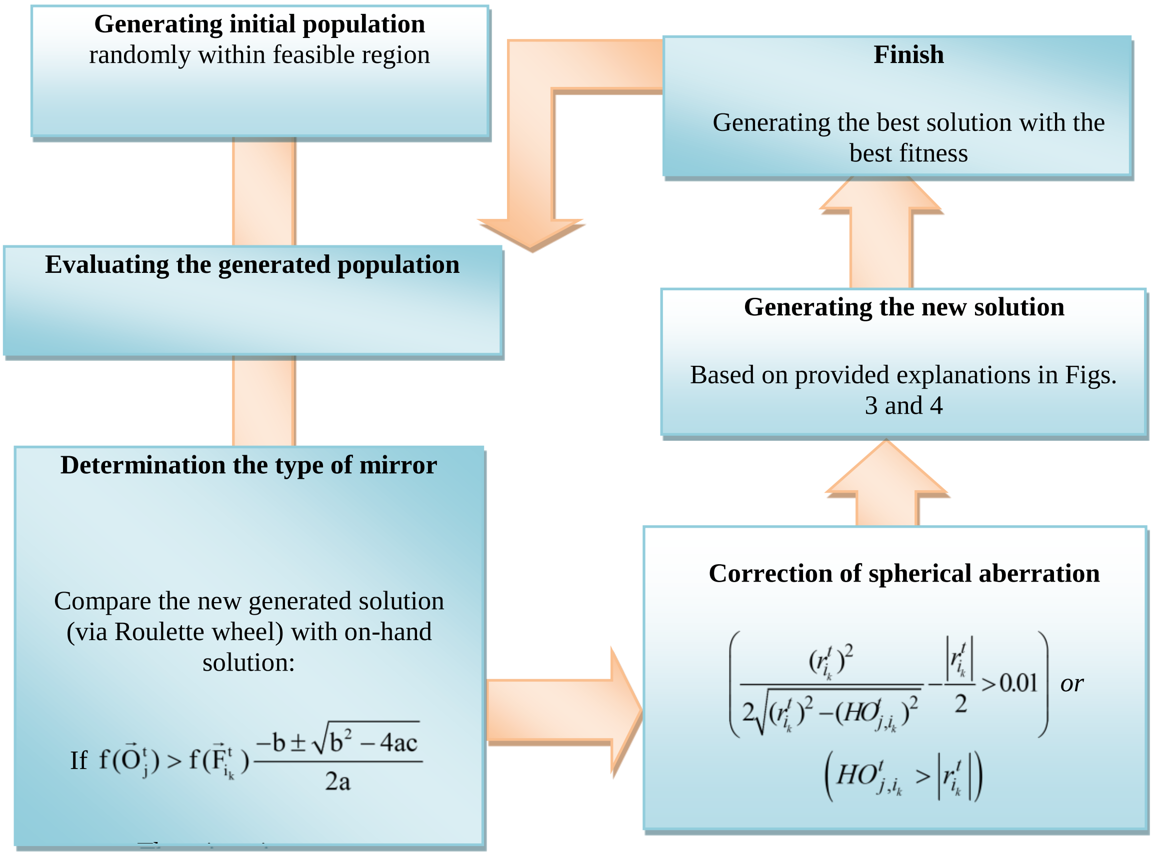


Fig. 6. *Flowchart of basic OIO algorithm.*

Since the main axis passes through $\vec{F}_{i_k}^t$ point in the search space ($\vec{F}_{i_k}^t$ is randomly selected from the current solution set or population, conditionally upon that $f(\vec{F}_{i_k}^t)$ should be different from $f(\vec{O}_j^t)$), while the artificial mirror radius of curvature is $r_{i_k}^t$. Mapping the artificial image position into the solution space generates an artificial image position $\vec{I}_{j,i_k}^{t}$ in the search space which be considered as a new solution of the problem [14].

5. Experimental Results and Discussions

Regarding the QAP, it should be pointed out that the QAPLIB library (http://www.seasupenn.edu\qaplib) contains approximately 140 different sample instances which usually are used by researchers for comparison of different methods. The size of these samples ranges from 12 to 256 and there are only 4 samples larger than 100. Most of these instances are already solved by relatively good heuristic algorithms such as Tabu Search method.

The biggest limitation of this study is the lack of an optimal response for comparison. Because in the online library of permutation problems solved, only the final values of the best answers are inserted, meaning neither the solution used to obtain that value, nor the time needed to reach that answer, nor even the reference article that It has earned it, no mention of it. So, the comparison is very difficult and time-consuming. The second major limitation of this research is the inadequacy of the files containing the reference information. Therefore, the best practice that the present researcher has succeeded in coding is to solve this problem and convert these single-column matrices into square matrices and correctly add the original diameter values, Suitable only for symmetric problems and incapable of asymmetric matrices. Therefore, asymmetric matrices can also be modified and inserted into the OIO by finding better methods or spending a lot of time.

