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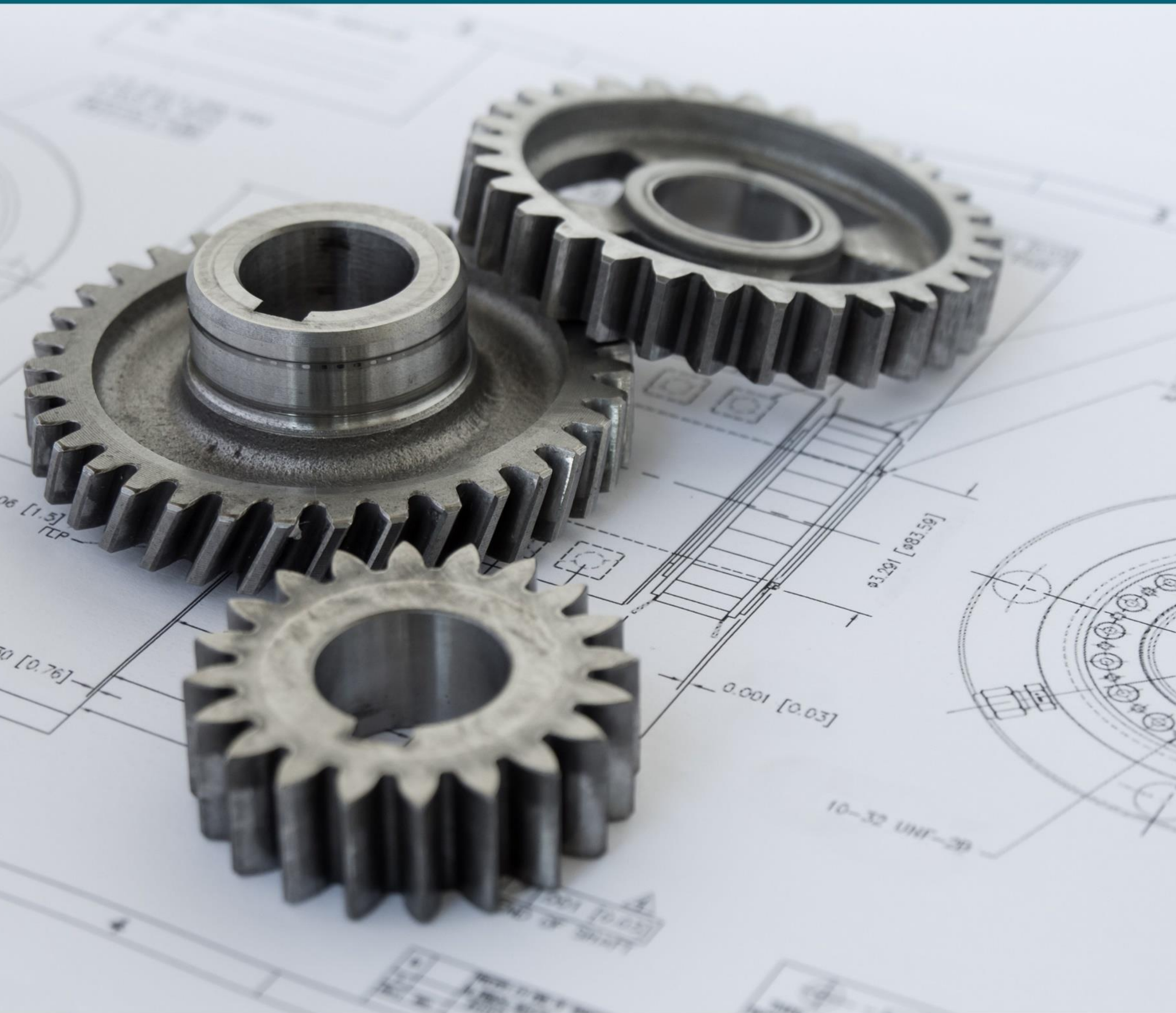

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Paper Type: Research Paper

Lean at Home: Applying RCA Techniques to Home Projects

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Abstract

Root cause analysis techniques are often applied to problems in the workplace; however, they may also prove very useful when applied to home projects. This research explores the application of root cause analysis techniques in three home projects: (1) Cause-and-Effect diagram to remove Palmetto bugs in a condo dwelling, (2) Five Whys method to repair a sunroof water leak in a car, and (3) Fault Tree Analysis to repair a Toro string trimmer that starts, then dies. The effective use of root cause analysis techniques can have a meaningful impact on resolving home project issues resulting in a restoration of homeowner issues as well as reduced homeowner anxiety, repair time, and repair cost.

Keywords: Cause-and-effect diagram, Fault tree analysis, Palmetto bugs, Toro string trimmer.

1 | Introduction

The Lean philosophy has universal applications crossing all manufacturing and service sectors in the world. Womak et al. [1] is credited with coining the term 'Lean', named after the world-famous Toyota Production System, which focused on waste elimination, shorter lead times, frequent changeovers, smaller lot sizes, and reduced inventories. Though initially implemented in the automotive industry with great success, the adoption of Lean production systems soon spread across many manufacturing industries worldwide. Today, the Lean philosophy is used in many service industries as well, such as healthcare, yielding similar successes as experienced in the industrial world.

Lean can also be practiced at home and the literature reveals scant research in this area. Among the many facets of the Lean philosophy is the use of Root Cause Analysis (RCA) techniques. The RCA toolkit contains numerous analytical methods and techniques, such as the seven basic tools of quality; namely, Cause-and-Effect diagram, Check Sheet, Control Chart, Histogram, Pareto chart, Scatter diagram, and Run chart [2]. Additional RCA techniques include Brainstorming,

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Nominal Group Technique, Delphi Technique, Cause Map, Five Whys, Designed Experiments, Failure Mode Effects Analysis, and Fault Tree Analysis, to name a few.

This research reveals four examples of how RCA techniques (i.e., Cause-and-Effect diagram, Five Whys, Failure Mode and Effects Analysis (FMEA), and Fault Tree Analysis (FTA)) can be used to streamline problems encountered with home projects. More specifically, these techniques are used to identify possible causes leading to a failure and once all possible causes are identified, steps are then taken by the homeowner to negate these causes, thus providing a remedy for the home projects.

2 | Literature Review

The success of the Lean philosophy is well-published in industrial applications; however, its diverse applicability among other economic sectors is just as significant.

Recent examples of the use of Lean concepts abound in the healthcare industry. Lean Six Sigma methodology such as the DMAIC and PDCA methods have been used to reduce the length of stay in a hospital's emergency department [3], reducing the percentage of patients who Leave Without Being Seen (LWBS) [4], and reducing the incidence of unplanned surgery cancellations [5]. Smith et al. [6] was used to improve scheduling time of home health nurses. A focus on "waste elimination" lead to improvements in streamlining the steps required in a hip fracture surgery process [7]. Standardizing the registration and triage processes lead to improved patient care in a hospital emergency department [8].

The versatility of Lean principles has been adopted by other service industries, such as in project management endeavors [9], the use of DMAIC methods to improve and sustain humanitarian efforts [10], the use of numerous Lean concepts to improve the dining experience at full-service restaurants [11], and in education by mistake-proofing teacher mistakes at a magnet high school [12].

A wide assortment of RCA techniques is a crucial facet under the Lean umbrella towards problem resolution. Otitigbe [13], Giol [14], and Root and Small [15] suggest using a team-based approach when utilizing Cause-and-Effect diagrams to find all possible root causes leading to an effect. The team should consist of various stakeholders, both past and present. Munro and Savel [16] employed the Five Whys approach to improve critical care by identifying problematic variables to describe relationships and provide a premise for novel interventions. Moaveni and Chou [17] utilized the Five Whys method as an assignment in a civil engineering classroom at Northwestern University, whereby students used this technique to identify what they did incorrectly on homework assignments and then corrected them.

Additional RCA examples include developing a seven-step RCA process to prevent Built-In-Test (BIT) false alarms during the design and development phase [18], using designed experiments to determine the best variable settings for desired results in the manufacture of corrugated sheet stock [19], and using of the seven tools of quality by internal auditors [20]. Fishbone diagrams, Pareto charts, and the Five Whys method were used to detect errors during the preparation of histology slides [21], and for determining various causes of cost deviation in highway construction projects [22].

FMEA has been employed to reduce the risk of failures in casting and machining processes of vehicle rear spring brackets [23] and the production of automotive leaf springs [24]. FMEA has also been used to evaluate the process for recognizing and referring children exposed to domestic abuse [25] identifying vulnerabilities in a paperless radiotherapy department [26] and anesthetic equipment [27], and for identifying and prioritizing barriers in behavior health workflows [28].

Modifications of FMEA include a five-step process called Health Care Failure Mode Effect Analysis (HFMEA) [29], the use of fuzzy Adaptive Resonance Theory (ART) for clustering failure modes in the edible bird nest industry [30], and the use of Design Failure Mode and Effect Analysis (DFMEA) to identify possible causes of warranty claims [31].

FTA has been used to identify potential causes of the transport of hazardous chemicals [32] and [33], to estimate the number of refrigerant exposures of both service technicians and vehicle occupants in the U.S. [34], and in the evaluation of a drilling mud pump system [35].

Techniques used in conjunction with FTA include the use of fuzzy neural networks to diagnose faults in the Internet of Things (IoT) of aquaculture [36], and using Monte Carlo simulation to analyze allowable effluent Biochemical Oxygen Demand (BOD) in a wastewater treatment plant [37].

3 | How RCA Techniques Can Be Applied to Home Projects

Example 1. Cause-and-Effect Diagram to Rid Home of Palmetto Bugs.

This example involves the sudden appearance of Palmetto bugs in a condo in Marietta, Georgia and it could apply to many homes in the state of Georgia. A Palmetto bug (in *Fig. 1*) looks like a large cockroach and can trigger an individual's startle reflex upon first sight.

Fig. 1. Palmetto bug.

Palmetto bugs can appear in any building, not just in the home. *Fig. 2* shows a deceased Palmetto bug on the floor in a copy room in the workplace.

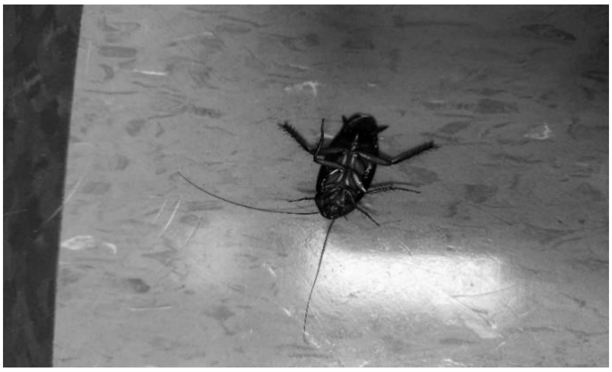

Fig. 2. Palmetto bug in the workplace.

Palmetto bugs are common in Georgia, particularly in the warm summer months. These bugs thrive living in warm external environments and wooded areas surrounded by pine trees and shrubs, so the summer heat in Georgia provides an excellent climate for Palmetto bugs. The observance of Palmetto bugs had become an incessant problem in the condo unit despite numerous measures that were implemented to prevent the entry of these bugs. A cause-and-effect diagram in *Fig. 3* was used to identify all possible causes of Palmetto bugs entering the condo.

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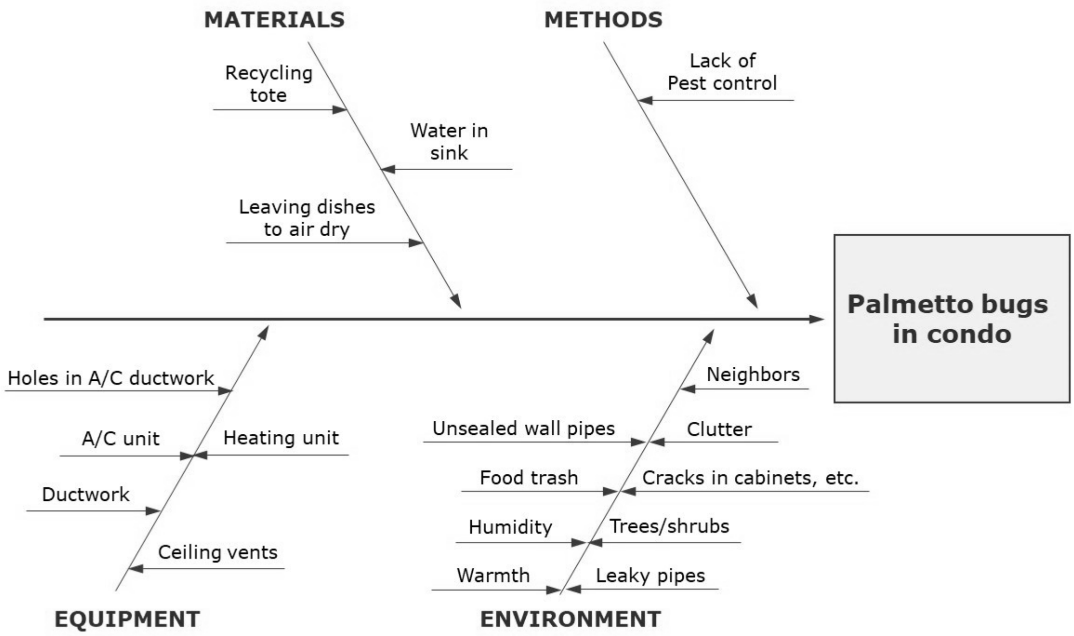

Fig. 3. Cause-and-effect diagram of palmetto bugs in condo.

An analysis of Materials in the condo revealed that a recycling tote, water left in the kitchen sink, and leaving dishes to air dry provided locations for Palmetto bugs to hide once inside the condo. Under Methods, although the homeowner sporadically sprayed the condo unit with a pesticide, the lack of applied pest control on a consistent basis contributed to the prevalence of these bugs in the condo unit.

An examination of the environment revealed that Palmetto bugs can enter the interior of a building, or condo, through any cracks in the building exterior, including the building foundation, open windows and doors, etc. Once inside a building, Palmetto bugs can penetrate the building infrastructure, then enter any room through wall cracks that are evident, but also through cracks that are not evident, such as cracks behind kitchen and bathroom cabinetry. They can enter through uninsulated pipes and holes in the wall such as under a bathroom vanity or kitchen sink, and unsealed areas around electrical outlets. This particular condo unit was the middle unit on the end of a building. It was known that two other homeowners, specifically one homeowner with an adjacent condo unit and the other homeowner immediately below, were what could best be described as "hoarders." This becomes fertile nesting ground for insects such as Palmetto bugs.

Under equipment, an inspection of the A/C unit outside the condo building revealed no abnormalities; however, a hole was discovered in the upper corner of the ductwork above the heating unit inside the condo. No anomalies were detected upon inspection of the water heater. The homeowner reached an epiphany one day in the sunroom while mulling over other possible ways a Palmetto bug could enter the condo. A Palmetto bug dropped down in front of the homeowner and landed on the carpet. Looking up at the source, the homeowner discovered that the Palmetto bug dropped down from the ceiling vent (See *Fig. 4*).

Fig. 4. Sunroom ceiling vent.

Example 2. Solutions.

The following action steps were implemented and the Palmetto bug problem is now under control; that is, a Palmetto bug now appears in the condo on rare occasions rather than on a regular basis:

- *The recycling bin has been removed and dishes are now washed in the dishwasher.*
- *The homeowner has hired a professional exterminator to use a commercial-grade pesticide on a monthly basis (See Fig. 5). Additionally, the homeowner continues to spray on a weekly basis using a name brand pesticide (See Fig. 6).*
- *All wall cracks have been patched and painted, including behind kitchen and bathroom cabinetry.*
- *All holes around pipes and all gaps around electrical outlets have been filled with insulation (See Fig. 7 and Fig. 8).*
- *Both "hoarder" neighbors eventually sold their units and moved out. Afterwards, both units were remodeled and the new homeowners do not appear to be hoarders.*
- *The hole in the corner of the A/C ductwork has been repaired (See Fig. 9). Roach powder has been applied along the floor perimeter inside the heating unit/water heater room.*
- *Storm door screening now covers the openings in all ceiling vents in the condo (See Fig. 10).*

Fig. 5. Professional exterminating co.

Fig. 6. Homeowner pest control.

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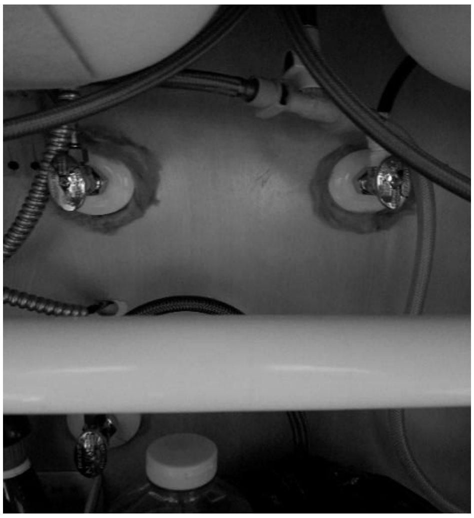

Fig. 7. Holes Sealed under kitchen sink.

Fig. 8. Holes sealed under bathroom vanity.

Fig. 9. Ductwork hole repaired.

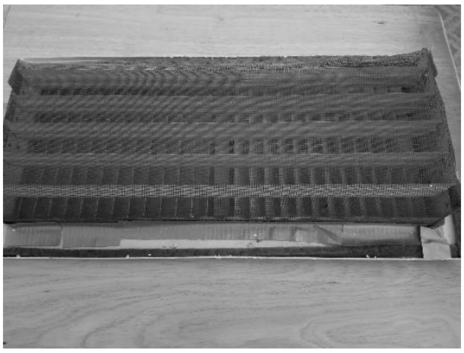

Fig. 10. Screened ceiling vent.

Example 3. Five Whys: Sunroof water leak floods floorboard in car.

Five Whys is a technique used to identify the root cause of a problem by asking "Why?" five times (more or less depending on the problem). The answers to each successive Why delves deeper towards identifying the root cause of the problem.

The floorboard in a 2001 Acura TL was flooded after a big rainfall one day. The car owner thought the car windows had been inadvertently left slightly open during the rainfall, resulting in the flooded floorboards. The next time it rained, the car windows were checked and were fully closed. After the rain ended, however, the floorboards were flooded again. Puzzled, the car owner opened the sunroof and discovered the water well filled with water and debris, such as pine needles and dirt, and that the drain ports were clogged.

Once both drain ports were cleaned, the driver's side floorboard flooded once again after the next rainfall. However, the passenger's side floorboard remained dry. After further investigation inside the car, it was discovered that the drain tube from the sunroof drain port had become unattached from the boot tube as shown in *Fig. 11*.

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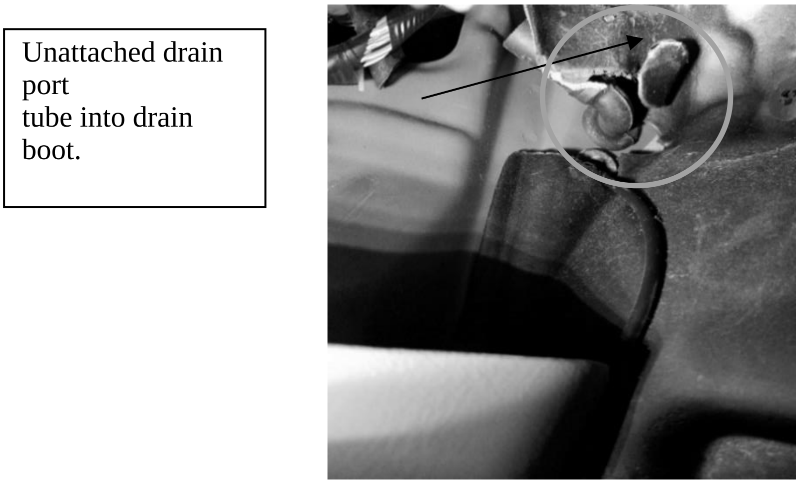

Fig. 11. Driver's side interior drain hose.

The Five Whys diagram in *Fig. 12* illustrates the analysis for this problem.

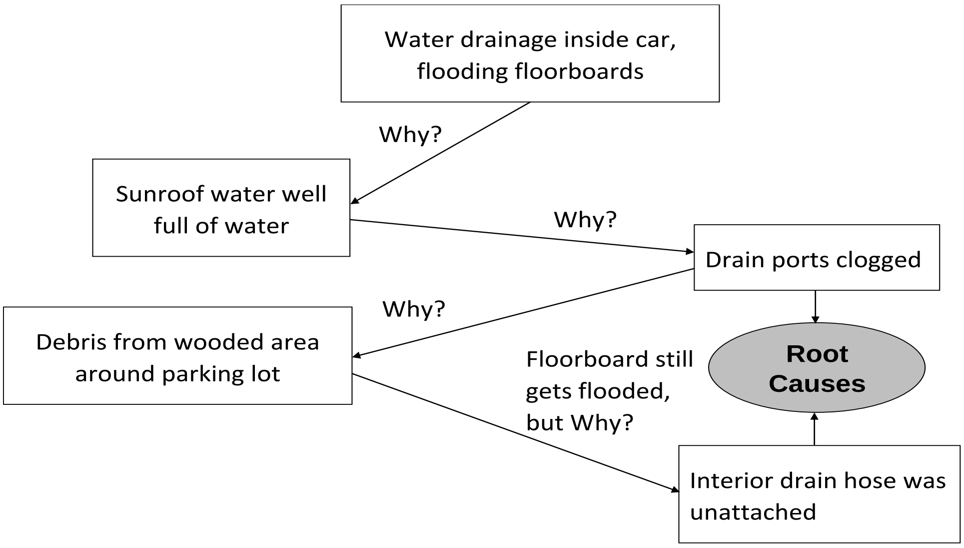

Fig. 12. Five Whys diagram for flooded floorboards in car.

Example 4. Solution.

Two solutions were required to repair this problem. First, the sunroof was opened in order to clean the water well and drain ports from water build-up and debris as shown in *Fig. 13* and *Fig. 14*.

The car is normally parked in a condo community parking lot surrounded by pine trees and bushes. Pine needles and debris were discovered all over the car, including in the sunroof water well.

Fig. 13. Driver's side sunroof drain port.

Fig. 14. Passenger's side sunroof drain port.

Second, a new drain port tube was inserted into the drain boot (in *Fig. 15*) to allow water runoff to channel from the sunroof through the windshield pillars and through the drain boot to the outside of the vehicle.

Fig. 15. Interior drain hose repaired.

Example 5. Fault Tree Analysis: Why Toro String trimmer starts, then dies.

FTA is a top-down, deductive failure analysis technique in which an undesired state of a system is analyzed. In identifying the potential factors affecting the undesired state, FTA also allows for the evaluation of risk.

The Toro string trimmer (in *Fig. 16*) ran for many years. Then, as in many cases involving equipment, one day it unexpectedly didn't run and the troubleshooting process began. The homeowner used a Fault Tree Analysis to determine possible root causes and, in this particular example, there were many (See *Fig. 17*).

Fig. 16. Toro string trimmer.

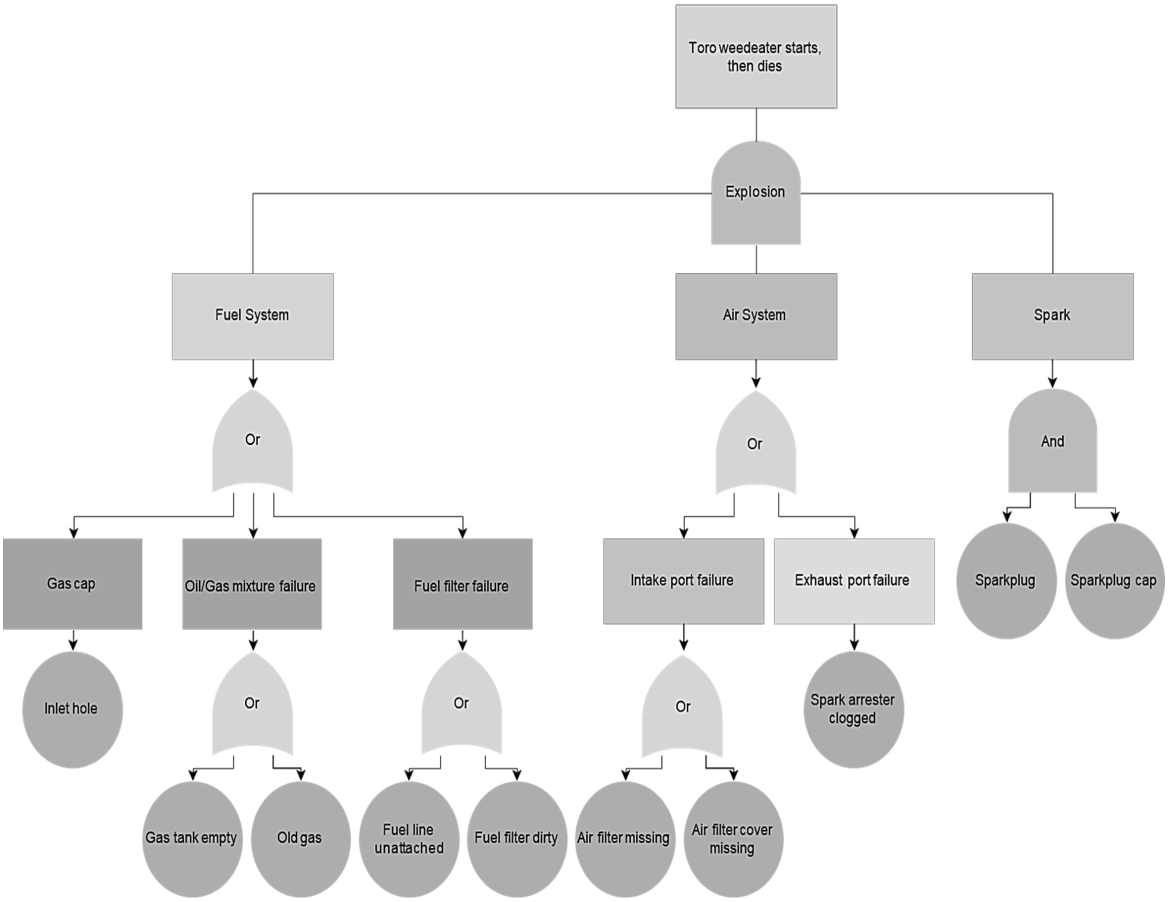

Fig. 17. Fault tree analysis of Toro string trimmer starts, then dies.

The Toro string trimmer's ability to start and run consistently is based on three things working in harmony: the fuel system, the air system, and the spark system.

The fuel system must have a gas cap with a clear inlet hole. Additionally, the gas/oil combination in the fuel tank must have the proper 50:1 gas/oil mixture level and the gas must be fresh gas. The fuel filter must be clean with the fuel line attached. The carburetor must be clean of dirt and debris. If any of these potential causes is compromised, the string trimmer could die shortly after starting.

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Regarding the air system, the air intake must have a clean air filter and an air filter cover for proper operation. For air exhaust, the spark arrestor must be clean of any debris. If the air filter or air filter cover is missing, debris could enter the string trimmer causing it to die suddenly after it starts. If the spark arrester is dirty or clogged, this prevents proper air exhaust and could cause the string trimmer to die shortly after it starts.

The sparkplug must have the proper 0.025” gap, be clean of any residue, and it must be capped by the sparkplug boot cover. If the sparkplug is dirty and/or it is not capped by the sparkplug boot cover, the string trimmer will not start.

Example 6. Solution.

- *The gas cap and inlet hole were thoroughly cleaned.*
- *The carburetor was disassembled and thoroughly cleaned. (See Fig. 18)*
- *A 50:1 gas/oil mixture was used in the gas tank with fresh gas.*
- *The fuel filter was replaced by a new one. (See Fig. 19)*
- *An air filter and air filter cover were purchased to replace the missing air filter and air filter cover.*
- *The spark arrestor was cleaned on a wire wheel.*
- *The sparkplug was replaced with a new sparkplug, properly gapped at 0.025".*

Once these steps were taken, the Toro string trimmer ran like new.

Fig. 18. Carburetor disassembled and cleaned.

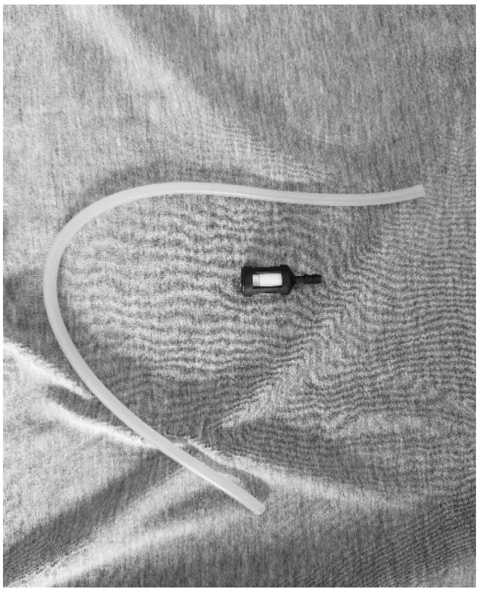

Fig. 19. New fuel line and fuel filter.

4 | Conclusions

The cause-and-effect diagram was used to list all possible causes of Palmetto bugs getting into a condo unit. This provided an impetus for corrective actions. By identifying and addressing all of the possible causes, the problem is now under control. A Palmetto bug is now observed in the condo on rare occasions rather than every day. The five Whys approach aided in deducing the root causes of sunroof rainfall water runoff onto the floorboard of a car. The sunroof water well and drain ports were thoroughly cleaned and a new driver’s side interior drain tube was attached to the boot drain hose to prevent water from draining onto the floorboard. Fault Tree Analysis was used to determine possible causes for why a Toro string trimmer would start, then die. Several causes were shown under the primary branches of Fuel System, Air System, and Spark, which must work in harmony for the string trimmer to start and run. Corrective actions were taken to negate these causes and the string trimmer now runs like new. The three root cause analysis techniques utilized in this research provides evidence that they can be successfully applied to home projects. Each technique provides a structured approach for consideration of all possible root causes leading to improved troubleshooting and problem resolution. This, in turn, can lead to a revival of previously out-of-service equipment as well as reduced aggravation, repair expense, and repair time for the do-it-yourself homeowner.

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Paper Type: Research Paper

Stochastic Cost Modeling for Second-hand Products' Optimum Warranty Period and Upgrade Level

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Abstract

After the second-hand products return to the second-hand markets, either goods, parts, or materials are reused or disposed. For optimum sales, these products are under the process of determining the optimal warranty period and warranty policy, so that both the seller's profit and the consumer's profit are met. Therefore, estimating the warranty costs for future claims on second-hand products is very necessary. In this paper, a hybrid stochastic model is presented to improve the reliability of second-hand products under the free repair replacement warranty policy to determine the level of upgrade, with the aim of reducing warranty costs. Using the upgrade actions, the model, based on three approaches, minimal-perfect repair, virtual age, and improvement factor, is developed for estimating the warranty costs of second-hand products. The contribution of this research is the application of three upgrade approaches in the warranty cost model simultaneously to estimate the warranty costs and optimum upgrade level more realistic. Finally, under different product lifetime, a numerical example and sensitivity analysis are provided. Evaluation is presented in four lifetimes for second-hand products ranging from one to four years for which the optimum upgrade level and the warranty period are determined. The results show that the higher the level of second-hand product upgrades to a certain extent, the higher the savings, but the more upgrades are not cost-effective for second-hand products.

Keywords: Warranty, Second-hand product,Warranty costs,Stochastic modeling, Upgrade.

1 | Introduction

During the past 10 years, the market share of the second-hand products has rapidly increased due to the fact that the highly developed technologies have resulted in a longer product life and a higher price of the new product. Warranty for Second-hand product is now provided by dealers in home appliances, personal computers, automobiles, etc. The warranty offers useful product information and leads to customer confidence in the quality of the product and serves as a

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promotional tool for sellers or dealers to provide different types of warranty policies for their used products. Warranty policy is a statement of compensation offered by the dealer in the event of failure. A better warranty policy usually provides a higher product quality and an assurance to buyers. Warranty provides assurance to customers and signals product reliability. In fact, the laws, regulations and standards for second-hand items aim to guarantee that they will operate as expected at least for a specific warranty period [1]. Hence, in order to promote sales as well as to improve the reliability of the second-hand product, dealers usually perform various maintenance actions not only at the pre-sale point but also during warranty period [2]. According to the definitions provided, the warranty improves the quality of the product (from the customer's point of view) and guarantees repair, replacement or refund in case of the faulty product.

Due to some problems may occur for these products, such as uncertain durability and performance, dealers tend to carryout upgrade actions on the used products to restore the failed products during the warranty period. The mail goal of the dealers is to minimize warranty costs. To reach this goal, numerous methods and approaches have been developed to obtain optimal upgrade level and warranty period. On the one hand, the warranty period is determined by indicators such as the product usage period, product lifetime, and reliability and maintenance history. On the other hand, the quality of returned products plays a significant role in product warranty decisions. Therefore, the warranty period may differ for each product categories. It is also true in general that users of second-hand products have more reason to be concerned about potential product failures than do purchasers of new products and may thus want longer warranty periods. From the dealer's perspective, however, the longer the warranty period, the higher risk of a product failure and thus the higher the cost of the warranty [3]. Thus, the determination of optimal warranty period can play an active role in the costs incurred.

The main objective of this paper is to investigate a stochastic cost model for second-hand products under given warranty costs and failure rate structures. The presented model incorporates different upgrade actions (explained in section 3.1) and results in the derivation of the optimal expected upgrade level to minimize the dealer's expected costs per second-hand product during the different warranty periods. The model makes a useful contribution to the reliability improvement literature as it captures different upgrade actions occurred in reality. In our study, the product lifetime distribution is assumed to be the Weibull distribution for which an appropriate choice of the parameter values results in a flexible model for the failure rate distribution.

The remainder of the paper is organized as follows: in Section 2, we present related literature review of warranty for second hand products and a brief overview of upgrade actions so as to set the background for the main contribution of the paper. In Section 3, first, we discuss upgrade actions then we formulate a new upgrade-based costing stochastic model. In Section 4, we numerically analyzed the proposed model using a real case study. Finally, the concluding remarks are given in Section 5.

2 | Literature Review

Although the number of papers in the fields of new products' warranty, reliability and maintenance ([4]-[6]) is considerable, the number of similar surveys on second-hand products is limited. This brief review of the literature mainly emphasizes and discusses the warranty policies, upgrade actions and the cost modeling of second-hand products.

In 1999, Murthy and Chattopadhyay [7] present a classification on the second-hand products warranty. They calculate the average warranty costs using product lifetime and various warranty policies. Chattopadhyay and Murthy [8] introduce new warranty policies for second-hand products and develop statistical models for cost analysis. In the presented policy, both the buyer and the seller participate in the warranty costs, and if a product is damaged during the warranty period, each buyer and seller will be responsible for a share of it. Chattopadhyay and Murthy [9] set the optimal upgrade level for second-hand products under the warranty policy. Shafiee et al. [10] present a sustainable improvement approach for the second-hand items under the free repair/replacement policy to determine the optimal upgrade level in which uncertainties are reduced and the remanufacturing process of second-hand items is also improved. Yu and Peng [11] analyze the impact of remanufactured product quality, including initial product design, component mismatch, and reassembly errors, on the warranty costs of these products by modelling reliability and warranty costs under free rectification lifetime warranty policy. Saidi-Mehrabad et al. [12] use two strategic approaches to decide on reliability improvement strategies for second-hand products sold under different warranty policies. These approaches are achieved when the reduction in warranty costs outweighs the additional costs of upgrade actions.

A review of warranty cost models is conducted by Aksezer [13] with an emphasis on the analysis of failures of second-hand vehicles considering their age, function and maintenance records. Chukova and Shafiee [14] develop a model of one-dimensional warranty cost estimates for second-hand products, in which warranty policies are examined and compared. Yazdian et al. [15] present a combined mathematical-stochastic model to determine the optimal price and warranty period for the remanufacturing of second-hand products. This approach improves the reliability of the second-hand products to reach higher quality levels. Park et al. [3] propose a cost analysis for second-hand products to determine an optimal warranty period. The authors introduce the concept of full refund rather than replacement in order to deal with the situation when a failed product cannot be repaired within the warranty period. Lu and Shang [16] propose a warranty mechanism for the second-hand electronic market.

Other studies on the second-hand products warranty are related to two-dimensional warranty approaches. There are three main approaches for cost modeling under a two-dimensional warranty; "Two-variable distribution function" approach, "linear combination" approach and the "performance rate as a random variable" approach.

Dealers in second-hand products typically upgrade the product prior to sale to minimize the warranty costs, which would be incurred during the warranty period, as well as to enhance the product value by improving its performance and reliability [17]. Upgrade actions on second-hand products are with the aim of restoring the products to working condition, reducing its failure rate during the warranty period, increasing the remaining life of the product, etc. [18]. Therefore, dealers of second-hand products often have the option to upgrade an item to improve its reliability. It is notable that upgrade is worthwhile only if the cost of upgrade is less than the expected reduction in the warranty cost. Detecting defective parts, they are divided into two categories, repairable and non-repairable. Repairable parts are repaired and upgraded. If these actions are not cost-effective, they are recycled or destroyed. If the repair is done, the reliability returns to pre-failure condition or increases. Thus, the failure rate decreases during the warranty period. The following three upgrade action strategies are considered in this study:

- *A minimal repair makes a minor improvement, and the product operational state is "as bad as old."*
- *An imperfect repair restores the item to an operational state that is between "as good as new" and as bad as old.*
- *A perfect (complete) repair makes the item's operational state as good as new [19].*

The higher the quality of supplied second-hand products, the stronger the application of each strategy, from the first to the third, will be. On the other hand, the seller bears more cost. Increasing the reliability of the second-hand products is related to the products entered the second-hand market with a specific warranty period and upgrade actions.

Due to the limited number of articles in the field of upgrade actions on used products, in this section, a review of the research background related to improving the reliability of second-hand products until 2019 is done. An example of basic research in which the proposed model has been used in many reliability studies is the paper by Hussain and Murthy [20]. They proposed a stochastic model based on the upgrade coefficient in improving reliability of the second-hand products. One of the most important articles increasing the reliability of second-hand products offered with warranty policies is presented by Chattopadhyay and Murthy [9]. Given that the supply of second-hand products under different warranty policies leads to additional costs for sellers, they tend to take steps to improve and enhance the reliability of these products before offering so that they can bear lower costs during the warranty period. Shafiee et al. [21] and [22] develop two costing models for warranty with upgrade actions. The first model is based on the combination of virtual age and minimal repair approaches, and the second model is based on a change in the improvement coefficient approach and based on the upgrade level. By the two proposed models, the sum of warranty costs and the optimal upgrade level are determined. They also proposed other models in the form of joint determination of warranty period, upgrade level and price of second-hand products under different warranty policies [23] and [24]. Soltani Neshan and Asgharizadeh [25] present a model for estimating warranty costs after upgrade actions using a combination of the virtual age approach and minimal repairs along with pricing for the second-hand biochemical equipment. The result of the model is a reduction in failure costs during the warranty period under the above approaches. A cost modeling is also developed by Darghouth et al. [26] to determine the optimal upgrade level for second-hand products under the free repair replacement policy with periodic maintenance and repairs during the warranty period. A pre-sale upgrade model is developed for repairable used products sold with two-dimensional warranty policies by Su and Wang [27]. Two kinds of two-dimensional warranty regions, including rectangular and L-shaped, are considered. The optimal upgrade policy is determined to minimize the dealer's total expected servicing cost. Yazdian et al. [15] present a model for simultaneously determining the optimal price, warranty and the upgrade level of second-hand products by the virtual age approach while taking into account price and warranty based on demand. Wang et al. [28] proposed a model for determining the optimal upgrade level in second-hand production equipment with and without maintenance programs. Zhang et al. [29] presented a stochastic model for minimizing short-term warranty costs for two-component systems. Wang et al. [30] developed an upgrade model for sophisticated second-hand products under the of free repair-replacement policy.

From the warranty literature reviewed, second-hand products' warranty is offered under two approaches: warranty after upgrades and warranty without upgrades. In this paper, a hybrid stochastic model is presented to improve the reliability of second-hand products under the free repair replacement warranty policy, which aims to determine the optimum level of upgrade, as a decision variable, with the aim of reducing warranty costs. Where the modelling proposed by Shafiee et al. [31] is developed, and a new approach to estimating warranty costs under the upgrade approach is presented. *Table 1* compares the model developed in this research and the model presented by Shafiee et al. [31]. The contribution of the presented model is that we integrated the three upgrade actions including "virtual age", "improvement coefficient" and "minimal perfect repair" actions. The proposed integration helps modeling almost all real situations occurred during the second-hand products' upgrade.

Table 1. Comparing the proposed model and the existing approach.

Authors	Virtual age	Improvement coefficient	Minimal- perfect repair
Shafie et al. [23]	☒	☐	☒
The presented model	☒	☒	☒

3 | Modeling Warranty Costs under the Upgrade Approach

This section first analyzes and describes upgrade approaches. Then, a new upgrade-based costing stochastic model is also developed as a contribution to the reliability improvement literature. The warranty costs are determined based on the upgrade approaches in the developed model. All the steps that the seller can take to sell the second-hand product in the best possible way are presented. The model considers both the seller and the buyer's profits simultaneously. This study supports dealers' decision making on how much to invest in reliability improvement programs.

There exist different approaches for modelling reliability improvement programs in the literature. But in general, the three main approaches, "minimal-perfect repair", "improvement factor" and "virtual age" approaches are considered:

- ***Minimal- perfect repair Approach:*** *In this approach, a second-hand product is returned to the 'good-as-new' state and fully repaired with probability p. As well, a used product is minimally repaired and restored to its functioning state, with probability 1-p. This approach is proposed by Brown and Proschan [32] developed in later years.*
- ***Improvement factor Approach:*** *In this approach, improvement coefficient indicates the effect of upgrade actions on the second-hand product in reducing the failure rate during the warranty period. The coefficient is considered between 0 and 1. For which, 0 indicates the ineffectiveness of the action and 1 shows the complete refurbishing of the second-hand product.*
- ***Virtual Age Approach:*** *Each improvement action on a second-hand product leads to its rejuvenation depending on the upgrade level. It is the basis of the virtual age approach proposed by Kijima [33], in which the modelling is based on the virtual age of the product after the upgrade action.*

The second-hand product's age is reduced to a virtual value of ν t at time t after the upgrade at a, usually expressed as ka in *Eq. (1)*. *K* is the 'virtual age parameter' (*0≤k≤1*). There are only a few references in the literature that deal with the estimation of parameter *k*.

$$\nu_t = \varkappa a + (t - a) \qquad t \geq a. \tag{1}$$

3.1 | Warranty Cost Modeling for Second-hand Product with Upgrade Action

Based on the approaches presented for the second-hand products upgrade, in this section, a hybrid stochastic model of existing approaches has been developed to find the cost and determine the optimal warranty period and the upgrade level. The proposed model is designed to facilitate the use of each approach separately, combining the three approaches and applying the characteristics of each. The model is based on the models presented by Shafiee et al. [31] described in the literature. They presented two models. One is based on the combination of two approaches of virtual age and minimal-perfect repair and the other is based on the improvement coefficient approach. During the upgrade action, a second-hand product can be in the range of minimal to perfect repair to improve the reliability, which is not 100% effective. Also, it may reduce its age after the upgrade action. Therefore, in the real world, a combination of three approaches to estimating warranty costs seems reasonable and practical. The model presented in this section is obtained by combining these three approaches in the form of a new model described below:

Let *T* be a random variable describing the failure time of the product with cumulative distribution function *F(t)* and the density function *f(t)= dF(t)/dt*. The Rate of Occurrence of Failures (ROCOF) is the Non- Homogeneous Poisson Process (NHPP) based on *Eq. (2)*.

$$\Lambda_T(t) = \frac{f_T(t)}{1 - F_T(t)}. \tag{2}$$

On the basis of minimal-perfect approach, when the product is under repair, the failure rate turns to the failure rate before repair, $\Lambda_T(t)$. In complete repair the failure rate decreases to $\Lambda_T(t-a)$. It is assumed that failures occur based on point process.

Based on 'Minimal-perfect repair approach', we assume that, for given p the used product undergoes a 'complete repair' with probability p and a 'minimal repair' with a probability *(1-p)*. Thus, the rate of occurrence of failures of a second-hand products with age "a" after an upgrade action of level u is given by *Eq. (3)*.