The sample instances of the QAPLIB library could be classified into four main categories. Considering this feature, the problems of the first category have relatively less dominancy of flow, while the real-world problems have more dominancy of flow. To cover the structure of the distance matrix, the dominancy of dd distance can be correspondingly defined. In order to analyze the QAP, one should consider that there are multiple optimum solutions in a large number of samples, owing to symmetric structure of the problem (such as sample class ii) at the maximum distance from each other, i.e., n. So, in such conditions, the distance to the nearest optimal point should be measured. Since the exact number of optimum solutions for samples of QAP are not given, a relatively big arbitrary number is taken into account as the number of optimal solutions (the best ever known solutions). In this research, a great amount of efforts is made to have a comprehensive selection taking the following criteria into account:

- Considering different sizes of problems to evaluate the proposed algorithm (to the possible extent for this algorithm and current research) through factors such as solving speed in small and large instances, exhaustion power in large cases and determination of tuning impact for each of the used parameters in final solution.
- Taking different types of data so as to enhance the applicability of the research.
- Applicability of the problem in terms of type, values of variables, and size.
- Solving feasibility of the problem in terms of needed computational time and power/ability of the algorithm.
- Standardizing the current problem to be solved by meta-heuristic population-based algorithms based on existing records.

In solving all selected samples of QAP using OIO algorithm, its initial tuning is first evaluated in different statuses and the best-obtained result accompanied by its parameters are registered in the research. Moreover, to check the proximity of the gained solutions to the best ones, a comparison is conducted between the obtained solutions and the best ever obtained ones (QAPLIB).

Table 2. *Solving QAP by OIO algorithm.*

No.	Abbreviation	No. of locations	No. of evaluations	No. of rays	No. of mirrors	OIO solution	Best solution of QAPLIB
1	*Chr12a*	*12*	*1500*	*30*	*1*	*9552*	*9552*
2	*Chr15a*	*15*	*2000*	*30*	*1*	*9896*	*9896*
3	*Chr22a*	*22*	*3000*	*30*	*1*	*7818*	*6156*
4	*Chr25a*	*25*	*4000*	*30*	*1*	*4525*	*3796*
5	*Bur26a*	*26*	*4000*	*35*	*1*	*5537890*	*5426670*
6	*Kra30a*	*30*	*5000*	*35*	*1*	*90552*	*88900*
7	*Esc32a*	*32*	*5000*	*35*	*1*	*150*	*130*
8	*Ste36a*	*36*	*5000*	*50*	*1*	*9736*	*9526*
9	*Esc64a*	*64*	*5000*	*50*	*1*	*153*	*116*

The obtained results gained from solving the selected problems of symmetric QAP using OIO algorithm are reported in Table 2. These results comprise two sets of information pertaining to tuning of OIO algorithm;

- The first category contains parameters of pre-tuning of the algorithm such as "Num of Light Point", where it is assumed that each object shines "Num of Light Points" light rays to mirror, i.e. "Num of Light Points" random numbers are generated for each object. The minimum and maximum number of mirrors in which the objects are positioned in front of them, and also the number of evaluations should be tuned in each iteration.
- The second category includes the obtained results of running the algorithm such as the spent computational time to obtain the optimum solution, the obtained optimum solution and the successfulness amount of the proposed algorithm measured through comparing its results with the already reported optimum solutions in the literature. These values show the performance of the proposed OIO algorithm in solving QAP to be able to evaluate it. As can be seen, the obtained results of the OIO algorithm are reported just next to the best ever found solutions in the QAPLIB library to show the performance of the proposed OIO algorithm. In this research, depending on the obtained solution of OIO algorithm, the gained permutation of the OIO (which is shown in fig 7 to Fig. 14) is reported separately for each of the sample problems.