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On the other hand, according to Hussain and Murthy [20], the improvement coefficient, which is a function of the second-hand products' upgrade, is defined as *Eq. (4)*. p' is the effectiveness of the upgrade action value. If p' is equal to 1, the upgrade action is quite effective.

$$\Lambda_T(t,u) = p \times \Lambda_T(t-a) + (1-p) \times \Lambda_T(t) \qquad t > \alpha. \tag{3}$$

$$\delta_u = (1-e^{-\varrho u}) \times p' \qquad p' \in \{0,1\}. \tag{4}$$

This parameter is considered due to the uncertainty in the effectiveness of improvement upgrade actions and to create conformity to the real world. Therefore, the parameters p and p' can be considered as random variables. According to the related literature, one of the most suitable random distributions for these two parameters is the Beta distribution ([34] and [35]). Because this function accepts finite values and can model the results realistically and experimentally. In addition, the number of parameters of this distribution for estimation is limited, which is defined as *Eq. (5)* [24]. Its mathematical expectation is equal to *Eq. (6)*.

$$\begin{aligned} \beta_p(p) &= \frac{1}{\beta(\alpha_1,\alpha_2)} p^{\alpha_1-1}(1-p)^{\alpha_2-1}, \delta_u = (1-e^{-\varrho u}) \times p', \\ \beta_p(p) &= \frac{1}{\beta(\alpha_1,\alpha_2)} p^{\alpha_1-1}(1-p)^{\alpha_2-1}, \delta_u = (1-e^{-\varrho u}) \times p'. \end{aligned} \tag{5}$$

$$E(p) = \frac{\alpha_1}{\alpha_1+\alpha_2}, \qquad E(p') = \frac{\alpha_1'}{\alpha_1'+\alpha_2'}. \tag{6}$$

The random variable p is related to the value of u. It is assumed that based on the nonlinear *Eq. (7)* and *Eq. (8)* that Hussain [36] presented, a_1, b_1, a_2, b_2, and also a'_1, b'_1, a'_2, b'_2 can be calculated through logarithmic regression.

$$\alpha_1 = a_1 u^{\alpha_2}, \alpha_1' = a_1' u^{\alpha_2'}. \tag{7}$$

$$\alpha_2 = b_1 u^{b_2}, \alpha_2' = b_1' u^{b_2'}. \tag{8}$$

For modeling warranty costs using the combined approach, it is possible to calculate the average warranty costs of a second-hand product with the age of "*a*" to compare the costs in the case of upgrade and without upgrade under the free replacement warranty policy. The savings from upgrade actions are also obtained from the difference in costs with and without the upgrade actions.

3.1.1 | Costs of upgrade actions for second hand product

If the recovery of second-hand products leads to their upgrade, an investment, equal to the upgrade cost, is made through the seller or the manufacturer. The costs are usually in the form of initial costs for start-up, upgrade costs, equipment procurement costs, overhead costs, parts replacement and etc.

Assume that $C_u(a)$ is the amount of capital spent by the seller to improve the quality and performance of the second-hand product with the age of "*a*" and upgrade level of *u*. Using the model of Pongpech and Murthy [19], the cost of upgrade action, which is a non-linear function of product age and upgrade strategy, is formulated as *Eq. (9)*.

$$C_u(a) = \frac{\alpha \times u}{1-\exp[-\phi(a-u)]}. \tag{9}$$

In this model, ϕ and a are two positive parameters estimated using the past history of the product. If $u = 0$, no upgrade action is made on the second-hand product, and the cost is 0, and if $u = a$, the product undergoes complete repair and the cost is maximum. Therefore, the higher the upgrade level is, the higher the upgrade costs will be.

3.1.2 | Modeling warranty costs without upgrade action

E [*Nw (a)*] indicates the expected number of claims for a second-hand product with age of "a" over the warranty period w based on *Eq. (10)*.

$$E(N_w(a)) = \int_a^{a+w} \Lambda_T(t)dt. \tag{10}$$

Therefore, the seller's expected warranty cost over the warranty period is given by *Eq. (11)*.

$$E(C_w(a)) = \bar{C}\int_a^{a+w} \Lambda_T(t)dt. \tag{11}$$

3.1.3 | Modeling warranty costs with upgrade action

Suppose E [N_w *(a, u)*] represents the number of claims that occurred for a second-hand product with age "*a*" over the warranty period w with upgrade value of u (age reduction following the upgrade action). Based on *Eq. (12)*, the expected warranty over the warranty period w is equal to *Eq. (13)*.

$$E(N_w(a,u)) = \int_a^{a+w} \Lambda_T(t,u)dt. \tag{12}$$

$$\begin{aligned} E(C_w(a,u)) = {} & \bar{C}\int_a^{a+w}\int_0^1\int_0^1 p\times\Lambda_T(t)\times\delta_u\times B(p)\times B(p')dpdp'dt \\ & + \\ & \bar{C}\int_a^{a+w}\int_0^1\int_0^1 (1-p)\times\Lambda_T(t)\times\delta_u\times B(p)\times B(p')dpdp'dt. \end{aligned} \tag{13}$$

Where $\overline{C}$ is the average warranty cost over the warranty period. Based on the modeling by Hussain [36], the improvement coefficient (*δu*) is modeled as *Eq. (14)* with minimal changes in the failure parameter.

$$\delta_u = (1-e^{-\varrho u})\times p'. \tag{14}$$

The improvement factor is function of *u* (upgrade value) in which ϱ is a positive parameter, and the *p′* is between 0 and 1, indicating the effectiveness of the improvement action. If *p = 1*, the upgrade action has been quite effective.

By placing *Eq. (14)* and *Eq. (6)* in *Eq. (13)* we have *Eq. (15)* and by placing *Eq. (7)* and *Eq. (8)* in *Eq. (15)*, the expected warranty costs over the warranty period based on the combination of three approaches is given by *Eq. (16)*.

$$\begin{aligned} E(C_w(a,u)) = {} & \bar{C}\times E(p)\times E(p')\int_a^{a+w} \Lambda_T(t-a)\times(1-e^{-\varrho u})dt \\ & + \\ & \bar{C}\times E(1-p)\times E(p')\int_a^{a+w} \Lambda_T(t)\times(1-e^{-\varrho u})dt. \end{aligned} \tag{15}$$

$$E(C_w(a,u)) = \frac{(1-e^{-\varrho u})}{a_1 u^{a_2} + b_1 u^{b2}} \times \frac{a'_1 u^{a'_2}}{a'_1 u^{a'_2} + b'_1 u^{b'_2}} \times [a_1 u^{a_2} \times \int_a^{a+w} \Lambda_T(t-a)dt + b_1 u^{b_2} \int_a^{a+w} \Lambda_T(t)dt. \tag{16}$$

3.2 | Average Savings in Warranty Costs with and without Upgrade Action

It has already been mentioned that upgrade actions reduce the seller's costs over the warranty period. This leads to cost savings for the seller. The amount of this savings is equal to the difference between the warranty costs with the upgrade program and without the upgrade program (*Eq. (17)*).

$$E(C_w(a,u,w)) = E(C_w(a,u)) + C_u(a). \tag{17}$$

There are two conditions for analyzing upgrade costs:

1) Cost-effectiveness of upgrade action

Upgrade actions on second-hand products cost-effective product will be cost-effective if the average amount of savings resulting from upgrade programs exceeds the upgrade costs *(Eq. (18))*.

$$E(CS(a,u)) = E(C_w(a)) - E(C_w(a,u)). \tag{18}$$

This means that the seller's investment in upgrading a second-hand product should not exceed the savings.

2) Determining the minimum cost and the level of optimal upgrade.

To determine the best upgrade level, for the known warranty periods, it is necessary to determine the total cost. According to *Eq. (19)* it is equal to the sum of the warranty and the upgrading costs of the second-hand products.

$$E[CS_u(a)] > C_u(a). \tag{19}$$

By minimizing the above equation, if the upgrade action is cost-effective, the optimal upgrade level can be achieved in different warranty periods. Because, the higher the level of upgrade, the higher the initial investment on the one hand, and the reduction of warranty costs on the other hand. Therefore, the necessary balance must be settled between the two.

4 | Numerical and Sensitivity Analysis

The case study intended for this research is the air conditioner. The air conditioning system consists of 17 main components, including thermostat, motor, filter, compressor, coil, and several by-products that do not play a critical role in the product's operation. The main parts are the main ones because they suffer the most damage. In case of failure over the warranty period, "free repair replacement" policy, the system undergoes repairing the parts or replacing them.

In order to determine the statistical distribution of failures of second-hand products sold during the warranty period, the relevant information from the representatives of this product with a known brand located in Italy has been collected for analysis. The collected data regards customer claims about product failure during the warranty period. Of the 4212 upgraded products distributed over 18 months, 1307 products were accompanied by failure claims. Thus, about 30 percent have been damaged during the warranty period. The type of warranty is “free repair – replacement”. Number of products damaged again after repair are excluded from this study due to lack of information.

Among the registered claims, the engine and compressor parts have the highest number of failures. Accordingly, in order to determine the failure distribution function of these parts within the warranty period, EASY-FIT software is used to find the appropriate distribution function. Different distributions (more than 50 types) were tested on the data. Finally, among the existing distributions, based on the warranty literature and due to the increasing-decreasing trend of engine failure rate, Weibull distribution is considered as the most appropriate distribution function for the main components failures. The results of the k-s test at different levels of reliability indicate a good fit for this distribution on failure data.

In the present study, an estimation of the Weibull distribution parameters for the engine obtained as $\lambda = 0.56$ and $\beta = 2.8$ using MATLAB software.

4.1 | Stochastic Model: Determining Minimum Cost and Optimal Upgrade Level

The seller sells his products under a "free repair replacement" policy and offers his second-hand products after repair with a one-year warranty period. Due to the high costs of the warranty period, the seller tends to better control and optimize the costs of the warranty period by finding the optimal upgrade level. In this study, the costs are considered as the sum of the warranty period costs and the upgrade investment costs. According to the selected Weibull distribution as the failure time distribution and by placing its failure rate function, *Eq. (19)* becomes *Eq. (20)*, and the total cost function is obtained based on *Eq. (21)*.

$$E(C_w(a,u)) = \frac{(1-e^{-\varrho u})}{a_1 u^{a_2} + b_1 u^{b2}} \times \frac{a_1' u^{a_2'}}{a_1' u^{a_2'} + b_1' u^{b_2'}} \times [a_1 u^{a_2} \times \lambda^{\beta} + ((a+w)^{\beta} - a^{\beta}) \times b_1 u^{b_2}. \tag{20}$$

$$E(C_w(a,u)) = \frac{\delta \times u}{1-e^{-\theta(a-u)}} + \frac{(1-e^{-\varrho u})}{a_1 u^{a_2} + b_1 u^{b2}} \times \frac{a_1' u^{a_2'}}{a_1' u^{a_2'} + b_1' u^{b_2'}} \times [a_1 u^{a_2} \times \lambda^{\beta} + ((a+w)^{\beta} - a^{\beta}) \times b_1 u^{b_2}. \tag{21}$$

To determine the optimal upgrade level, the above equation is minimized. Due to the complexity of this function, and the impossibility of using derivation methods, the optimization code of this function has been created using a network search algorithm. Defining the variable in the specified interval, the algorithm searches in this interval to find the optimal value. Accordingly, the proposed algorithm searches the function at the midpoints of the intervals to find the points with the lowest costs.

Also, in order to find the parameters in the upgrade cost function (δ and θ) - *Eq. (9)*, beta distribution parameters (a_1, α_2, β_1, β_2) - *Eq. (16)* and also the parameters of improvement coefficient (ϱ) - *Eq. (7)* and *Eq. (8)* logarithmic regression method is utilized using the existing records of the used products.

The information obtained is as follows:

$\beta=2.8$, $\lambda=0.56$, $\delta=5.01$, $\theta=0.31$, $\rho=0.8$, $a1=2.13$, $a2=1$, $b1=3.04$, $b2=1$, $a1'=2.13$, $a2'=1$, $b1'=3.04$, $b2'=1$.

The results of warranty cost optimization and the optimal value of u (u*) for a range of values of w and a are given in *Fig. 2*. Analyses are presented in four different ages, from 1 to 4 years. In second-hand products at age 1, it is not cost-effective to upgrade the products. Thus, they have no feasible solution. For other periods, for example, a product at age 4 and a warranty period 1.5 years, the optimal upgrade level is 1.44 years. This means that the second-hand product at its current age needs to be rejuvenated 1.44 years to meet the lowest cost. That is, the age of the product should be reduced from four years and rejuvenated accordingly. After all, the product is offered to the market with a 1.5 years free repair replacement warranty.

Warranty costs will increase if the product age and the warranty period increase. For example, in a given age of 2 years and a 1year warranty period, the upgrade rate is 0.43 years, and for a three years old product with the same warranty period, the upgrade level is 0.76 years. On the other hand, for a given product with a 1.5 years warranty, the upgrade value is 0.60, and the same product with a warranty period of 2 years has the upgrade value of 0.73. Besides, the higher the upgrade level as the age of the product increases, the higher the savings. Also, all the saving amounts are higher than the investment amounts. *Fig. 1* and *Fig. 2* show the expected savings and the upgrade level incurred versus the improvement level for a second-hand product sold with the past age from 2 to 4 years with a warranty period of 3 years.

Although, the higher the level of upgrade, the greater the amount of savings. The more upgrades than a certain extent does not lead to more savings. Therefore, selling second-hand products after upgrade actions is more cost-effective than selling them without upgrading.

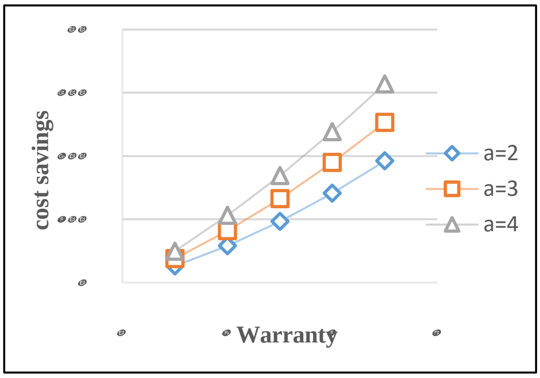

Fig. 1. Average savings of upgrade action.

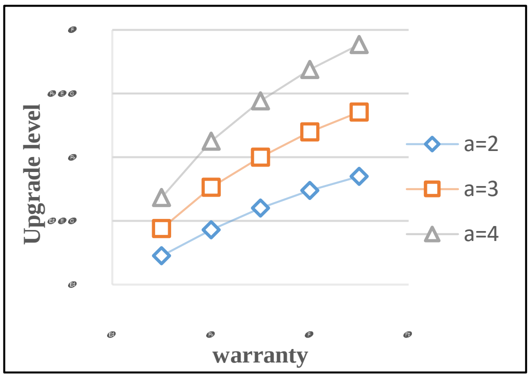

Fig. 2. Optimum upgrade levels in different ages.

4.1.1 | Sensitivity analysis

In this section, we illustrate our results and investigate the effects of relevant parameters on the optimal upgrade level and warranty cost saving numerically. For the purpose of the numerical evaluation, each time, we fix repair costs in different values and calculate the results by several combinations of the repair costs together with different warranty periods and product ages measured in years. *Fig. 1* shows the average investment and warranty costs and savings resulting from upgrades for different warranty periods of 6 months to 3 years. The results indicate that upgrade action is not cost-effective within the age of less than one year. In addition, the average warranty costs increase with the length of the warranty period. The sensitivity of the total cost model with the average change in repair costs for \$ 120, \$ 140 and \$ 160 for second-hand products with the age of 1 to 4 years and warranty periods of 6 months to 2 years, is presented in *Fig. 2*. The results show that although increasing the warranty period and upgrade actions cost the seller more and consequently a higher price for the buyer, but in the long run, these actions will reduce the warranty costs after the end of the warranty period. Therefore, it is better to take the upgrade actions on the products whose performance is more important than their appearance for the customer. In fact, the higher the repair costs, the higher the upgrade under the same conditions. It is due to the use of higher quality parts when performing a second-hand product improvement operation. The level of second-hand product upgrade depends on factors such as product age, the severity of the failure, second-hand product market, and related costs. The higher the repair costs, the higher the quality of the parts used, or the higher the cost of the repair. Therefore, upgrade actions are more cost-effective for high-value products. In addition, the upgrade actions are more cost effective when the second-hand products have higher past age. Therefore, it is suggested to focus on upgrading second-hand products with longer past age and high value. Also, it is better to take upgrade actions for products that have more possibility to increase the warranty period.

5 | Conclusion

Despite the competitive environment for second-hand products markets, reusing these products has become an attractive investment market. In order to sell better, these products are subjected to an optimal upgrade level in order to provide the seller's profit on the one hand and the consumer's profit on the other hand. In this paper, a new hybrid stochastic model is developed to model the estimation of the minimum warranty costs of second-hand products under the "free repair replacement" warranty policy to find the optimal upgrade level. The structure of the model is based on combining three upgrade approaches, including "improvement factor approach", "minimal-perfect repair" and "virtual age". Under given cost structures, the optimal upgrade action strategy is identified, leading to minimization of the sellers' expected warranty costs. A case study is conducted to evaluate the validity of the model. The results show that warranty length, the past life of second-hand products, and the upgrade level have all significant effects on the warranty costs and sellers' expected profit considering all occurrences in the real world. The results showed that although increasing the warranty period and the upgrade level imposes higher costs on the seller and consequently a higher price, but in the long run, these improvements actions lead to a reduction in warranty costs after the end of the warranty period. In fact, the higher the repair costs, the higher the upgrade under the same conditions will be. Therefore, given the upgrade costs, upgrade actions are more cost-effective for high-value products. Furthermore, the longer the life of the second-hand product, the more cost-effective the upgrade actions, and the higher the savings.

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Proposing an Integrated Model for Evaluation of Green and Resilient Suppliers by Path Analysis, SWARA and TOPSIS

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Abstract

The main purpose of this paper is to identify the traditional, green and effective resilience criteria in the performance of green and resilient suppliers and their ranking with path analysis, SWARA and TOPSIS combined approach in Fanavaran Petrochemical Company. The research method is applied in terms of goal and descriptive-survey in terms of data collection. By a comprehensive review of the literature, first a set of key performance criteria and sub-criteria (traditional, green, and resilience) were extracted. Then, using the path analysis approach, the effectiveness of these criteria was evaluated in Fanavaran Petrochemical Company. The statistical population included 55 experts of the mentioned company, which due to the limited size of the population, all members were considered as the research sample. The path analysis result showed that all identified criteria affect the company's supplier's performance. Then, using new SWARA decision-making technique and also the opinions of 30 experts, the criteria and sub-criteria were evaluated and their weight (importance) was extracted. In the final evaluation of the main criteria, the criterion of "resilience" was in the first rank, the criterion of "green" in the second rank and the criterion of "traditional" in the last rank. Subsequently, due to the sensitivity of the ranking of green and resilient suppliers in the company, using the TOPSIS decision-making technique and based on the extractive weight of the criteria, seven suppliers of the company were evaluated by the experts and the final ranking of the suppliers in terms of performance was determined. Thus, the proposed approach of this research provides a valuable conceptual framework for company' managers to improve the situation of the suppliers in terms of the environmental issues and resilience. Also, the development and improvement of traditional criteria and selection of suppliers of the company based on green standards and resilience were the main goals of this research.

Keywords: Performance, Green and resilient suppliers, Path analysis, SWARA, TOPSIS.

1 | Introduction

In recent years, the advent of new technologies and shifts in global markets have necessitated the supply chain management, so that different organizations use supply chain management inevitably to create and maintain their competitive position. Some company strategies in managing business that can be implemented are Supply Chain Management (SCM).

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Supply chain includes all activities related to the flow and exchange of goods and services ranging from the consumable raw material stage to the final product stage. In addition to the material flow, these interactions include information flow and financial discussions [11, 34]. This system acts as a connection between the inputs of an organization and its outputs.

The primary goal of a chain is to provide and meet the customer needs in the value production process. The purpose of each chain is to maximize its total value production in a defined time interval. Chain profitability is the total profit that should be divided in all stages of the chain. Therefore, the success of a chain is defined in terms of its profitability, and supply chain management requires managing the flows between and within the steps of a chain to maximize its total profitability.

Nowadays, sustainable development depends on the optimal preservation and use of limited and non-replaceable resources in countries. Various measures have been taken by governments to deal with this issue including the use of environmental-friendly raw materials in production and industrial centers, reducing the use of fossil and petroleum resources and reusing waste [19].

Observing government regulations to meet the environmental standards and growing consumer demand for green products in the supply chain, which cover all activities related to the flow of goods from the raw material stage to the delivery of goods to end consumers, including the flow of information across the chain, have led to the emergence of a new concept of “green supply chain management” that includes the stages of the product life cycle from design to recycling [17].

Srivastava [40] defines green supply chain management as considering the environmental issues in supply chain management, such as product design, material selection and sourcing, final product delivery to the customer, and product management after consumption and its useful life. Accordingly, green supply chain management is similar to the product life cycle. Global organizations are always seeking to achieve competitive advantage through the creation of innovations and new approaches. Some of these organizations gain competitive advantage by improving the environmental performance by adhering to environmental laws and standards, enhancing customer knowledge and reducing negative environmental impacts on their products and services.

Selection of supplier refers to the issue of multi-criteria decision making in evaluating the performance of suppliers by several criteria with the aim of purchasing items from the most suitable supplier. Despite the importance of price, other evaluation criteria affecting the efficiency and productivity of the production environment and the overall costs of companies, such as timely delivery and other elements should also be considered. Ha and Krishnan [15] updated these criteria and added several traditional criteria. According to their research, the most traditional criteria for business performance in supplier evaluation include quality, cost, and delivery. The general green standards also include the environmental management systems for resource consumption, the environmental design, and waste management [29]. Purchasing management usually considers traditional and green performance appraisal criteria and may ignore resilience criteria [24] and [29]. Resilience in the sense of system capability to adapt efficiency and expected and predictable performance disorders and return the system to the natural process, is considered as an essential aspect of any supply chain management system [29].

In the present study, a new approach has been proposed to integrate the three main and fundamental criteria for evaluating suppliers (traditional business, green and resilience criteria). Despite extensive studies, none of them met all the criteria simultaneously with the combined approach to path analysis and decision-making techniques. Considering the research gap, this study has examined all the performance criteria (traditional, green and resilience) simultaneously in selecting the suppliers of Fanavaran Petrochemical Company. The petrochemical industry is one of the leading and biggest industries by employment that, as a feeder for other sectors of the industry, can play a key role as the engine of the country’s economy. Iran has an effective role in the petrochemical industry due to its vast oil and gas resources and being located on the world’s energy highway. On the other hand, due to the high added value of the petrochemical industry, this industry has a high position in the national economy and is considered a competitive advantage. Also, according to the upstream and downstream connection of this industry with other industries, the expansion of its capacities makes it possible to have a positive impact on the growth and economic development of the country. In addition, considering the effects of the petrochemical industry on the environment and its various pollutants, the development of this industry is not possible without considering the above-mentioned cases and will have irreparable consequences for the country.

Fanavaran Petrochemical Company (Public Joint Stock) was established on 1998/4/28 and registered with registration number of 2017.02 in the Office of Corporate Registration and Industrial Property. The company is active in the establishment, startup and operation of methanol, acetic acid and carbon monoxide units for export, utilization in the petrochemical industry and downstream industries and is consistent with the industry's major export goals, policies, employment creation, transfer of technical knowledge, specialist training and production of high value-added petrochemical products from natural gas. Given the importance of green supply chain management at Fanavaran Petrochemical Company, the necessity for operational implementation of this system in this industry is obvious. For doing so, at first, designing and explaining the model of key factors affecting the implementation of green supply chain management in Fanavaran Petrochemical Company is essential. Main purposes of present research are: 1. Identification the effective factors on implementation of GSCM at the Company in Iran by using statistical methods 2. Evaluating the factors by using a new method of MADM by topic SWARA and finally ranking them based on weight and importance.

SWARA is one of the new methods of MCDM which was used in 2010 to develop analysis of the differences between the criteria. In SWARA, each expert ranks the criteria at first. The most important criterion is scored one and the least important one receives low score. Finally, the criteria are prioritized according to average values of the relative importance [2].

To this end, this research aims to answer the basic question: what factors have a significant impact on the implementation of green supply chain management in Fanavaran Petrochemical Company and how is the rank (weight & importance) of these factors?

The organization of the remaining sections in this paper is as follows: Section 2, reviews the research literature and proposes conceptual model; Section 3, presents the research questions; the research methodology is presented in Section 4. Section 5, presents the findings of research; finally, conclusion and recommendations are presented in Section 6.

2 | Literature Review and Conceptual Model

In this section, review of literature and background is discussed in two parts: 1- traditional and green performance criteria, 2- resilience criteria in supplier evaluation and then an integrated conceptual model including selection criteria for green and resilient suppliers is presented.

2.1 | Traditional and Green Criteria

Most previous studies have focused on evaluating and selecting suppliers based on traditional performance criteria and fewer studies have identified green performance and resilience criteria and evaluation based on these criteria [29] and [13]. On the other hand, in none of the previous researches, all the functional criteria (traditional, green and resilience) have been studied simultaneously. Therefore, the selection of suppliers by decision makers is based on only one of these criteria. Tirkolaee et al. [6] presented a novel hybrid method using fuzzy decision making and multi-objective programming for sustainable-reliable supplier selection in two-echelon supply chain design. This paper applies the Fuzzy Analytic Network Process (FANP) method to ranking criteria and sub-criteria, the fuzzy Decision-Making Trial and Evaluation Laboratory (DEMATEL) is applied to identification of the relationships among the main criteria, and the fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to prioritizing the suppliers. After prioritizing the suppliers, the obtained weights are considered as the input of a tri-objective model designed to optimize the proposed supply chain. Vafaei et al. [43] examined the mediating role of innovation and sustainable process management on the relationship between sustainable supply chain management and sustainable competitive advantage. The statistical population consists of 20 companies involved in the production of banking equipment. Since the size of the statistical population is very small, structural equations model and partial least squares approach were used to analyze the research data and to test the hypotheses of the research. The results showed that there is a positive and significant relationship between sustainable supply chain management and sustainable competitive advantage. Moreover, it was found that innovation and sustainable process management variables play a mediating and moderating role on the relationship between sustainable supply chain and sustainable competitive advantage. Amani et al. [4] identified barriers to green supply chain acceptance using Fuzzy DEMATEl Technique. Extracted factors in this research were Outsourcing, technology, knowledge, finance and support. Govindan and Sivakumar [14] developed an integrated multi-criteria decision-making and multi-objective linear programming approach as an aid to select the best green supplier. Gandhi et al. [12] evaluated the important factors associated with the successful implementation of GSCM. This paper proposes a DEMATEL approach to develop a structural model for evaluating the influential factors among recognized factors. To show the real-life applicability of the proposed DEMATEL based model, an empirical case study of an Indian manufacturing company is conducted. Research findings indicate that Top Management Commitment, Human Technical Expertise, Financial Factors, has obtained the highest influential power for accomplishing the successful GSCM adoption. Conclusions and implications for managers are also discussed. Tyagi et al. [42] identified seven green criteria (including saving energy, design for environment, waste minimization, reuse of hazardous waste, awareness about green concept, information sharing regarding environmental regulations and proper mode of transport) and three mutually important alternatives namely as suppliers, web-based technologies and advanced manufacturing technologies. On the basis of considered criteria and alternatives, a hierarchy type performance model has been developed and analyzed using Fuzzy TOPSIS approach to select the best alternative in order to improve the performance of GSCM system. The findings suggested that alternative 'web-based technologies' is more desirable among considered alternatives and insert a significant role in enhancing the green supply chain performance of an industry.

Given the literature review, traditional key criteria in evaluating suppliers' performance are: Costs (TC1); quality (TC2); delivery reliability (TC3); performance history (TC4); turnover (TC5); lead time (TC6); operating capacity (TC7) were considered [29]. Also, key green criteria in evaluating suppliers' performance were proposed as *Table 1* [29] and [41]).

Table 1. Key green criteria in evaluating suppliers' performance.

Adherence to the required standards (GC1)	Government participation and regulations and standards. Materials and compliance with the standards required for the purchase of raw materials. Designing products to reduce energy and material consumption. Green production and packaging. Procurement, distribution and reverse logistics.
Compliance with environmental issues (GC2)	Recycling and reuse of waste inside and outside the company. Environmental certificates. Total environmental quality management.
Management Commitment and employees and customer requirements (GC3)	Human resources employment management and technical Assistance. Top management commitment. Customer requirements.

2.2 | Resilience Criterion

Supply chain management involves a variety of complex set of activities established by unpredictable events. For doing so, improving the resilience of the supply chain is essential for the management of strong disorders. The concept of resilience, the ability of a company or supply chain to resist and improve at the same time against disruptions, is very important from supply chain management perspective. Supplier disruptions can impose significant losses on the entire supply chain by cutting off supply flows. Resilience criteria refer to the ability of suppliers to deal with risks and unexpected and unpredictable events that affect performance of the suppliers [29]. A review of the literature shows that studies are limited to using quantitative approaches to solving supplier selection problems. Mitra et al. [27] identified several criteria in selecting resilient suppliers. Haldar et al. [16] developed a fuzzy multi-criteria decision-making approach by considering the degree of importance of scientific indicators in the form of linguistic variables formulated with triangular and trapezoidal fuzzy numbers. Sahu et al. [37] proposed a supportive decision support system for suppliers using the Vikor method and taking into account general criteria and resilience. Pramanik et al. [32] suggested a fuzzy multi-criteria decision-making approach to develop and select resilient suppliers. Mohammed et al. [29] evaluated Green and Resilient Supplier Performance using AHP-Fuzzy TOPSIS Decision-Making Approach. They ranked suppliers with respect to their Traditional, Green and Resilience characteristics (TGR). A set of criteria/sub-criteria were identified within a unified framework and their relative importance weighted using the Analytical Hierarchy Process (AHP) algorithm. In addition, the suppliers were evaluated and ranked based on their performance towards the identified TGR criteria using the fuzzy TOPSIS algorithm through a real case study. The study provides a noteworthy aid to management who understand the necessity of building supply chain resilience while concurrently pursuing 'go green' responsibilities.

Following the review of the literature, the resilience criteria in evaluating the performance of suppliers were proposed as *Table 2*.

Table 2. The resilience criteria in evaluating the performance of suppliers.

Redundancy (RC1)	Mohamed et al. [29]; Kamalahmadi et al. [22]
Agility of Supply Chain (RC2)	Mohamed et al. [29]; Purvis et al. [33]; Rajesh and Ravi [35]
Complexity (RC3)	Carvalho et al. [8]; Blackhurst et al. [7]
Visibility of Supply Chain (RC4)	Kamalahmadi et al. [22]; Rajesh and Ravi [35]
Flexibility (RC5)	Rajesh and Ravi [35]; Jayaram et al. [21]

2.3 | The Integrated Conceptual Model

In this study, considering the extensive review of literature, the main and key criteria (traditional, green, resilience) in evaluating the performance of suppliers were identified and extracted. *Fig. 1* shows the conceptual model of the hierarchical series of criteria (traditional, green and resilience) and sub-criteria for selecting suppliers.

3 | Research Questions

Given the proposed conceptual model, the following questions arise:

- *What are the factors affecting the implementation of "traditional, green and resilient" suppliers integrated performance evaluation in Iranian petrochemical company?*
- *What is the importance (weight) of the identified factors in this company?*
- *What is the performance rating of the company's suppliers in terms of all criteria (traditional, green and resilience)?*
- *What effective solutions can be provided for managers to deal with the obstacles to the establishment of an integrated system and to improve the status of this system for implementation in the Iranian petrochemical company?*

4 | Research Methodology

This study is an applied research in terms of purpose and descriptive-survey in terms of data collection. This research, as survey studies, systematically describes the current situation through a questionnaire tool and studies its characteristics. Data collection was performed through questionnaires and interviews. This research was conducted for studying the impact of identified factors and finally extracts the final factors in Fanavaran Petrochemical Company's Green Supply Chain Management System with mean statistical test. The statistical population consisted of the experts and managers and specialists with useful experience and expertise in this system (55 people). All members of the community were participated because of their limitations. In sum, the research approach proposed in this study was developed in four stages. In the first step, the traditional and green key criteria and sub-criteria and effective resilience on the supply chain management system were identified and presented as a conceptual model. In the second step, using the path analysis approach, the final criteria affecting the integrated supply chain management system in Fanavaran Petrochemical Company were specified. Then, in the third step, the weight (importance) of the criteria and sub-criteria was calculated and extracted using the SWARA decision-making technique. Finally, in the fourth section, using the TOPSIS decision-making technique, the suppliers of the company were evaluated and ranked based on performance.

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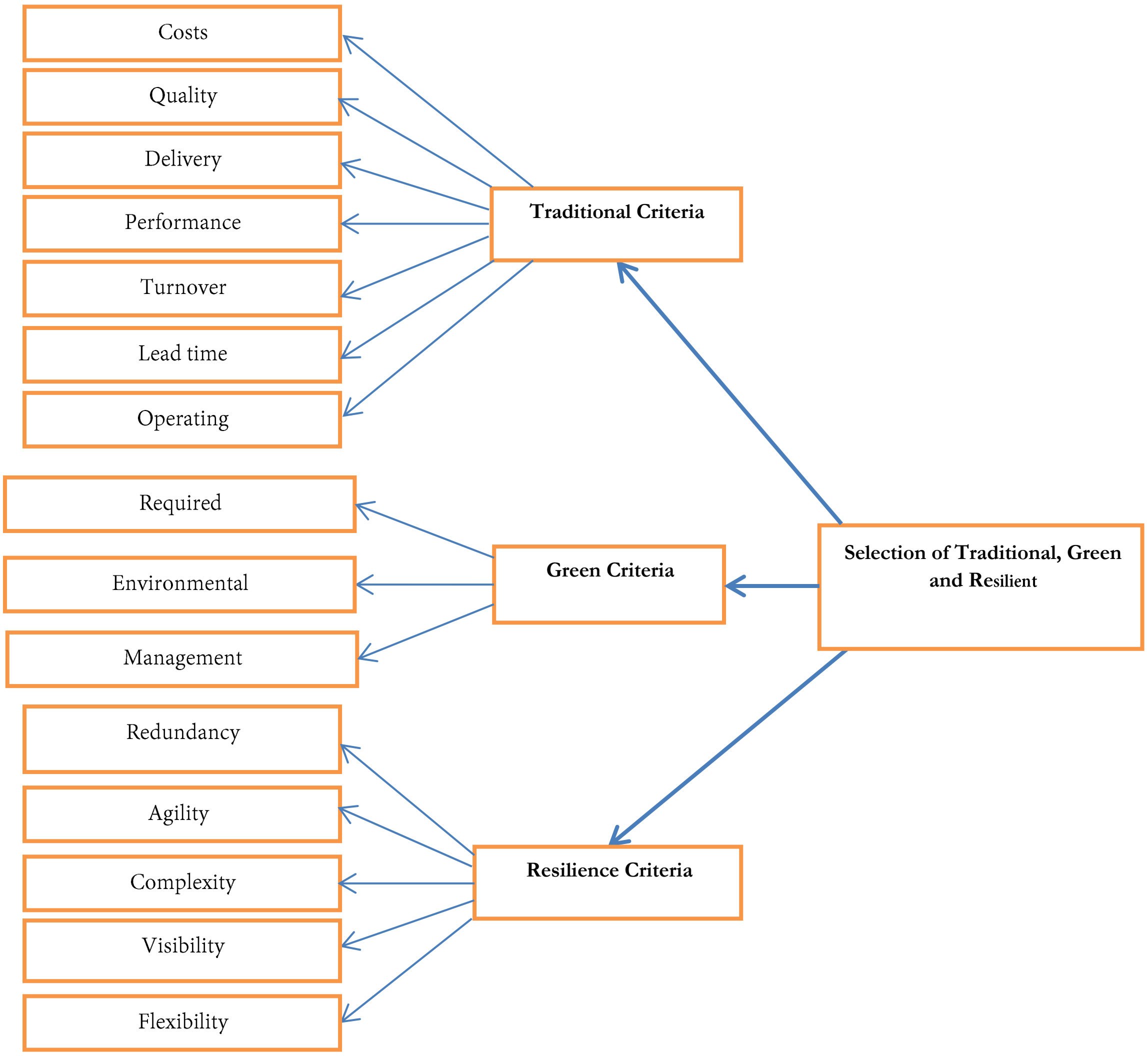

Fig. 1. The conceptual model of the hierarchical series of criteria (traditional, green and resilience) and sub-criteria for selecting suppliers.

In order to implement the SWARA decision making technique, the viewpoints of at least 10 experts of the studied company (30 people in this study) were used in a second questionnaire designed for this purpose. This type of sampling is a non-probability random sampling method and usually 10 to 20 people are considered sufficient. In this study, the researchers achieved this number of experts with theoretical saturation in the field because theoretical saturation occurs when the data that helps to define a class characteristics is no longer entered into the research and all comparisons are made. In fact, these experts are all first rank managers of Fanavaran Petrochemical Company and are fully knowledgeable on the subject. Then, these criteria were entered into the present research questionnaire and were given to experts to express their views on the importance of criteria in terms of impact. Next, using the SWARA technique steps, the questionnaire data were analyzed to weight these key criteria and rank them. In this technique, the expert assesses the calculated weights. In addition, each expert specifies the importance of each criterion according to tacit knowledge, information and experience. Then, according to the average value of the group's ranks obtained by experts, the weight of each criterion is determined. Therefore, in this study, the interviews of 20 Iranian Industries experts were used. The weight of each criterion indicates its importance. Measuring of weight is an important topic in many issues of decision-making. SWARA is one of the weighting methods in which professionals play an important role in the calculation of their weight and final assessment [2].

In order to rank the performance of green and resilient suppliers, the TOPSIS technique is used using the opinions of 30 experts. This technique is one of the most cost-effective compensating methods for alternative rankings. This technique is a compromise subgroup of compensatory models. In this method, the m option is evaluated by n index and each issue can be considered as a geometric system including m point in a subsequent n space. This technique is based on the concept that the selected option should be the shortest distance from the solution of the positive ideal (best possible state, A_i^+) and the longest distance from the solution of the negative ideal (worst possible solution, A_i^-). It is assumed that the utility of each indicator is uniformly increasing or decreasing. In analyzing complex multi-criteria problems, TOPSIS is a well-known technique used to rank options by scoring them to arrive at a desirable solution. The main advantages of this method compared to similar methods such as AHP is that if the decision criteria include reducing cost and purpose or increasing profit, this method easily finds the ideal answer, which is a combination of the best values to meet all criteria. *Fig. 2* shows the flowchart methodology presented in this study.

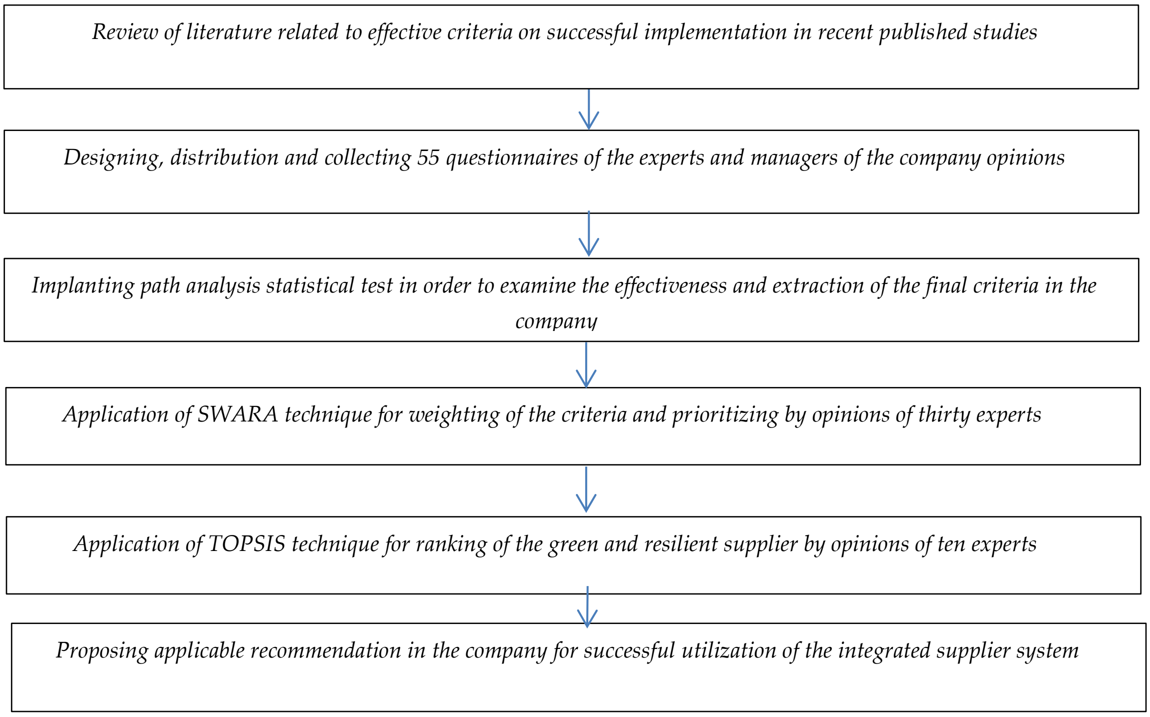

Fig. 2. Research methodology flowchart.

In summary, the steps for conducting this research are as follows:

- *Identify key factors affecting the integrated performance evaluation system of "traditional, green and resilient" suppliers with a comprehensive overview of the literature.*
- *Testing the proposed conceptual model to confirm the impact of key factors affecting the integrated system in the Iranian Fanavaran Petrochemical Company through interviewing, distributing and collecting the company's expert opinion questionnaire and using the path analysis approach and finally extracting the final research model in the company.*
- *Distribution and collection of questionnaires among the company's experts in order to implement the SWARA technique and extract the weight of the factors.*
- *Distribution and summarization of the company's experts' opinion questionnaire and use of the TOPSIS technique to rank the company's suppliers.*
- *Proposing effective solutions and suggestions for managers to strengthen key factors affecting the suppliers integrated performance evaluation system and prevent or refer to the barriers of establishment and implementation of this system in the company.*

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5 | Findings

5.1 | Path Analysis and Study of the Effect of Key Factors in Fanavaran Petrochemical Company

After designing the conceptual model, using SPLS software, the path coefficients between the determined relationships were calculated. The values of the coefficients mentioned in *Table 3* indicate that the reliability and validity of the research factors are appropriate. In other words, the components (factors) specified in *Table 3* are well able to measure the concept under consideration.

Table 3. The reliability and validity of the research factors.

Factors	Alpha Coefficient	AVE	CR
Costs	0.738	0.784	0.792
Quality	0.763	0.785	0.806
Delivery reliability	0.634	0.662	0.681
Performance history	0.702	0.759	0.764
Turnover	0.629	0.646	0.683
Lead time	0.761	0.794	0.832
Operating capacity	0.716	0.742	0.773
Required standards	0.749	0.783	0.803
Environmental issues	0.683	0.695	0.739
Management Commitment	0.778	0.795	0.839
Redundancy	0.692	0.728	0.756
Agility	0.751	0.784	0.829
Complexity	0.612	0.636	0.687
Visibility	0.706	0.738	0.796
Flexibility	0.637	0.684	0.718

The output values of the T-test are shown in *Fig. 3*.