7	5	12	2	1	3
9	11	10	6	8	4

Fig. 7. *Permutation of problem "Chr12a" with OIO algorithm.*

5	10	8	13	12
11	14	2	4	6
7	15	3	1	9

Fig. 8. *Permutation of problem "Chr15a" with OIO algorithm.*

1	4	6	20	2	13	3	19
17	11	18	16	12	10	5	8
9	7	15	22	14	21	p	P

Fig. 9. *Permutation of problem "Chr22a" with OIO algorithm.*

16	25	24	23	22
15	17	18	19	21
6	8	13	14	20
5	7	12	9	11
4	3	2	1	10

Fig. 10. *Permutation of problem "Chr25a" with OIO algorithm.*

3	1	17	21	11	23	2	18	13
4	9	15	10	7	16	19	6	26
5	8	14	20	22	24	12	25	p

Fig11. *Permutation of problem "Chr26a" with OIO algorithm.*

14	4	30	27	18	15
23	29	24	11	20	17
33	16	12	8	21	1
22	10	13	9	7	19
5	2	28	25	6	26

Fig12. *Permutation of problem "Kro30a" with OIO algorithm.*

5	3	18	26	8	31	12	2
7	27	11	9	13	29	22	30
16	10	15	19	32	14	25	21
1	17	4	28	20	23	6	24

Fig. 13. *Permutation of problem "Esc32a" with OIO algorithm.*

14	3	31	35	18	25	36	32	6
16	20	11	26	30	4	12	7	28
23	8	17	1	22	34	19	33	24
2	27	10	15	9	29	5	21	13

Fig. 14. *Permutation of problem "Ste36a" with OIO algorithm.*

As already explained, to solve QAP using OIO algorithm one should first tune the parameters such as number of evaluations, number of mirrors in each problem and also number of shone rays to mirror by each object. As could be seen from the obtained results, the OIO algorithm has acceptable performance compared to the online library of QAP problems named QAPLIB and can well tackle other hard combinatorial optimization problems.

Table 3. *Performance of OIO algorithm in solving sample problems of QAP.*

No.	**Abbreviation**	**No. of locations**	**OIO solution**	**Best solution of QAPLIB**	**Percent of successfulness**
1	*Chr12a*	*12*	*9552*	*9552*	*100*
2	*Chr15a*	*15*	*9896*	*9896*	*100*
3	*Chr22a*	*22*	*6098*	*6156*	*100*
4	*Chr25a*	*25*	*4265.8*	*3796*	*88.98*
5	*Bur26a*	*26*	*5537890*	*5426670*	*97.99*
6	*Kra30a*	*30*	*90056.89*	*88900*	*98.71*
7	*Esc32a*	*32*	*132.68*	*130*	*97.98*
8	*Ste36a*	*36*	*9635*	*9526*	*98.86*
9	*Esc64a*	*64*	*122*	*116*	*95.08*

In Table 3, the performance of the OIO algorithm is shown through reporting the percent of its successfulness to reach the optimum solution. As could be seen, the OIO algorithm can well solve both small instances (with 12 and 14 locations) and large problems (with 64 locations). So, it is highly recommended for solving other sample problems of QAP.

Regarding the spent computational time of such problems, one can conclude that the proposed algorithm has appropriate performance since OIO can reach an acceptable solution within one minute for sample problems with the size up to 22 facilities. So, using OIO in solving QAP with such sizes are recommended, Table 3 shows that the spent computational time of solving QAP sample instances increases by growth of the problem size since by such increment, the number of evaluations and even number of mirrors, employed for assessing, increase.

Also, Table 4 shows the computation time required to obtain the precision of operation required for the optical algorithm and its other components to solve double allocation problems separately so that the reader can clearly see the computation time of the algorithm and even Compare this time with other algorithms and choose the best algorithm in terms of time.

Table 4. *Time of OIO algorithm in solving sample problems of QAP.*

No.	Abbreviation	No. of locations	OIO time (s)
1	*Chr12a*	*12*	*8/9*
2	*Chr15a*	*15*	*3/20*
3	*Chr22a*	*22*	*65/6*
4	*Chr25a*	*25*	*95/8*
5	*Bur26a*	*26*	*116/36*
6	*Kra30a*	*30*	*102*
7	*Esc32a*	*32*	*185*
8	*Ste36a*	*36*	*359*
9	*Esc64a*	*64*	*1289/9*

Extensive studies on the quadratic allocation problem have been carried out so far using precise and innovative methods. In this study, we have compared the problems solved through the optical algorithm with the traditional genetic algorithm and the intelligent one, and presented the results numerically in Table 5.