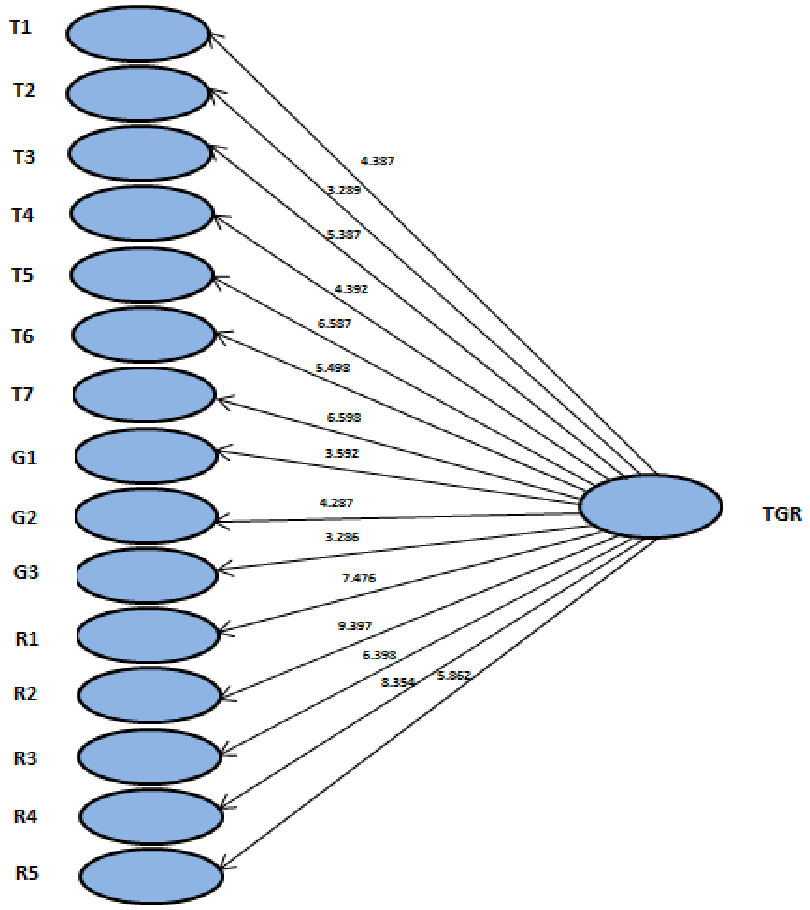

Fig. 3. Values of the T-test.

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As *Fig. 3* shows, the T-value values for all paths are higher than the absolute standard value of 1.96 (outside the specified range) and depict that the identified factors are well measured in the integrated supply chain (traditional, green and resilient) management system influence the company. In other words, this shows the significance of all factors and relationships between variables at the 95% confidence level, and finally all model relationships are acceptable. The results (*Fig. 4*) also show that the relationship between all variables is very high and significant.

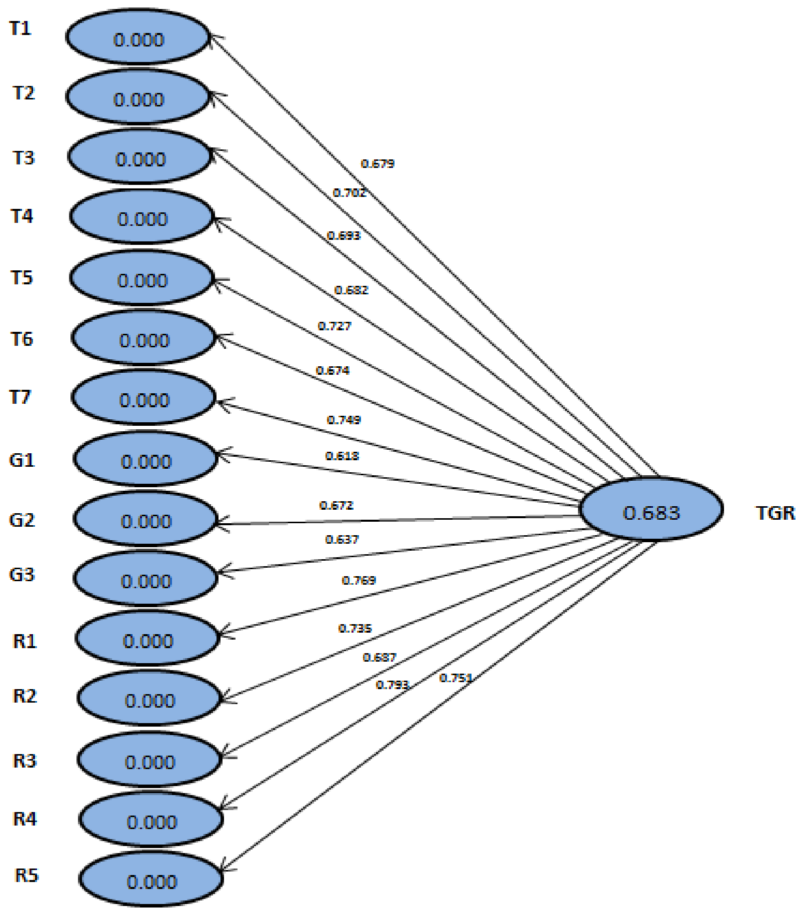

Fig. 4. The path coefficients and factors coefficient.

Therefore, according to *Fig. 4* and the path coefficients, the positive and strong impact of all factors in the management of the traditional, green and resilient integrated supply chain of this company are confirmed. Finally, after calculating all the fitting criteria of the measurement models and the structural model of the research, GOF should be calculated. The criterion shown by GOF is a number between zero and one, if it closes to one, it means the higher fitness of the model. Three values of 0.01, 0.25 and 0.36 have been introduced as weak, medium and strong values for GOF [9], are obtained from the redundancy of the mean values of the adjustment coefficient and the average of the additional values for the endogenous structures of the model. By calculation of the software output, the GOF value was 0.317, which is a good value and indicates the strong fitness of the structural model, so the GOF of the model is also confirmed.

5.2 | SWARA Technique and Calculating the Weight (Importance and Rank) of the Criteria and Sub-Criteria

In order to calculating of weight, the factors, SWARA (Stepwise Weight Assessment Ratio Analysis) technique is used. In this method, the expert assesses the calculated weights. In addition, each expert specifies the importance of each criterion according to tacit knowledge, information and experience.

Then according to the average value of the group's ranks obtained by experts, the weight of each criterion is determined [1]. SWARA is one of the weighting methods in which professionals play an important role in the calculation of their weight and final assessment [2]. In this study, the interviews of 30 Iranian experts in Fanavaran Petrochemical Company were used. The weight of each criterion indicates its importance. Measuring of weight is an important topic in many issues of decision-making.

In this section, using SWARA technique, the criteria and sub-criteria are evaluated and their weight was determined in four sections. All computational steps were presented in the first section and a summary of the final output of the sub-criteria and criteria was given below. The middle tables were neglected due to the high volume of calculations.

5.2.1 | Determining the weight of sub-criteria related to the traditional criterion

For this purpose, seven traditional sub-criteria effective on the evaluation of suppliers in Fanavaran Petrochemical Company were extracted, which are given in *Table 4*.

Table 4. Keys traditional sub-criteria effective on the evaluation of suppliers.

Traditional sub-criteria	T1	T2	T3	T4	T5	T6	T7
Description	Costs	Quality	Delivery reliability	Performance history	Turnover	Lead time	Operating capacity

Then, 30 expert opinions were evaluated to examining the factors. Fanavaran Petrochemical Company (*Table 5*) was used in this field.

Table 5. Information of experts.

Group	Classification	Number
Age	Lower of 40 years	5
	Between 40 to 50 years	7
	Between 50 to 60 years	15
	Upper of 60 years	3
Position	Managers	8
	Assistant and Engineers	22
Education	Diploma	--
	Bachelor	11
	Master	17
	P.H.D.	2
Record of service	Lower of 10 years	3
	Between 10 to 20 years	7
	Between 20 to 25 years	12
	Upper of 25 years	8
Sexuality	Male	28
	Female	2

In the following, the step-by-step and executive procedure of this technique for calculating the weight of the sub-criteria and their ranking is explained.

Implementation of Steps.

Step 1. By dividing the number of opinions on each sub-criterion by the number of experts (30), the percentage of opinions on each sub-criterion was calculated (*Table 6*).

Table 6. Percentage of opinions and rank of each sub-criterion.

Key sub-criterion	T1	T2	T3	T4	T5	T6	T7
Description	Operating capacity	Lead time	Turnover	Performance history	Delivery reliability	Quality	Costs
Number of opinions	17	27	23	14	8	19	11
Percentage of opinions	0.57	0.90	0.77	0.47	0.27	0.63	0.37
Rank	4	2	2	5	7	3	6
						Number of experts	30

Step 2. Sort the sub-criteria in order of importance in *Table 7*.

Table 7. Sort of sub-criteria in order of importance.

Rank	1	2	3	4	5	6	7
Sub-criterion	T2	T3	T6	T1	T4	T7	T5
Percentage of opinions	0.90	0.77	0.63	0.57	0.47	0.37	0.27

Step 3. Calculate the relative difference of each subscale's opinion on the next subscale, s_j, for each subscale (other than the first); a number as s_j does not belong to the first subscale, and S_2 equals 0.90-0.77=0.13 (*Table 8*).

Table 8. The value of s_j.

Sub-criterion	T2	T3	T6	T1	T4	T7	T5
s_j	….	0.13	0.13	0.07	0.10	0.10	0.10

Step 4. The growth rate of k_j is equal to 1 for the first sub-criterion and $1 + s_j$ for the other subscale. These values are given in *Table 9*:

Table 9. k_j Growth values for each sub-criterion.

Sub-criterion	T2	T3	T6	T1	T4	T7	T5
k_j	1	1.13	1.13	1.07	1.10	1.10	1.10

Step 5. Set the recovered value of the first sub-criterion (T1) that set the $q_1 = 1$ and by dividing q_j from the previous sub-criterion to k_j of that sub-criterion, we calculate the values of q_j for the other sub-criteria; for example, $q_1 = 1$ and $k_2 = 1.13$.

So: $q_2 = \frac{1}{1.13} = 0.88$ and $q_3 = \frac{q_2}{k_3} = \frac{0.88}{1.13} = 0.78$.

The extracted values of q_j are given in *Table 10*.

Table 10. Values of q_j for each sub-criterion.

Sub-criterion	T2	T3	T6	T1	T4	T7	T5
q_j	1	0.88	0.78	0.73	0.66	0.60	0.55

Step 6. Divide the q_j by their sum to calculate the weight of each sub-criterion. For example, w_1 equals to:

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$$w_1 = \frac{1}{5.21} = 0.192.$$

The weight of the following criteria is given in *Table 11*:

Table 11. The weight of each sub-criterion.

Sub-criterion	T2	T3	T6	T1	T4	T7	T5
w_j	0.192	0.169	0.150	0.140	0.127	0.116	0.105

Step 7. Finally, the weight of the sub-criteria was presented in *Table 12* after sorting.

Table 12. The weight of the sub-criteria after sorting.

Key sub-criterion	T1	T2	T3	T4	T5	T6	T7
Description	Operating capacity	Lead time	Turnover	Performance history	Delivery reliability	Quality	Costs
w_j	0.140	0.192	0.169	0.140	0.127	0.116	0.105

According to *Table 12*, the second sub-criterion, "quality", has been extracted with the highest weight as the most important traditional sub-criterion. Also, the third sub-criteria (delivery reliability) and the sixth (pre-order time) are in the next ranks in terms of importance in evaluating the traditional performance of suppliers. The fifth sub-criterion (trading volume) was also identified with the lowest weight as the least important sub-criterion in the traditional evaluation of the company's suppliers.

Determining the weight of the sub-criteria related to the green criterion.

Step 1. Step 1 output is presented in *Table 13*:

Table 13. Percentage of opinions and rank of each sub-criterion.

Green key sub-criterion	G1	G2	G3
Description	Required standards	Environmental issues	Management Commitment
Number of opinions	23	14	19
Percentage of opinions	0.767	0.467	0.633
Rank	1	3	2

Steps 2-6. The final output of these steps is given in *Table 14*:

Table 14. Values of s_j, k_j, q_j and w_j.

Rank	1	2	3
Sub-criterion	G1	G3	G2
Percentage of opinions	0.767	0.633	0.467
s_j	---	0.133	0.167
k_j	1	1.133	1.167
q_j	1	0.882	0.756
w_j	0.379	0.334	0.287

Step 7. The output of this step is also presented in *Table 15*:

Table 15. The weight of the sub-criteria after sorting.

Sub-criterion	G1	G2	G3
w_j	0.379	0.287	0.334

As can be seen in *Table 15*, the first sub-criterion, "compliance with the required standards" with the highest weight, has been extracted as the most important green sub-criterion. Also, the third sub-criteria (commitment and management of employees and customer needs) and the second (observance of environmental issues) are in the next ranks in terms of importance in evaluating the traditional performance of suppliers.

Determining the weight of the sub-criteria related to the resilience criterion.

Step 1. The output of this step is given in *Table 16*:

Table 16. Percentage of opinions and rank of each sub-criterion.

Resilience key sub-criterion	R1	R2	R3	R4	R5
Description	Redundancy	Agility	Complexity	Visibility	Flexibility
Number of opinions	22	26	11	18	14
Percentage of opinions	0.73	0.87	0.37	0.60	0.47
Rank	2	1	5	3	4

Steps 2-6. The final output of these steps is given in *Table 17*:

Table 17. Values of s_j, k_j, q_j and w_j.

Rank	1	2	3	4	5
Sub-criterion	R2	R1	R4	R5	R3
Percentage of opinions	0.87	0.73	0.60	0.47	0.37
s_j	---	0.13	0.13	0.13	0.10
k_j	1	1.13	1.13	1.13	1.10
q_j	1	0.88	0.78	0.69	0.62
w_j	0.252	0.222	0.196	0.173	0.157

Step 7. The output of this step is also presented in *Table 18*:

Table 18. The weight of the sub-criteria after sorting.

Sub-criteria	R1	R2	R3	R4	R5
w_j	0.222	0.252	0.17	0.196	0.173

As can be seen in *Table 18*, the second most important sub-criterion, agility, is the most important sub-criterion for resilience, and the first (surplus) and fourth (obvious) sub-criteria are the next most important in evaluating the resilience performance of suppliers. The fifth sub-criteria (flexibility) and the third (complexity) ranked last.

Determining the weight of key criteria (traditional, green and resilience).

In this section, in order to evaluate the key criteria and extract the weight of the criteria and rank them, the SWARA technique was used. The output of the executive steps is presented in *Tables 19, 20,* and *21*:

Table 19. Percentage of opinions and rank of each sub-criterion.

Description	Traditional (T)	Green (G)	Resilience (R)
Number of opinions	16	20	28
Percentage of opinions	0.533	0.667	0.933
Rank	3	2	1

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Table 20. Values of s_j, k_j, q_j and w_j.

Rank	1	2	3
Sub-criterion	R	G	T
Percentage of opinions	0.933	0.667	0.533
s_j	---	0.267	0.133
k_j	1	1.267	1.133
q_j	1	0.789	0.697
w_j	0.402	0.318	0.280

Table 21. The weight of the sub-criteria after sorting.

Criterion	T	G	R
w_j	0.280	0.318	0.402

According to *Table 21*, the third criterion is "resilience" with the highest Weight as the most important criterion, the second criterion (green) in the second place and finally the first criterion (traditional) in the last rank in terms of importance in the integrated evaluation of the suppliers' performance. This output (the final weights of the main criteria) is considered in the ranking of the company's suppliers as the input of the TOPSIS decision-making technique, which is presented in the following steps of this technique.

5.2 | Ranking of the Company's Suppliers based on the Weight of the Criteria with TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the well-known methods for classical MCDM. TOPSIS technique, as one of the known classical MCDM methods, was first developed for solving a MCDM problem. The underlying logic of TOPSIS is to define the ideal solution and negative ideal solution. The ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution is the solution that maximizes the cost criteria and minimizes the benefit criteria. In short, the ideal solution consists of all the best values attainable of criteria, whereas the negative ideal solution is composed of all the worst values attainable of criteria. The optimal alternative is the one which has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution [44]. In this study, in order to rank the Fanavaran Petrochemical Company, the TOPSIS decision-making technique including the following six steps has been used:

Step 1. Conversion of the D decision-making matrix to the ND matrix based on Euclidean norm.

$$r_{ij} = \frac{r_{ij}}{(\sum_{i=1}^{m} r_{ij})^{1/2}} \qquad (j = 1, \ldots, n)$$

In order to implement the first step, in the beginning decision matrix was extracted from the opinions of 10 experts of the mentioned company and finally the last aggregation matrix was presented as *Table 22*.

Table 22. Collective matrix of expert opinions.

Decision-Making Matrix (DM)	C1	C2	C3
A1	10	160	4
A2	12	173	5
A3	15	129	7
A4	8	159	1
A5	9	191	6
A6	13	183	8
A7	8	179	3
w_j	0.28	0.318	0.402
Alpha	29.10	446.61	14.14

In the w_j line, the weight of the main criteria (traditional, green and resilience) extracted from the final output of the SWARA technique is given. The following is a ND matrix in *Table 23*:

Table 23. Normal Decision-Making Matrix.

Normal DM	C1	C2	C3
A1	0.3436	0.3583	0.2828
A2	0.4123	0.3874	0.3536
A3	0.5154	0.2888	0.4950
A4	0.2749	0.3560	0.0707
A5	0.3092	0.4277	0.4243
A6	0.4467	0.4098	0.5657
A7	0.2749	0.4008	0.2121

Step 2. The matrix of the balanced scale is obtained by assuming the vector w:

$$V = N_D . W$$

Where V is the balanced scale matrix and W is the diameter matrix of the weights obtained for the criteria. *Table 24* shows the balanced ND decision making:

Table 24. The balanced ND decision making.

The Balanced ND DM	C1	C2	C3
A1	0.0962	0.1139	0.1137
A2	0.1155	0.1232	0.1421
A3	0.1443	0.0919	0.1990
A4	0.0770	0.1132	0.0284
A5	0.0866	0.1360	0.1706
A6	0.1251	0.1303	0.2274
A7	0.0770	0.1275	0.0853

Step 3. Identify the solution to the positive ideal and the solution to the negative ideal as follows:

$$A^{+} = \{(\max V_{ij}, J \in j_1), (\min V_{ij}, J \in j_2), i = 1,2, \dots, m\}$$

$$A^{-} = \{(\min V_{ij}, J \in j_1), (\max V_{ij}, J \in j_2), i = 1,2, \dots, m\}$$

$$A_i^{+} = \{v_1^{+}, v_2^{+}, \dots, v_n^{+}\}$$

$$A_i^{-} = \{v_1^{-}, v_2^{-}, \dots, v_n^{-}\}$$

For the positive elements of the criteria: $j_1 = \{j = 1,2, \dots . n\}$

For the negative elements of the criteria: $j_2 = \{j = 1,2, \dots .n\}$

According to the above relations, the values of the positive ideal solution and the negative ideal solution are calculated and given in *Table 25*:

Table 25. The values of the positive and negative ideal solution.

Values	C1	C2	C3
A+	0.1443	0.1360	0.2274
A-	0.0770	0.0919	0.0284

Step 4. Calculation of distances based on Euclidean norm:

$$d_i^+ = \left\{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2\right\}^{1/2}, \ (i= 1,2,\dots,m) \quad d_i^- = \left\{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2\right\}^{1/2}$$

Considering the above relations, the distance between positive and negative ideas is presented in *Table 26*.

Table 26. The distance between positive and negative ideas.

Distances	d+	d-
A1	0.1254	0.0902
A2	0.0909	0.1241
A3	0.0525	0.1834
A4	0.2113	0.0214
A5	0.0810	0.1491
A6	0.0201	0.2083
A7	0.1575	0.0671

Step 5. Calculate the relative proximity of the option to the ideal solution as follows:

$$c_i = \frac{d_i^-}{(d_i^- + d_i^+)}, (i = 1,2, \dots , n)$$

If $A_i = A_i^+$ then $d_i^+ = 0$. So the closer the option is to the ideal solution, the closer it will be to one.

According to the above relation, the relative proximity of the options to the ideal solution is calculated and given in *Table 27*.

Table 27. The relative proximity of the options to the ideal solution.

Proximity	CC
A1	0.4182
A2	0.5770
A3	0.7774
A4	0.0918
A5	0.6480
A6	0.9121
A7	0.2978

Ranking options: At this stage, the options (suppliers) are ranked based on c_i the values from largest to smallest. The results are shown in *Table 28*.

Table 28. Ranking options.

	Rank
A1	5
A2	4
A3	2
A4	7
A5	3
A6	1
A7	6

6 | Conclusion and Recommendations

Nowadays, SCM has received a lot of attention in several organizations. Customers demand more environmentally-friendly products. There is a growing pressure from strict government norms on industries due to increased environmental disruption, shortage of material resources, and increased levels of pollution (essentially carbon footprints). Green supply chain management is a powerful way to compare an organization with competitors. This is the latest technique to improve supply chain management capabilities. Many companies have taken green steps in their day-to-day management. Green supply chain provides operational and financial benefits to an organization and at the same time benefits the sustainability of the work environment. Green supply chain management has a competitive advantage and improves the economic situation of an organization. It refers to improve the environmental performance of a product and process at every stage of the organization, for example purchasing, manufacturing, marketing, and presentation. Effective implementation of green supply chain management leads to reduced waste and environmental pollution, optimized resource utilization, and costs' reduction. The ultimate goal of selecting suppliers is to choose the right suppliers according to the resilience capabilities of the supply chain of companies. Just as suppliers are an irreplaceable vital resource, choosing a better supplier can help to build resilience to decrease supply chain risks as a whole. For this purpose, at first, by reviewing the literature and interviewing and discussing with the experts and purchasing managers of the mentioned company, the performance criteria have been extracted and proposed. Then, the suppliers of Fanavaran Petrochemical Company were evaluated and selected based on traditional, green and resilience criteria. To this end, after identifying and extracting the key sub-criteria related to each criterion with a comprehensive review of the literature, a questionnaire was designed and provided to 55 experts and managers of the company to express their views on the effectiveness of each of these criteria. After collecting the questionnaires, using the path analysis approach, the proposed conceptual model of the criteria was evaluated and tested. The results of this approach showed that all the proposed criteria are effective in implementing the integrated supply chain system (traditional, green and resilience) and evaluating the company's suppliers. Then, using the SWARA decision-making technique as one of the new techniques and gathering the opinions of experts, all the criteria and sub-criteria were compared and finally their weights were extracted. Based on the extraction weight, their importance (rank) was extracted and presented separately by table. Finally, based on the necessity and importance of ranking the company's suppliers of materials based on all criteria, the TOPSIS ranking technique was used. In order to implement this technique, the extraction weight of the criteria was applied to the TOPSIS technique as the input of the first stage through the SWARA technique. Then, based on the implementation steps, the suppliers of the company were evaluated and their ranking was extracted based on three key research criteria. The technical output showed that the sixth supplier of the best suppliers and the third supplier were in the next place (due to the preservation of research ethics and confidentiality of information, the names of the suppliers were not announced). The seventh and fourth suppliers also placed at last rank based on performance. Such generalization and interpretation were major limitations of this study. Studied sample in this research is a sample of employees, managers and experts in Fanavaran Petrochemical Company so that in finding generalization with other populations (including Petrochemical Companies), consideration on the cautious side is essential.

According to the conclusion of this research, the following recommendations are proposed to the company managers:

- *Based on the weight of the criteria and sub-criteria, sub-criterion "quality" as the most important traditional sub-criterion, sub-criterion "compliance with the necessary standards" as the most important sub-criterion green, sub-criterion "agility" as the most important sub-criterion of resilience and in the final analysis of general and key research the "resilience" criterion has been extracted as the most important criterion for evaluating the performance of suppliers. In order to realize these goals, in terms of quality, it is necessary to select suppliers by providing high quality materials among existing suppliers or to search and replace other higher quality suppliers. In terms of compliance with standards, improving government participation and regulations and standards, compliance with standards in the purchase of raw materials, machinery, equipment and tools, product design. Improving the agility and speed of suppliers, activities can also enhance suppliers' responsiveness to orders. Finally, improving resilience and adaptability in response to disruptions, and restoring it in selecting suppliers and tackling external disruptions and risks will reduce and enhance chain vulnerabilities.*
- *Based on the ranking of suppliers, it is recommended that the studied company reduce its communications and activities with the seventh and fourth suppliers (as the weakest suppliers) or replace them with other better suppliers in order to improve the performance of the supply recovery function. It will also increase the supply of more materials to sixth and third suppliers (as the strongest suppliers in terms of environmental and environmental issues).*
- *The proposed research approach will greatly assist the company's purchasing managers in achieving a green and productive purchasing strategy by evaluating suppliers.*
- *The proposed method of this research can be used in other companies as a tool to measure the unhealthiness of the supply chain in the components of green performance and resilience.*
- *Considering the uncertainty in the opinions of the experts of the studied company, it is recommended to use decision-making techniques in the fuzzy environment and by collecting and analyzing the opinions of experts in verbal phrases to conduct a more detailed study of the criteria and sub-criteria and extract the ranking of suppliers and compare it with the results of this research.*
- *Using other weighting techniques, such as AHP, ANP, BWM and etc. It is advised to re-extract the weight of the criteria and compare it with the results of this research and use other ranking techniques such as WASPAS, ELECTER, QUALIFLEX, PROMETEE, ORESTE etc., and rank the suppliers and compare them with the results of this study.*
- *Identify sustainability indicators in the supply chain and present a more comprehensive model based on four key criteria (traditional, green, resilience and sustainability) and implement and analyze the approach of the present study.*

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Optimization of Safety Audit Planning: A Case Study of Process Plant in India

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Abstract

Safety audit is a system for evaluating the adherence of an occupational safety and health plan to fulfill statutory requirements with prior planning. Safety audit planning is the predetermining activity of what, how, where, when and by whom the audit will be carried out to achieve its objective. Safety audit planning is a complex process and an anticipatory function which will provide audit activity on time with the least audit risk and cost by assigning the auditors with the required ability and skill level to the activities to be audited. In general, the safety audit planning is based on the professional judgment of a lead auditor by considering the qualitative characteristics of the auditors. However, the audit planning is possible by framing a mathematical model with an assumption that the auditors possess similar characteristics. The objective of the study is to optimize the safety audit planning by allocating auditors to activities using a linear programming model so as to minimize the audit cost; which is not possible with statistical tools. The study was conducted in an integrated cement plant located in India and the results of the study show that the number of hours and the activity of the auditors was specified with a minimum cost. Similar studies can be conducted to optimize time and cost of safety audit for industrial units involving more number of activities with large batch size of auditors by planning the audit process in advance.

Keywords: Occupational safety and health (OSH), Safety audit, Auditors, Linear programming.

1 | Introduction

Safety audit is a comprehensive assessment and appraisal of all components of safety management system to establish compliance with guidelines and standards [1]. The purpose safety audit is to detect nonconformities, renew certifications, and fulfill goals and objectives [2] Annual Occupational Safety and Health (OSH) audits are instrumental to ensure best safety practices in construction sites, which includes safety management system, employee protection and safety

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practices [3]. OSH audit is carried out by safety auditors or professionals in the domain of safety management system and the report prepared by the audit team is useful to initiate preventive/ corrective measures [4]. Safety audits assist in identifying failures within a system, process and the information gathered helps to determine the best course of corrective action [5]. Safety audits are basically of two types; structural and operational audits. Structural audits are based on available documentation and a consideration whether these documented activities meet the established criteria whereas operational audits also include interviews and observations to verify whether safety rules and routines are implemented in practice [6] and [7].Typically, the process of OSH audit includes; gathering evidence through systematic data collection, usually by reviewing documentation, conducting interviews and observing worksites; evaluating the evidence against audit criteria; and summarizing and reporting the results [8]. OSH management audits are used to evaluate workplaces' OSH management structures and processes. The audits typically determine whether the organization is compliant with one or more standards; such as its organizations own policies and procedures, applicable legislation and regulations.

Safety audit is a tool to measure the level of safety performance in any organization. The findings of safety audit are useful to conduct root cause analysis and preventive/corrective actions. Safety auditing was an awareness tool; for hazard identification and risk assessment, and behavior modification of employees. The OSH audit needs to focus not only on the physical hazards in the workplace but also on the safety culture at organization level which has an impact on overall safety performance. Organizations should ensure that all levels of management take notice on the use of audit as an important appraisal method in OSH management. The criteria for conducting OSH audits are revealing the findings of audits to employees, time frame/deadline for conducting audits and continuous review of OSH audits; the number of audits performed and reviewed in a given period [9]. The strength and weakness of safety management system can be noticed by continuously auditing and reviewing the management systems and operational procedures in order to achieve safety performance [10]. The quantitative results of audits are often used by organizations as performance measures [11]. Safety audit is the leading indicator while measuring the safety performance. Indeed, the safety audit may apply similar methodologies used in financial audits to mitigate safety risks within an organization.

The characteristics of safety auditors have an impact audit quality. The main attributes of safety auditors are independence, unbiased approach and competency. Lack of independence in conducting safety audit may lead to less likely to report irregularities, thereby impairing audit quality [12] and [13] Identifying independent and qualified safety auditors is of paramount importance in audit planning. The report of the safety auditors is considered a positive feedback loop and assists to reinforce the safety management system implementation. The debits of possible loss or injury situations should balance against the credits of adequate safeguards [14]. Auditing provides a means of exposing the organizations frequent and deliberate efforts to raise safety awareness and compliance.

In India, due to the growing awareness of safety in the industrial sector and introduction of various legislations for implementing directives of the statutory bodies, the need has been felt to formulate the Indian Standard 14489: Code of practice on OSH audit, which will give a guideline to audit safety aspects in the industrial and other units of concern [15]. While formulating the standard, utmost care has been taken to cover all the possible elements relating to safety. This standard doesn't include rating system but it gives qualitative analysis of safety elements. From the available literature and discussions with the safety professionals, it is noticed that the importance of OSH audit, attributes and selection of safety auditors were highlighted in the past; and the task of audit planning was not discussed. The gap in the literature motivated to conduct the present study with an objective to optimize safety audit planning by allocating auditors to OSH activities by developing a linear programming model so as to minimize the audit cost and time.

As the industries are framing their own safety policies and procedures in addition to complying the statutory requirements, the safety auditing has become vital to establish that the existing procedures are consistent with procedures/requirements. The industrial units are mainly accomplish safety through operational-level experience and with a low level of formalisation, the audits have a primary focus on formal documentation and gathering audit evidence at the strategic and tactical levels in the units [16]. In general, the audit organization allocate shorter timeframes for the conduct of an audit than are necessary in practice irrespective of type of safety audit. Allocating insufficient audit time may lead to rushing and consciously cutting sampling of activities or records which can dilute the accuracy of the audit. Safety audit planning is the process of balancing the time and cost with prior assessment and planning for determining the OSH practices are functional and effective.

2 | Methodology

In India, the need of safety audit is mainly to fulfill safety and statutory requirements. Indian Standard 14489 establishes audit objectives, criteria and practices, and provides guidelines for establishing, planning, conducting and documenting of audits on OSH systems at workplace. It provides guidelines for verifying the existence and implementation of elements of OSH system and for verifying the system's ability to achieve defined safety objectives. It is sufficiently general in nature to permit it to be applicable or adaptable to different kinds of organizations [15]. The need to perform an audit is determined by the client, taking into account of specified or regulatory requirements and any other pertinent factors. Significant changes in management, organization, policy, techniques or technologies that could affect the OSH system, or changes to the system itself and the results of recent previous audits, are typical of the circumstances to be considered when deciding audit frequency. Normally an external or third-party safety audit should be conducted once in two years and an internal audit may be organized once in every year.

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2.1 | Safety Audit Procedure (IS 14489)

The safety audit procedure as per the Indian standard is detailed below.

The lead auditor and the team of auditors have to adopt the procedure while conducting safety audit.

- *Constitution of Audit team (at least two members).*
- *Constitution of auditee representatives.*
- *Recording identification and brief history of the auditee industry.*
- *Deciding audit goals, objectives and scope.*
- *Drawing audit plan with time schedule.*
- *Holding opening meeting with the auditee.*
- *Study of process and applicability of safely laws and standards.*
- *Plant visit and noting observations.*
- *Examining records and documents.*
- *Filling checklists of audit points.*
- *Holding of closing meeting and discussing findings.*
- *Preparation and submission of audit report*
- *Report distribution for compliance.*
- *Compliance audit if required by the audittee or client.*
- *Visit for compliance audit and its report.*

The standard also specified the details of safety audit goals, objectives, scope, plan, and verification of records, checking applicability of safety legislations, plant visits, consolidation of observations and report preparation and submission.

2.2 | Study Area

The study was conducted in an integrated cement plant located in north east of India with a capacity of one million tons per annum. The present strength of employees of the plant is more than 2000 and the management is committed towards OSH and welfare of the employees. The plant is certified with ISO 9001, ISO 14001 and OHSAS 18001certifications. Safety department of the plant is actively involved in organizing the safety training programmes, conducting the mock drills and implementing the motivational schemes to create awareness towards OSH systems. The management of the plant strictly follow to conduct the safety audits to comply with legislations and to provide safe workplace for employees. The management of the plant organizes to conduct an external safety audit by engaging safety auditors to fulfill the statutory requirements. The scope of the audit involves auditing the various departments of the plant, preparation and submission of the report to the management for further action.

The management of the plant was decided to utilize the services of three safety auditors including a lead auditor to complete the audit. The auditors vary in their experience and qualifications but capable of conducting audit all the departments. The auditors are designated as Lead Auditor (LA), auditor 1 (*A1*) and auditor 2 (*A2*) respectively. The lead auditor in consultation with management has finalized the number of hours required to complete the audit, time required for auditing each department and the number of hours auditors are supposed to work in each department. The audit fee for LA, *A1* and *A2* are fixed as *Rs*.2500, *Rs*.2100 and *Rs*.1800 per hour respectively. The total audit time required for completion is 80 hours. Based on audit fee, number of audit departments and number of hours required for each department, a linear programming problem was formulated. The details of the departments and number of hours required to conduct as per estimate is presented in *Table 1*.

Table 1. Departments, activities and hours required for audit.

S/N	Department	Activity No	No of hours for audit
1	Process plant.	Activity 1	10
2	Mechanical equipment.	Activity 2	13
3	Electrical & Instrumentation Department.	Activity 3	11
4	Civil Engineering Department.	Activity 4	9
5	Safety, Health &Environment, Occupational Health Centre.	Activity 5	16
6	Human resources and Stores Department.	Activity 6	7
7	Supply and dispatch.	Activity 7	8
8	Audit report preparation/submission.	Activity 8	6

2.3 | Linear Programming Problem Formulation

Optimization techniques enable to achieve certain goals (cost or time minimization) by efficient use of resources and it is widely applied in the decision-making process. Linear programming is a method used in optimization techniques to obtain the optimal utilization of resources so as to minimize cost/time or maximize profit [17]. Linear Programming (LP) is a group of mathematical techniques that can obtain the very best solution to problems which have many possible solutions. In most of the situations, resources available to the decision maker are limited. With the help of linear programming those scarce resources are allocated in an optimal manner on the basis of a given criterion of optimality [18]. The structure of all LP Problem has three important components; decision variable (activities), objective function (goal) and constraints.

Linear programming technique is based on the following assumptions [19]:

- *The objective function must be defined accurately (maximization or minimization).*
- *Variables should be quantitative. Linear programming is not used for qualitative.*
- *Variables should be correlated with one another.*
- *Resources to be used should be limited.*

- *Established relations among variables should be linear.*
- *Dependent variables should be positive or zero.*

LP is one of the most popular technique to find best solution in variety of situations such as man power planning, production management, marketing, financial management, etc. [18]. The objective function and the constraints are shown in *Eq. (1)* and *Eq. (2)*, respectively. The LP problem can be expressed in compact form as follows:

Optimize (Maximize or Minimize)

$$Z=\sum_{j=1} c_j x_j \,. \tag{1}$$

Subject to constraints,

$$\sum_{j=1}^{n} a_{ij}\ x_j\ (\leq \text{or} = \text{or} \geq) b_{i;}\ i=1,2,3\ldots m \tag{2}$$

$$\text{and } x_i \geq 0;\ j=1,2,\ldots n \qquad \text{(Non negative Constraints).}$$

x_1, x_2, x_3......x_n are decision variables and contribution of each decision variable are c_1, c_2, c_3....c_n respectively. Technical coefficient and resource capacity is represented by a_{ij} and b_i respectively. The objective function is represented by Z.

In all LP problems, left hand side of constraints is either less than, or equal to or greater than right hand side [18].

2.3.1 | Application of LP problem formulation of study area

The objective of the present study is to allocate the auditors to audit activities so as to minimize the cost. In this context, the objective function is formulated and shown as *Eq. (3)*.

$$\begin{aligned}\text{Minimize}\quad Z=&2500x_{11}+2500x_{12}+2500x_{13}+2500x_{14}+2500x_{15}+2500x_{16}+2500x_{17}+2500x_{18}+\\&2100x_{21}+2100x_{22}+2100x_{23}+2100x_{24}+2100x_{25}+2100x_{26}+2100x_{27}+2100x_{28}+\\&1800x_{31}+1800x_{32}+1800x_{33}+1800x_{34}+1800x_{35}+1800x_{36}+1800x_{37}+1800X_{38}.\end{aligned} \tag{3}$$

Subjected to constraints,

1) Minimum number of hours for auditors. The minimum number hours of audit of LA, *A1* and *A2* are 15, 35 and 30 hours respectively, and the total audit hours are 80 hours. The *Eq. (4)* to *Eq. (7)* represent the constraints auditor wise.

$$\text{Lead Auditor; } x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}\geq 15. \tag{4}$$

$$\text{Auditor 1; } x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{28}\geq 35. \tag{5}$$

$$\text{Auditor 2; } x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}+x_{38}\geq 30 \quad \text{and} \tag{6}$$

$$\begin{aligned}&x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}+x_{21}+x_{22}+x_{23}+x_{24}+x_{25}\\&+x_{26}+x_{27}+x_{28}+x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+X_{37}+x_{38}\geq 80.\end{aligned} \tag{7}$$

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2) Activity wise time constraints (From *Table 1*). The constraints for eight departments are represented by *Eq. (8)* to *Eq. (15)*.

Activity 1: $x_{11}+x_{21}+x_{31}\geq 10.$ (8)

Activity 2: $x_{12}+x_{22}+x_{32}\geq 13.$ (9)

Activity 3: $x_{13}+x_{23}+x_{33}\geq 11.$ (10)

Activity 4: $x_{14}+x_{24}+x_{34}\geq 9.$ (11)

Activity 5: $x_{15}+x_{25}+x_{35}\geq 16.$ (12)

Activity 6: $x_{16}+x_{26}+x_{36}\geq 7.$ (13)

Activity 7: $x_{17}+x_{27}+x_{37}\geq 8.$ (14)

Activity 8: $x_{18}+x_{28}+x_{38}\geq 6.$ (15)

3) Minimum working hours required to conduct the audit by the LA on the basis of audit scope of the departments are represented by *Eq. (16)* to *Eq. (23)*.

LA: $x_{11}\geq 2.$ (16)

LA: $x_{12}\geq 1.$ (17)

LA: $x_{13}\geq 1.$ (18)

LA: $x_{14}\geq 1.$ (19)

LA: $x_{15}\geq 1.$ (20)

LA: $x_{16}\geq 1.$ (21)

LA: $x_{17}\geq 1.$ (22)

LA: $x_{18}\geq 2.$ (23)

Minimum working hours required to conduct the audit by the *A1* (Auditor 1) on the basis of audit scope of the departments are represented by *Eq. (24)* to *Eq. (31)*.

A1: $x_{21}\geq 3.$ (24)

A1: $x_{22}\geq 3.$ (25)

A1: $x_{23}\geq 3.$ (26)

A1: $x_{24}\geq 3.$ (27)

A1: $x_{25}\geq 3.$ (28)

A1: $x_{26}\geq 3.$ (29)

A1: $x_{27}\geq 3.$ (30)

A1: $x_{28}\geq 2.$ (31)

Minimum working hours required to conduct the audit by the *A2* (Auditor 2) on the basis of audit scope of the departments are represented by *Eq. (32)* to *Eq. (39)*.

A2: $x_{31} \geq 2$. (32)

A2: $x_{32} \geq 2$. (33)

A2: $x_{33} \geq 2$. (34)

A2: $x_{34} \geq 2$. (35)

A2: $x_{35} \geq 2$. (36)

A2: $x_{36} \geq 2$. (37)

A2: $x_{37} \geq 2$. (38)

A2: $x_{38} \geq 2$. (39)

3 | Results and Discussion

The formulated linear programming model consists of 24 variables and 36 constraints. The objective function and constraints are run in LINDO software to solve the linear programming problem. The output of the formulated model was tabulated and the consolidated results of safety auditor's allocation to auditing departments are shown in *Table 2*.

Table 2. Allocation of safety auditors to departments.

Auditors	Departments								Total
	1	2	3	4	5	6	7	8	
LA	2	1	6	1	1	1	1	2	15
A1	3	3	3	3	12	4	5	2	35
A2	5	9	2	5	3	2	2	2	30
	10	13	11	9	16	7	8	6	80

Output summary is shown in *Fig. 1*, the number of hours allocated to each auditor for a department and audit fee are mentioned in the summary. The minimum total cost is *Rs* 165000 (Rupees one lakh sixty-five thousand) to complete the audit of the plant.

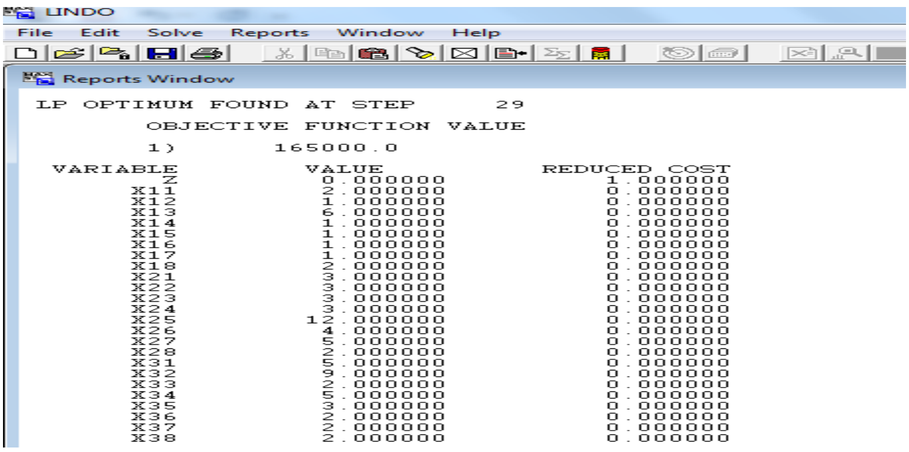

Fig. 1. Output summary of LINDO software.

From the *Table 2*, the lead auditor should work one hour each for auditing mechanical, civil engineering department, safety, health &environment, occupational health center and supply and dispatch and two hours each for process plant and audit report preparation/submission; and six hours for electrical and instrumentation departments. The auditor *A1* should work maximum for twelve hours to audit safety, health and environment, occupational health center; and the auditor *A2* should work for maximum of nine hours to audit mechanical equipment. It is also observed from the results that the safety, health

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&environment, occupational health center and mechanical equipment auditing require sixteen and thirteen hours respectively.

The audit fee for LA, *A1* and *A2* are *Rs*.37500, *Rs*.73500 and *Rs*.54000, respectively so that the total cost involved for audit will be minimum. The auditor *A1* fee is high among others due to expertise in conducting safety audit of cement plants. The lead auditor role is mainly to monitor the smooth conduct of audit, guide the audit team, discussions with the management and preparation of report with the assistance of the team members.

4 | Conclusion

A sound safety audit plan has a direct impact on the success of audit activity. Allocating the right auditors to the right department bestow to the accomplishment of safety audit with minimum cost. Conducting safety audit for an integrated cement plant require quality audit and allocation of auditors to different departments is crucial. The application of optimization techniques like linear programming is useful in this context instead of allocating the auditors randomly. The audit planning is a useful tool for the lead auditors to allocate specific audit work in the time frame of a particular department so as to avoid duplication of work. Similar studies are useful particularly for safety audit planning of refineries, fertilizer and pulp and paper industry due to presence of number of departments and also handling of hazardous chemicals.

The limitation of the study is the audit planning was conducted by framing a LP model with an assumption that the auditors possess similar characteristics but in reality, it may not so. The auditor's expertise in the domain of OSH is vital when the company's compliance with the legal requirements is to be evaluated. The results of the audit should be reliable which means that the different auditors should come to same conclusions. The audits conducted by different auditors should reach similar results when the same operation is audited under the same conditions. In this study, the auditors are allotted to conduct audit to different departments based on their experience and qualification which may lead to inaccurate results. In future, while conducting the similar studies the inter observer reliability is to be assessed as it is useful to examine the variation which occurs when one auditor performs multiple judgements at different times.