Table 5. *Comparison of OIO algorithm with genetic algorithm.*

Abbreviation	QAPLIB	Traditional Genetic Algorithm		Intelligent Genetic Algorithm		OIO Algorithm	
		solution	time	solution	time	solution	time
Chr22a	*6156*	*6202/5*	*75/9*	*6194/40*	*72/9*	*6098*	*65/6*
Chr25a	*3796*	*4356/20*	*96/8*	*4243/23*	*93/8*	*4265/8*	*95/8*
Bur26a	*5426670*	*5426670*	*117/3*	*5426670*	*115*	*5537890*	*116/3*
Kra30a	*88900*	*90317/99*	*150/7*	*90107/43*	*145/8*	*90056/89*	*102*
Esc32a	*130*	*134/13*	*190/9*	*132/8*	*192/8*	*132/68*	*185*
Ste36a	*9526*	*9668/12*	*354/8*	*9660/27*	*360/8*	*9435*	*359*
Esc64a	*116*	*126*	*1315/4*	*129*	*1357/9*	*122*	*1289/9*

In the figure above, a general comparison is made between the optics-based algorithm and the traditional genetic algorithm and the intelligent genetic algorithm, as well as the duration of each of these algorithms. As can be seen, the optics algorithm performs better than other meta-algorithms and is recommended to use. In solving these permutation problems through optical algorithm, precision and heuristic methods are remarkably time-saving and obtain better performance and success than other meta-heuristic algorithms, including genetic algorithms. Therefore, it is recommended to use this algorithm to solve permutation problems such as the round-trip seller problem and the double allocation problem. The results obtained for small sizes (10–12–12 equations) in the quadratic allocation problem equate to their exact values. So, for small size permutation problems are considered the optimal solution. For intermediate sizes (from 16 to 76 cities or equipment), the results were found to be in good agreement with the values recorded in the research record to date (not necessarily the optimal response) and There has been no better response to these issues than the ones listed in the library, but the OIO has come close to satisfying them. This is what is to be expected since most of the values in the library are likely to have been obtained using precise solutions. Also, solving large dual allocation issues (76 out of 120 cities) through OIO with a moderate difference of the best found so far.

Also, these answers, while still quite plausible and useful for numerous applications, are expandable, and by adding more operators to the optical-based algorithm, much better answers can be achieved. In many cases, it is necessary to obtain a response rather than the exact shortest route that reduces the cost of imposition as soon as possible. Therefore, it can be concluded that the quality of OIO responses is highly acceptable and desirable compared to other methods and is recommended for solving similar problems. On a larger scale, of course, there is the problem of reducing the efficiency of this algorithm, but by adding operators to the algorithm in the not-too-distant future we can intelligently improve the final response with a few changes. For example, by combining an algorithm such as simulation of refrigeration or banned search, one can easily improve this algorithm.

6. Conclusion

So far, several algorithms have been used to solve the optimization problems, including genetic algorithms, particle swarm, ant colony, and so on, depending on the type of problem, each of these algorithms performs well. The OIO algorithm is a population-based random search method that mimics the behavior of mirrors. OIO is an analytical process of mirror behavior that has not been used in hybrid permutation optimization problems such as round-trip vendor permutation problems and double allocation so far. Due to its good results and its efficiency in delivering the desired answers, it is recommended to use this algorithm in different optimization domains. Also, in this paper, the performance evaluation of Optics Inspired Optimization (OIO) algorithm in solving sample instances of the Quadratic Assignment Problem (QAP) was well discussed and assessed. According to the obtained results, in all of the conducted comparisons with the best outcomes of the literature, the proposed OIO algorithm showed acceptable performance compared to the other heuristic, meta-heuristic and exact techniques in terms of solution quality, efficiency and successfulness in solving permutation-based sample problems. Accordingly, using the OIO algorithm for solving permutation-based problems such as QAP is strongly recommended. In this paper, the comes about of this calculation appeared that it merges quickly to the worldwide minimum. However, any statement on the convergence behavior of OIO should be based on the theoretical foundations or a formal convergence proof. The performance of OIO can be further tested in the real world, engineering optimization problems addressed in the literature. Beside self-adaptive strategies or multiple populations could be employed in OIO as they are used in other evolutionary algorithms. As future work, we plan to apply OIO for the other well-known combinatorial problem instances. There are many meta-heuristics introduced recently. Applying parallel computation and tuning their parameters in run-time can be interesting and has the potential to improve the solution of existing NP-Hard combinatorial problems. The black box optimization function is an effective tool to evaluate the performance of the new algorithms. It consists of a wide range of benchmark problems. We plan to use this tool to evaluate the performance of our new algorithm on different problem domains.

In future research, it is suggested that OIO not only be used to solve other research problems in the operations and allocation and design issues of industrial units but also to improve its response by making a few changes. Another suggestion is that similar research, but more applicable to real-time salesman issues and double allocation to domestic organizations, is similarly resolved and the effects of this change on the performance of that part of the measurement organization. And the extent of improvement. Finally, it is suggested that by writing appropriate graphical codes for this algorithm, it will be possible to display the algorithm's motion trend and the amount of pathway left and right. This will help to investigate the process of convergence of the problem as well as provide the researcher with appropriate scheduling to solve a set of selected problems.

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