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Semi-obnoxious Backup 2-Median Problem on a Tree

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Abstract

In this paper, we discuss the obnoxious and semi-obnoxious version of the backup 2-median problem on a tree. In the obnoxious case of the 2-median problem, all vertices have negative weights, whereas in the semi-obnoxious model the vertices may have either positive or negative weights. In these two problems, we should find the location of two facility servers on the tree so that the sum of minimum weighted distances from vertices in the tree to the set of functioning servers is minimized. In the backup model, each facility server may probably fail. If a facility server fails, the remaining server should serve the clients. Vertex optimality is an important property for the 2-median problem. This property indicates that the set of vertices involves an optimal solution of the 2-median problem. We verify that the vertex optimality holds for the semi-obnoxious backup 2-median problem on a tree network. In the obnoxious 2-median problem, the set of leaves contains an optimal solution, we show that this property does not hold for the obnoxious backup 2-median problem.

Keywords: 2-median, Backup, Obnoxious, Semi-Obnoxious, Positive and negative weight.

1 | Introduction

Location theory is an important branch of transportation and communication. In location theory one looks for sites for one or more facilities that should interact with a set of existing users. These sites should be optimal concerning some performance measure. The p-median problem is a basic problem in this area of investigation. The goal in the p-median problem is finding the location of p facilities on edges or vertices of a given network G, such that the sum of weighted distances from all vertices in G to the closest facility is minimized.

Kariv and Hakimi [1] showed that the p-median problem on general networks is NP-hard. However, when the underlying network is a tree, they showed the solution of this problem can

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be found by an $O(p^2n^2)$-time algorithm. Tamir [2] improved the time complexity by presenting an $O(pn^2)$ algorithm. In the case $p=1$ on trees linear time algorithms is proposed by Hua [3] and Goldman [4]. Gavish and Sridhar [5] investigated an algorithm with time complexity $O(nlogn)$ for the case $p=2$ on trees. Charikar et al. [6] introduced a constant-factor approximation algorithm for the k-median issue. An uncertain model for single facility location problems on networks was presented by Gao [7]. Since the median problem is NP-hard, some heuristic methods were presented for finding the solution to this problem (e. g. see [8] and [9]).

Traditionally it has been assumed that such interaction is beneficial to the users, who want the facility as close as possible to them. This ignores that most facilities also have negative effects on the population, ranging from threat to lifestyles, such as parking troubles and noise around business centers or soccer stadium and then the facility is called obnoxious- to danger for health or life, such as lethal fumes close to chemical plants -and then the facility is called noxious. Although in recent years there has been an increasing interest in considering such obnoxious aspects, most existing models accommodate just one of the two aspects of the problem, and see the facility as either purely attractive or purely obnoxious.

When the facilities are semi-obnoxious, i.e., they provide both services and disservices, the models designed for purely desirable or purely undesirable facilities oversimplify things, and may yield solutions which are unrealistic due to either the enormous economic costs incurred or the social reaction they provoke against them. Instead, one can combine models of both types.

Although, conceptually the resulting model is just the combination of two existing models, things are more complicated from an algorithmic viewpoint. since such models cannot usually be coped with using the basic tools of the original models: to mention a few, semi-obnoxious facility models are as a rule multimodal, thus the standard techniques of convex analysis used for locating desirable facilities in the plane may be trapped in local optimal; moreover, the geometry of these problems is in general rather involved, thus the computational geometry approaches often used for the purely undesirable case may not be successfully adapted to this new context.

However, in recent years, semi-obnoxious facilities, i.e., facilities that desirable for a part of the clients, and undesirable for the other part of clients, have gained increasing interest. Research in the location of multiple semi-obnoxious facilities is limited, though there are a few instances available in the literature (e. g., Berman and Wang [10]; Coutinho-Rodrigues et al. [11]; Eiselt and Marianov [12]; Silva et al. [13]). Other types of the multiple semi-obnoxious facility models found in the literature are either discrete (see e.g., Colmenar et al. [14]; Song et al. [15]) or network-based models presented in [16] and [17].

Burkard and Krarup [18] were the first authors that assigned the positive and negative weights to desirable and undesirable clients in semi-obnoxious facility location models, respectively. They presented a $O(n)$ time algorithm for the semi-obnoxious 1-median problem on a cactus. A cactus is a connected graph with no two cycles has more than one vertex in common. Burkard et al. [19] proposed two models for the semi-obnoxious 2-median problem on trees. One model tries to minimize the sum of the Minimal Weighted Distances (MWD) from clients to servers, whereas the second one minimizes the sum of the Weighted Minimum Distance (WMD) from clients to the facilities. For the MWD-model on trees, they presented some properties of the problem to propose an $O(n^2)$ algorithm. They also, developed an $O(nlogn)$ algorithm for star graphs, and a linear time algorithm for paths. The WMD problem is harder than MWD to solve. Burkard et al. [19] find an $O(n^3)$ algorithm for WMD on trees. They showed that if the medians should locate only on the nodes, the complexity will be reduced to $O(n^2)$. These results were improved by Benkoczi et al. [20]. They showed that the MWD model of 2-median on a tree with positive and negative weights can be solved in $O(nlogn)$ time. For the WMD model they developed an $O(nhlog^2 n)$ algorithm, where h is the height of the tree. In the case that all vertices of the tree have negative weights, Burkard et al. [21] showed that an optimal solution of the MWD model of 2-median problem can be solved in a linear time. In the case that the underlying network is a cycle, Burkard and Hatzl [22] developed a linear and $O(n^2)$ algorithm for the MWD and WMD models of 2-

median problem, respectively. In the case $p=3$ Burkard and Fathali [23] gave an $O(n^5)$ algorithm for the WMD model on a tree and an $O(n^3)$ algorithm when medians are restricted to nodes. Zaferanieh and Fathali [24] considered the case of finding the semi-obnoxious median path in a tree.

In the classical models of location problems, the facility weights and their availability are known precisely. However, in most real-life emergencies, a customer (demand) is likely not to receive immediate service if his designated nearest emergency service (facility) is engaged in serving other patients at the time of incident (see, Karatas et al. [25]). This is especially common in large scale emergency location problems or in severely supplies restricted systems. Cases, where this is observed, include emergency ambulance operations, repair crews responding to critical equipment failures, utilization of a limited number of expensive medical devices, etc. (see, Weaver and Church [26]). Keeping in mind that the difference between life and death can be sometimes measured in minutes (or even seconds), decision makers should take into account the issue of congestions and queues and should consider providing backup supplies for demands. When a demand nearest (primary) facility is busy, a common approach is to serve the demand with an available backup facility.

Several cases of uncertainty have been defined and studied. Snyder and Daskin [27] investigated the reliability location model where servers may fail, and the clients allocated to these servers have to request service from other active servers. Wang et al. [28] developed a reliability-based formulation for the 2-median problem. In their model, each server may fail with a given probability and when a server fails, the other server should serve the clients. They called this model as the backup 2-median problem and presented an $O(n\log n)$ algorithm for finding the solution of this problem. Cheng et al. [29] developed an $O(n\log n+m)$ algorithm for the backup 2-median problem on block graphs, where m is the number of edges of the given tree. Fathali [30] investigated the backup multifacility location problem on the plane, and proposed an iterative method for solving this problem. Nazari and Fathali [31] and Nazari et al. [32] considered the reverse and inverse of backup 2-median problem on trees, respectively.

The utilization of backup service concept in emergency service systems continued in subsequent studies, including recent papers such as Araz et al. [33], Chanta et al. [34], Curtin et al. [35], Janosikova et al. [36] and Kordjazi and Kazemi [37], which mostly adopt a multi-objective optimization approach.

In this paper we investigate the backup semi-obnoxious and obnoxious 2-median problems with MWD objective function on trees. The vertex optimality verified for these models.

In what follows we describe the backup semi-obnoxious and obnoxious 2-median problems in Section 2. Some properties of these models on trees are given in sections 3 and 4, respectively.

2 | Problem Description

Let $T=(V, E)$ be a tree with n vertices. Each vertex v_i in the tree T has an arbitrary weight w_i. The weight w_i also called the demand at v_i. Let $d(x, y)$ be the length of shortest path between two points x and y in T. The 2-median problem asks to find a set of 2 vertices, $X_2=\{x_1,x_2\}$ called facilities, so that the sum of the weighted distances from the vertices to the closest facility in X_2 is minimized, i.e.,

$$\min F(X_2)=\sum_{i=1}^{n} w_i d(X_2, v_i). \tag{1}$$

For every edge $e(v,u)\in T$ let T_v be the subtree of T obtained by deleting edge $e(v,u)$ so that $v\in T_v$, and let $T_u=T\setminus T_v$. Moreover, for each subtree $T'\subseteq T$ we define $W(T')=\sum_{v_i\in T'} w_i$.

Let ϱ_1 and ϱ_2 be the failure probabilities of the two servers in the backup semi-obnoxious 2-median problem. In this paper, we suppose that $\varrho_1=\varrho_2=\varrho$. The backup semi-obnoxious 2-median problem, asks to find the location of servers such that the expected sum of distances from all vertices to the set of functioning servers is minimized, i.e., we want to minimize:

$$F(v_1,v_2)=(1-\varrho)\sum_{v\in V}\min_{\{1\leq i\leq 2\}} w(v)d(v,v_i)+\varrho\left[\sum_{v\in V} w(v)d(v,v_1)+\sum_{v\in V} w(v)d(v,v_2)\right]. \tag{2}$$

Let $\{u_1,u_2\}$ be a solution of the considered problem. Denote by m the midpoint of $P(u_1,u_2)$, where $P(u_1,u_2)$ is the shortest path between u_1 and u_2. In fact, m is a point in T such that $m\in P(u_1,u_2)$ and $d(u_1,m)=d(u_2,m)$. Let $e(x,y)$ be the edge containing m in $P(u_1,u_2)$. Without loss of generality, suppose that vertex x is on the path between u_1 and m. Moreover, if m is a vertex of T, we assume that it coincides with x.

Since $d(u_1,v_i)\leq d(u_2,v_i)$ and $d(u_1,v_j)>d(u_2,v_j)$ for any vertices $v_i\in T_x$ and $v_j\in T_y$, then all nonnegative weighted vertices in T_x and all negative weighted vertices in T_y are allocated to u_1. The rest of vertices are allocated to u_2. (If $m=x$ we assume that x and all other vertices at the same distance from u_1 and u_2 are allocated to u_1.)

Therefore, we can partition the set of vertices in V into two subsets V_1 and V_2 as follows:

$$V_1=\{v|d(v,x)\leq d(v.y),w(v)\geq 0\}\cup\{v|d(v,x)\geq d(v.y),w(v)\leq 0\}, \tag{3}$$

$$V_2=V\setminus V_1. \tag{4}$$

Such that the vertices in V_1 are served by u_1 and the remaining vertices are served by u_2. Let

$$D(V,u)=\sum_{v\in V} w(v)d(v,u). \tag{5}$$

Then the backup semi-obnoxious 2-median problem can be written as follows:

$$\begin{aligned} F(v_1,v_2)&=(1-\varrho)[D(V_1,v_1)+D(V_2,v_2)]\\ &\quad+\varrho[D(V_1,v_1)+D(V_2,v_1)+D(V_1,v_2)+D(V_2,v_2)]\\ &=D(V_1,v_1)+\varrho D(V_2,v_1)+\varrho D(V_1,v_2)+D(V_2,v_2). \end{aligned} \tag{6}$$

Note that, the obnoxious model is a special case of semi-obnoxious model, in which all vertices have negative weights. Therefore, in this case

$$V_1=\{v\in V|d(v,x)\geq d(v.y)\}. \tag{7}$$

3 | The Properties on Trees

The vertex optimality property is a basic property of the classical 2-median problem on a network with nonnegative weights. This property states that there exists an optimal solution on the set of vertices of T [38]. In the case that vertices may have either positive or negative weights, the vertex optimality does not hold as shown in [19]. However, on the tree networks, this property holds for MWD model due to Burkard et al. [19]. In the following lemma we show that this property also holds for the semi-obnoxious and obnoxious backup 2-median problem on trees.

Lemma 1. For the tree T, there exists an optimal solution of the semi-obnoxious and obnoxious backup 2-median problem on vertices of T.

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Proof. Since the obnoxious model is an especial case of semi-obnoxious model, we proof the lemma for the semi-obnoxious case.

Suppose, to the contrary, there isn't any optimal solution contains vertices of T. Let $\{m_1,m_2\}$ be an optimal solution of the considered problem on T, and m_1 be a non-vertex of the tree. Let $e(v_r,v_q)$ be the edge of the tree containing m_1. Let V_1 be the set of vertices of T that allocated to m_1. Consider the following function:

$$F(m_1,m_2)=D(V_1,m_1)+\varrho D(V_2,m_1)+\varrho D(V_1,m_2)+D(V_2,m_2). \tag{8}$$

Where $V_2=V\setminus V_1$. We have:

$$\begin{aligned}
&D(V_1,m_1)+\varrho D(V_2,m_1)\\
&=\sum_{v\in V_1\cap T_{v_r}} w(v)\big(d(v,v_r)+d(v_r,m_1)\big)+\sum_{v\in V_1\setminus T_{v_r}} w(v)\big(d(v,v_r)-d(v_r,m_1)\big)\\
&+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)\big(d(v,v_r)+d(v_r,m_1)\big)+\sum_{v\in V_2\setminus T_{v_r}} w(v)\big(d(v,v_r)-d(v_r,m_1)\big)\right]\\
&=\sum_{v\in V_1\cap T_{v_r}} w(v)d(v,v_r)+\sum_{v\in V_1\setminus T_{v_r}} w(v)d(v,v_r)\\
&+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)d(v,v_r)+\sum_{v\in V_2\setminus T_{v_r}} w(v)d(v,v_r)\right]\\
&+\left(\sum_{v\in V_1\cap T_{v_r}} w(v)-\sum_{v\in V_1\setminus T_{v_r}} w(v)+\varrho\left[\sum_{v\in V_2\cap T_{v_r}} w(v)-\sum_{v\in V_2\setminus T_{v_r}} w(v)\right]\right)d(v_r,m_1)\\
&=D(V_1,v_r)+\varrho D(V_2,v_r)\\
&+\left[2\left(\sum_{v\in V_1\cap T_{v_r}} w(v)+\varrho\sum_{v\in V_2\cap T_{v_r}} w(v)\right)-w\big(T_{V_1}\big)-\varrho w\big(T(V_2)\big)\right]d(v_r,m_1).
\end{aligned} \tag{9}$$

Let

$$K=\left[2\left(\sum_{v\in V_1\cap T_{v_r}} w(v)+\varrho\sum_{v\in V_2\cap T_{v_r}} w(v)\right)-w\left(T_{V_1}\right)-\varrho w\left(T(V_2)\right)\right]. \quad (10)$$

So

$$D(V_1,m_1)+\varrho D(V_2,m_1)=D(V_1,v_r)+\varrho D(V_2,v_r)+Kd(v_r,m_1). \quad (11)$$

Now if $K\geq 0$ since $d(v_r,v_j)<d(v_r,m_1)$ for every point v_j in $P(v_r,m_1)$ then $F(v_j,m_2)<F(m_1,m_2)$, which contradicts the optimality of $\{m_1,m_2\}$.

If $K<0$ since $d(v_r,v_j)>d(v_r,m_1)$ for every point v_j in $P(m_1,v_q)$ then $F(v_q,m_2)<F(m_1,m_2)$ which v_q and m_2 are vertices. But this contradicts with our assumption that $\{m_1,m_2\}$ is an optimal solution.

An efficient method for 2-median problem on a tree is edge deletion method. This method developed for the problem with positive and negative weights by Burkard et al. [19]. The edge deletion method has been also applied to find the solution of backup 2-median problem with positive weights on a tree by Wang et al. [28]. However, this method does not work for the semi-obnoxious 2-median problem. We can find the optimal solution, using vertex optimality property, in $O(n^3)$ time.

In the next section we investigate the obnoxious case where the weights of all vertices are negative.

4 | The Obnoxious Backup 2-Median

Note that the obnoxious problem is a special case of semi-obnoxious model, in which we don't have any vertex with positive weight. Therefore, the vertex optimality holds for obnoxious backup 2-median problem. In this section we consider the obnoxious model and discuss some properties for this case.

The obnoxious backup 2-median problem is equivalent to the backup 2-maxian problem. In the backup 2-maxian problem the weights of all vertices in tree are positive, i.e., $w(v)\geq 0$ for $v\in V$, and the following objective function should be maximized

$$F_1(v_1,v_2)=(1-\varrho)\sum_{v\in V} \max_{i=1,2} w(v)d(v,v_i)+\varrho\left[\sum_{v\in V} w(v)d(v,v_1)+\sum_{v\in V} w(v)d(v,v_2)\right]. \quad (12)$$

Due to Burkard et al. [21] the solution of 2-maxian problem on trees is the two end vertices of the longest path of tree. In Example 1, we show this property does not hold for the backup 2-maxian problem.

Example 1. Consider the tree depicted in *Fig. 1*. In the case $\varrho=0$ we have the 2-maxian problem, therefor, the two ends of longest path i.e., $\{v_1,v_4\}$, is the optimal solution with the value of objective function $F_1(v_1,v_4)=45.5$. However, in the case $\varrho>0$ the objective function for this solution can be obtained as follow

$$F_1(v_1,v_4)=(1-\varrho)45.5+\varrho(25.5+25.5)=45.5+5.5\varrho. \quad (13)$$

Now consider the solution $\{v_1,v_3\}$. The value of objective function for this solution is

$$F_1(v_1,v_3)=(1-\varrho)43+\varrho(25.5+36.5)=43+19\varrho. \quad (14)$$

If $0.185<\varrho\leq 1$ then $F_1(v_1,v_3)<F_1(v_1,v_4)$. Therefore, the longest path is not the optimal solution in this case. *Fig. 2* shows the value of objective functions for three solutions $\{v_1,v_4\}$, $\{v_1,v_3\}$ and $\{v_4,v_3\}$ for varying values of ϱ.

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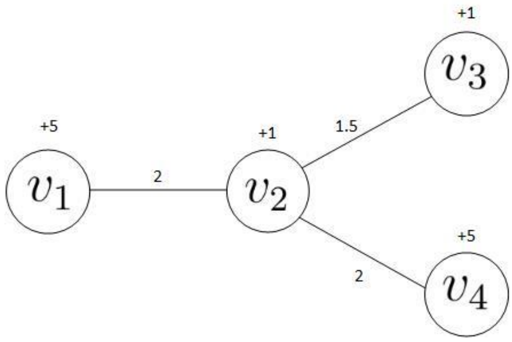

Fig. 1. The optimal solution of backup 2-maxian problem contains no both two ends of longest path of the tree.

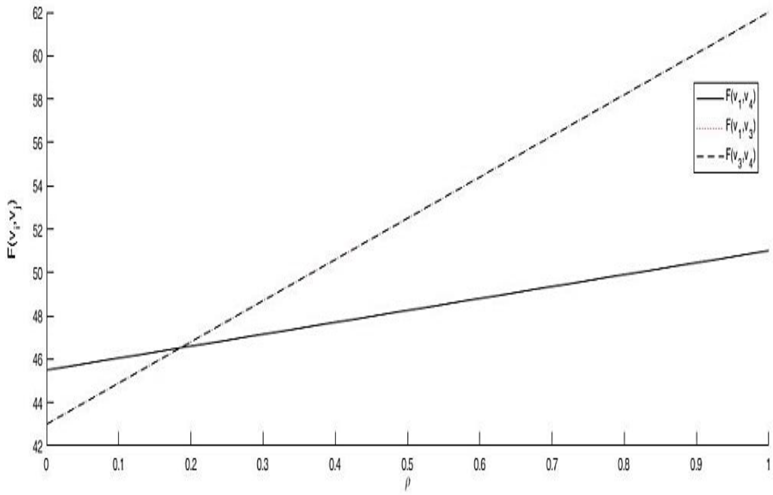

Fig. 2. The value of objective functions in example 1 respect to $0 \leq \rho \leq 1$.

Ting [39] showed that the optimal solution of 1-maxian problem is on the leaves of the tree and the optimal solution of 2-maxian, as mentioned previously in this section, is the two end vertices of the longest path. However, the following example shows that an optimal solution is not necessarily on the set of leaves of the tree anymore.

Example 2. Consider the path depicted in *Fig. 3*. The value of objective function for the solution $\{v_1,v_5\}$ is $F_1(v_1,v_5)=(1-\varrho)(7+52)+\varrho(15+53)=59+9\varrho$. On the other hand, the value of objective function for $\{v_2,v_5\}$ can be calculated as:

$$F_1(v_2,v_5)=(1-\varrho)(5+52)+\varrho(18+53)=57+14\varrho. \tag{15}$$

Therefore, if $59+9\varrho\leq 57+14\varrho$ or $\frac{2}{5}<\varrho\leq 1$, then $F_1(v_2,v_5)>F_1(v_1,v_5)$. Thus, the end vertex v_1 is not a part of solution anymore.

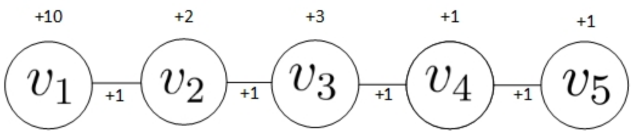

Fig. 3. A path with 5 vertices.

Fig. 4 shows the value of objective functions for the solutions $\{v_1,v_5\}$ and $\{v_2,v_5\}$ for varying values of ϱ.

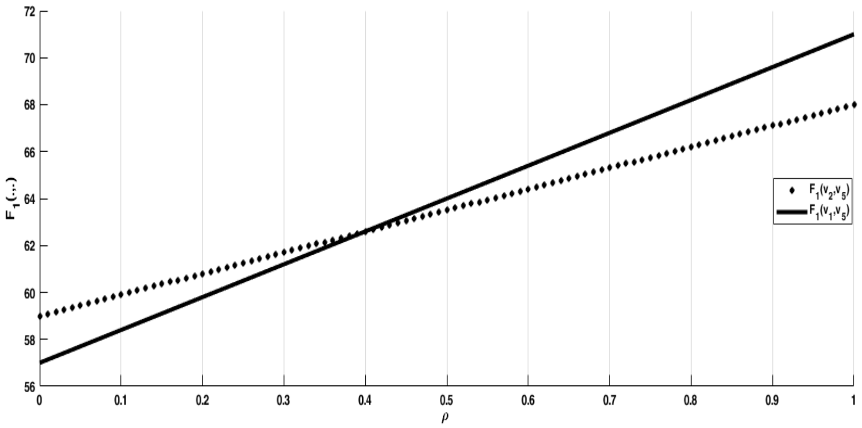

Fig. 4. The value of objective functions in example 2 respect to $0 \leq \rho \leq 1$.

To find the optimal solution of backup 2-maxian problem, we should compare the objective function of $\binom{n}{2}$ solution. Thus, the time complexity is $O(n^3)$.

In summary, using the obtained results in this paper, we will see that just the vertex optimality property holds for the both obnoxious and semi-obnoxious backup 2-median problems on a tree. However, the optimal solution of these problems can be found in polynomial times.

5 | Summary and Conclusion

In this paper we studied the semi-obnoxious and obnoxious versions of the backup 2-median problem on trees. In the classical 2-median problem we should find the location of two servers on the tree so that the sum of the minimum weighted distances from all vertices to the closest severs is minimized. In the semi-obnoxious case, every vertex has either a positive or a negative weight, whereas in the obnoxious model all vertices have negative weights. In these two problems the vertices with positive and negative weights are allocated to the closest and farthest servers, respectively. In the backup models the servers may fail and if a server fails, the functioning server should serve the clients. We investigated the vertex optimality for the semi-obnoxious and obnoxious backup 2-median problem on trees. For the obnoxious model it has been shown that the optimal solution may not happens on the leaves of the tree.

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Paper Type: Review Paper

A Short Survey on Face Recognition

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Abstract

Face recognition has received a great deal of attention over the past few years and has become one of the most researched and spoken topics. It is a kind of automated biometric distinguishing approach that recognizes an individual based on their facial characteristics. The main aim of face reorganization is to implement the system for a particular face and distinguish it from a large number of stored faces with some real-time variations as well. Face recognition is in trend these days, the main reason being its efficiency and vast applications in day to day life. Most all the Telecom companies provide an option to unlock the phones by recognizing the face, which is a time saver and gives protection from theft as well. There are many more such applications of this technique that will be discussed in this paper along with the methods used for face recognition.

Keywords: Face recognition, Face detection, Knowledge-based, Feature-invariant, Template matching, Appearance-based.

1 | Introduction

The enhancement of science and technologies has made life more comfortable than in older days. The emerging technologies like neutrosophic shortest path [1]-[5], transportation problem [6]-[8], uncertainty problem [9]-[14], fuzzy shortest path [15]-[19], Powershell [20], wireless sensor network [21]-[28], computer language [29] and [30], neural network [31], routing [32], image processing [33] have made the products more intelligent and self-healing based. Smart city applications like smart water [34] and [35], smart grid, smart parking, smart resource management, etc. are based on IoT and IoE technologies [36]-[39]. In recent years due to the advancements in the technical world, Artificial Intelligence has been a much-spoken topic which has led to a rise in technological standards. If not for AI, face recognition would not have been a go-to feature in our lives. Facial recognition is a technique used to identify a face from a video or an image or a pre-stored database [40]. Face recognition technology has become vital since it has proved its

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efficiency for augmented security without alerting the person. With the help of an automated face recognition system, it is easy to identify and recognize humans that have found numerous applications such as identifying a criminal, surveillance systems, verification of credit cards, identifying at ATMs, and many more which indeed could lead to a good background check. In the present scenario, it is one of the most convenient ways of identifying a person as there is no physical contact involved in it. There are chances that our security can be hampered due to security hacks such as at times passwords can be stolen also instances are there where they are forgotten, keys or cards can be stolen or misplaced then it is very difficult to recognize one's identity but it is not in the case of face recognition, as biological traits of a person cannot be forged, misplaced, stolen or forgotten [41]. Also using face recognition systems are non-disruptive and causes no health risks.

The human face plays a great significance in our social interactions, portraying one's identity. The salient features of a human face are eyes, nose, mouth, chin, forehead, lips which differentiate one from another and these are the basis of the recognition.

There are various techniques, algorithms, and methods used to recognize a face. Some of the methods are discussed in this paper.

There are 3 functions involved in the face recognition process [42] and [43]:

- *Face detection: face detection also known as facial detection is an AI-based computer technology that is applied to find and identify a human face from digital images.*
- *Feature extraction: feature extraction consists of segmentation, image rendering, and scaling of face prepared for identification. It is mainly reducing the data set and preparing for face recognition.*
- *Face recognition: facial recognition is when the detected face is analyzed and compared from the pre-stored databases.*

The basic flow of the face recognition is shown in the below diagram.

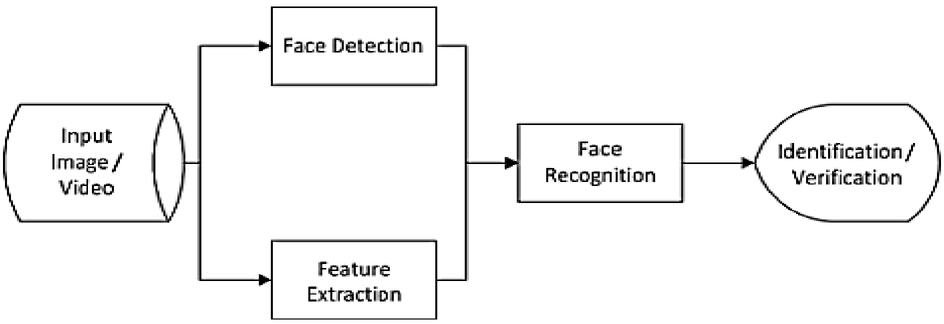

Fig. 1. Flow of face recognition.

2 | Review of Face Recognition Methods

Over the years extensive work has been done on face recognition techniques, methods. It has become one of the most favorite topics of research or study by the researchers. There are many methods and algorithms used to detect and recognize a human face. A few of them are discussed below [44]-[46]:

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- *Knowledge-based methods.*
- *Feature-invariant methods.*
- *Template matching methods.*
- *Appearance-based methods.*

2.1 | Knowledge-Based Methods

In knowledge-based methods our knowledge or understanding of the human face is encoded. These methods are termed rule-based methods. These models apprehend our understanding of faces and transfer it into a set of rules. Constructing a suitable set of rules is a big task with these methods as facial traits of human beings remains the same but the facial features of one can differ from another. For example, the color of the skin varies like a person may have dark under eyes when compared to the color of the cheeks and forehead. These methods are developed specifically for face localization, which focuses on determining the position of the image of a single face. There can be a possibility if the rules are too detailed which can confuse the model and hence in turn produce negative outputs. The solution is to construct hierarchical knowledge-based methods to avoid these problems.

2.2 | Feature-Invariant Methods

Feature variant methods tries to find the mutable features of a human face irrespective of its position or angle. This algorithm aims to find primitive features that exist even the lighting conditions, the pose or viewpoint are varying and finally locate faces based upon these results [47]. It is assumed that humans can easily detect objects and faces irrespective of the background condition such as varying poses or different lighting conditions, but the same is not with the systems. They have to be trained to identify different images or faces regardless of their background conditions. When compared to the knowledge-based methods, the feature invariant approach initiates at the feature extraction process. The knowledge-based methods derive information from the complete image and are fragile to the complex background and other influencing factors.

2.3 | Template Matching Methods

In Template matching methods, the input images are compared and analyzed with the saved patterns of human face and features. In template matching methods we must decide the template's features, shape, and size prior [48]. As the name suggests, a template of the face can be modeled by edges. The distinguishing features of the human face are face, eyes, and nose. In these methods, the facial features are recognized for a test face they are equated with the images stored in the database which returns matching scores via normalized cross-correlation. The global score is obtained by accumulating and integrating the scores of the different features. After the scores are assessed, a test face is allotted to the face class with the maximum score [49]. This can a long process as a face has to be compared with all the stored faces and may not be 100% accurate as it compares the template and finalizes the results on the matching score. Also, this method is limited to the frontal of the faces, hence the side faces images cannot be analyzed.

2.4 | Appearance-Based Methods

Appearance-based methods have received noteworthy attention from a broad range of research areas like pattern recognition, machine mining, biometrics, and computer vision [50]. It is easy for human beings to recognize the faces easily but it is a big challenge for the automated system. Appearance-based methods depend upon techniques from machine learning and statistical analysis to identify the admissible characteristics of an image [47]. Principal Component Analysis (PCA) and Independent Component Analysis (ICA) [40] and [42] are the widely used techniques for the appearance-based face recognition approach.

PCA is a method that depicts a group of sample data whereas, ICA is a method that is similar to PCA except that the division of the elements is created to be non-Gaussian.

3 | Applications of Face Recognition

Face recognition is used for two most important tasks:

Verification (1-to-1 matching) [51]-[53]: It is confirming, whether an unknown individual's identity is what he/she claims to be with the presented image. In simple words, verification means "Are you... You?"

Identification (1-to-n matching) [51]-[53]: It is confirming, whether an unknown individual's identity is what he/she claims to be with the presented image, compared against the stored data in the database. In simple words, identification is "Who Are You?"

For the above stated two purposes, there are innumerable implementations of face recognition, a few of them discussed below [40] and [45]:

- *Security [41]: email authentication, access-controlled airports, seaports, building, and computers.*
- *Surveillance [41]: a lot of CCTVs installed can be monitored to look for missing persons especially children, criminals, and offenders. These help in fraud detection and hence reduce the crime rate.*
- *Video games: labeling of the faces in the game.*
- *Healthcare: keeping patients' information confidential is an utmost priority, which can be achieved with face recognition systems. The details of the patients will only be revealed to the authorized doctors and no one else.*
- *Validate identity at the ATMs: in case of any theft at the ATMs, it is easy to look for the thief with the installed cameras.*
- *General Identity verification [41]: registration for elections, passports, national IDs, drivers' licenses, employee IDs.*
- *Attendance marking [54]: after the faces are verified and successfully recognized the attendance is marked on the server.*
- *Criminal Identification: with the increase in crimes at an alarming rate it is face-recognition systems that come to the rescue as the person may forge its identity but cannot play with its facial traits.*
- *Unlocking the phones: face recognition is a contactless feature that unlocks the phone and hence provides additional security from theft. There are many apps available that provide the user with this feature.*
- *Retail and Marketing: with the help of this technology there has been an added advantage to the shopkeepers as they get to cross verify and validate the customers' background with the data previously stored. Also, this has prevented shoplifting.*
- *Hospitality: We all know how vast the hospitality industry is. With the face recognizing systems the hotels can record the customers' fondness and can provide them with better protection. Based on their preference they can provide them with automated check-in and check-out.*

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3.1 | Pros and Cons of Using Face Recognition Systems

3.1.1 | Advantages

- *Improved security: With the help of facial biometric security systems, the security system can be enhanced as every individual entering the premises will be accounted for.*
- *High accuracy: With advanced technology, facial recognition systems are highly reliable. Whether day or night, light or dark these systems work with full accuracy.*
- *Fully automated: These systems are fully automated, earlier to grant permission to a person entering the building we depended on security staff to do this job. But as technology advances, this work is taken over by these systems.*

3.1.2 | Disadvantages

Everything comes with its disadvantages and advantages, hence this is the case with these systems.

- *Camera angle: To correctly detect and recognize a person the camera angle plays an important role, they have to be in the proper alignment to capture, process, and analyze the data.*
- *Data storage: The biggest problem with these systems is that an enormous amount of data has to be stored to perform its job well.*

4 | Conclusion and Future Works

With this paper, we are able to capture the necessity of face recognition in the real-time scenario. We discussed the methods used and the vastly used applications of face-recognizing systems, and how they are proving to be a boon to human life, and why are they the most searched, discussed, and the hot topic for everyone. Given the current scenario, where the world is highly affected by the ongoing pandemic and has seen vast destruction worldwide. We are going to take this research forward and implement it in unlocking the cars without any physical contact hence, contactless car unlocking system.

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Layerwise Finite Element Approach for the Bending Analysis of Bi-Directional Functionally Graded Layered Plates

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Abstract

In this paper, layerwise finite element analysis for the bending behavior of two-dimensional functionally graded layered plates with different boundary conditions is presented. The plates consist of three layers; a functionally graded layer embedded between ceramic and metal isotropic layers. The layerwise approach is based on the third-order shear deformation theory for the middle layer, while the first-order shear deformation theory is used for both the upper and lower isotropic layers. A quadrilateral 8-noded element with 13-degrees of freedom per node is used for this purpose. The present results show very good agreements with the published results for similar problems in literature solved by other methods of plates consist of either single or layered functionally graded plates.

Keywords: Two-dimensional functionally graded materials, Bending, Plates, Higher-order, Layerwise, Finite element.

1 | Introduction

Cracking and delamination, which are often observed in conventional multilayer composite materials, are one of the major observed drawbacks comparing to Functionally Graded Materials (FGM). FGM were discovered back in 1984 were a group of material scientists in the Sendai area of Japan introduced the concept of a new type of material named it as functionally gradient material as a means of preparing thermal barrier materials materials [1]. Since then, the use of FGM in a wide range of applications has been introduced especially in the fields of biomaterials serving as an artificial bone in the human body and aeronautics [2].

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To study FGM structure behavior, wide ranges of 2D theories are applied such as the Classical Plate Theory (CPT), First-order Shear Deformation Theory (FSDT), Higher-order Shear Deformation Theory (HSDT), Zig-Zag and layerwise approach. Irfan [3] conducted a comprehensive overview of the recent research trends in Finite Element (FE) formulations for analyzing sandwich plates carried out since 2000. Various theories that are based on FSDT, HSDT, mixed solid-shell elements, Zig-Zag and global-local theories were presented. They concluded that a major portion of research efforts are being employed to develop the FE formulations based on different HSDT rather than Zig-Zag theories since large computational efforts are needed to develop the later approach comparing to the former one.

The FSDT theory, which assumes a constant shear strain value throughout the thickness of the plate, was widely used in literature. Khorramabadi et al. [4] studied analytically the effect of using FSDT and TSDT on the free vibrational behavior of FGM plates using Navier's solution method. Singha et al. [5] studied the nonlinear behaviors FGM plates under transverse distributed load using plate-bending FE. The resultant error from using FSDT encouraged researches to use HSDT, especially when dealing with thick plates. Moita et al. [6] studied using FE approach the linear and nonlinear static behavior of FGM plate shell structures using TSDT. The FE models were based on a non-conforming triangular flat plate shell element with 3-nodes and either 8 or 11-Degrees of Freedom (DoF) per node. Čukanović et al. [7] proposed a new shape function containing hyperbolic cosine function to study bending analysis of thick and moderately thick square and rectangular plates as well as plates on Winkler–Pasternak elastic foundation subjected to a sinusoidal transverse load. Efraim [8] derived an empirical accurate correlation formula between the frequencies of FGM plates and the constituent materials. Thang et al. [9] investigated the non-linear static analysis of thin curved panels with FG coatings under combined axial compression and external pressure using the classical shell theory. Nguyen et al. [10] analyzed non-uniform polygonal cross-sections for thin-walled FG straight and curved beams using higher order approach. Lezgy-Nazargah et al. [11] studied the static, free vibration and dynamic response of FG piezoelectric material beams using an efficient three-nodded beam element.

In general, the layerwise approach is introduced to ensure the continuity of the displacement fields across the interfaces of composite and FGM layered plates. Yas et al. [12] applied layerwise FE formulations to analyze FGM cylindrical shells with finite length subjected to dynamic load. Pandey and Pradyumna [13] also used a layerwise approach based on the assumption of the FSDT theory in each layer imposing continuity of displacements at each layer's interface. Nikbakht et al. [14] used the full layerwise method to analyze the elastic bending of FG plates up to yielding.

Many other types of research have used available commercial software such as ABAQUS, MATLAB and Maple to study the behavior of FGM plates. Kurtaran [15] published an article studying the effect of the plate's shape on the vibration and modal analysis for FGM plates using MATLAB software. Martínez-Pañeda [16] used ABAQUS software to analyze stresses of FGM plate using two schemes: nodal based gradation and Gauss integration point-based gradation. Al-Hawamdeh et al. [17] used the Maple software program to study the static behavior of a layered beam structure made of FGM material embedded between metal and ceramic layers.

Applications of FGM may require the variation of mechanical properties to be in more than one direction, i.e., 2D-FGM. A 2D-FGM may consist of more than two constituent materials where usually 2D-FGM outperforms ordinary FGM. Hedia [18] conducted a comparison study between both 1D-FGM and 2D-FGM for the backing shell of the cemented acetabular cup. Nemat-Alla [19] made a comparison study between 1D-FGM and 2D-FGM plates under super high temperatures using the FE approach where his work indicated that 2D-FGM had high capabilities to reduce thermal and residual stresses more efficiently comparing to conventional FGM. Similar to 1D-FGM, 2D-FGM may be analyzed using any of the previously discussed theories. Joshi and Kar [20] recently studied the bending behavior of 2D-FGM using FE approach that was developed using the APDL platform. Asemi et al. [21] investigated the static analysis of 2D-FGM plates based on the 3D theory of elasticity. Nguyen and Lee [22] analyzed flexural-torsional vibration and buckling of thin-walled 2D-FG beams with different cross sections. Nguyen and Lee [23] also investigated the static behaviors and interactive geometric interpretation for modeling of thin-walled 2D-FGM beams using Vlasov's theory. Lezgy-Nazargah [24] analyzed fully coupled thermo-mechanical 2D-FGM beams using NURBS FE. Lezgy-Nazargah and Meshkani [25] also studied the static and free vibration analyses of FGM plates rested on two-parameter elastic foundations using mixed FE approach.

Although many research areas regarding the FGM plates have been already covered, none of the research areas in literature covered the part of analyzing 2D-FG layered plates using the layerwise FE approach. In this paper, the layerwise FE methods will be used to investigate the bending behavior of a 2D-FG layer embedded between ceramic and metal isotropic layers with both TSDT for the 2D-FG layer and FSDT for the isotropic layers. Furthermore, special case studies of the aforementioned plate problems will be investigated and compared with the existing literature.

2 | Mathematical Model

The model under study consists of a 2D-FGM embedded between homogenous materials (metal and ceramic). The middle layer consists of a 2D-FGM that contains four distinct material constituents, two ceramics C_1 and C_2 at its upper surface and two metals M_1 and M_2 at its lower surface. Plate geometry involves mainly individual layer thicknesses h_1, h_2 and h_3 for the bottom, middle and upper layers, respectively, besides, plate's length and width represented by a and b, respectively, *Fig. 1*.

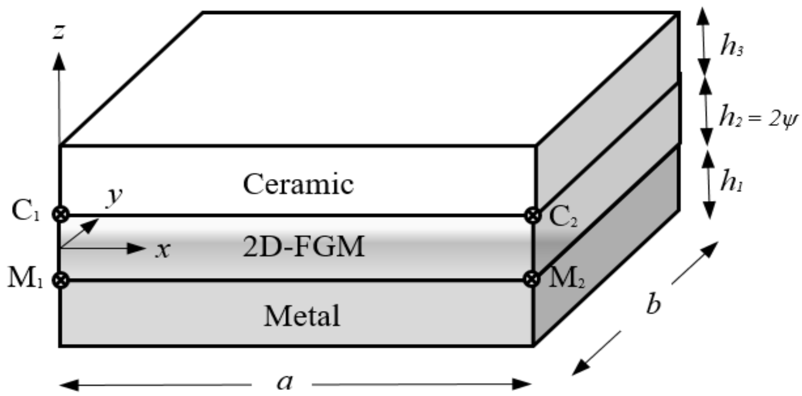

Fig. 1. Geometry of the 2D-FGM layered plate.

The variation of the volume fraction distribution for each material in the 2D-FG layer can be written as [21].

$$V_{C_1}=\left(1-\left(\frac{x}{a}\right)^{n_x}\right)\left(\frac{z}{h_2}+\frac{1}{2}\right)^{n_z}. \tag{1}$$

$$V_{C_2}=\left(\frac{x}{a}\right)^{n_x}\left(\frac{z}{h_2}+\frac{1}{2}\right)^{n_z}. \tag{2}$$

$$V_{M_1}=\left(1-\left(\frac{x}{a}\right)^{n_x}\right)\left(1-\left(\frac{z}{h_2}+\frac{1}{2}\right)^{n_z}\right). \tag{3}$$

$$V_{M_2}=\left(\frac{x}{a}\right)^{n_x}\left(1-\left(\frac{z}{h_2}+\frac{1}{2}\right)^{n_z}\right). \tag{4}$$

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Where *nx* and *nz* are non-zero parameters known as volume fraction exponents. These exponents represent constituent distributions in *x* and *z*-directions. V_{C_1}, V_{C_2}, V_{M_1} and V_{M_2} are the volume fractions of the material constituents *C1*, *C2*, *M1* and *M2*, respectively. *Fig. 2* and *Fig. 3* represent the through-thickness and the through-width volume fraction variations for different values of nz and nx.

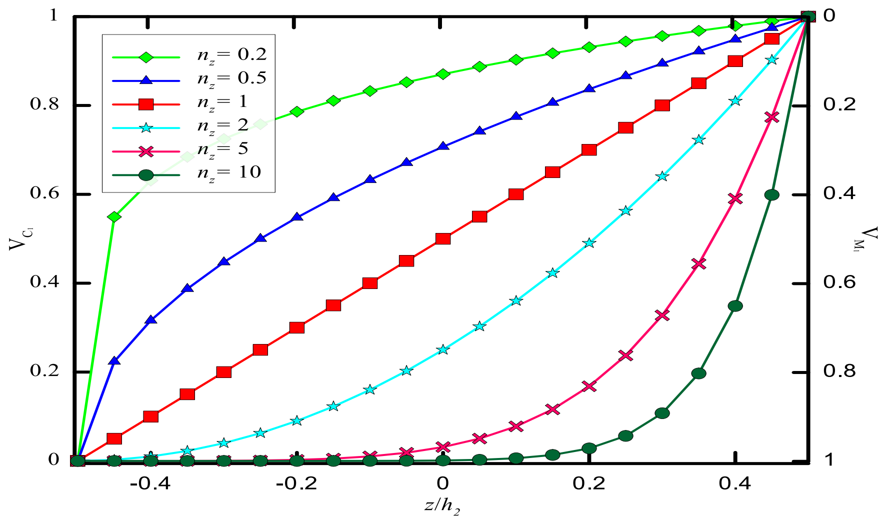

Fig. 2. Through-thickness volume fraction variation of both m_1 and c_1 material constituents for different values of n_z and a value of zero for n_x.

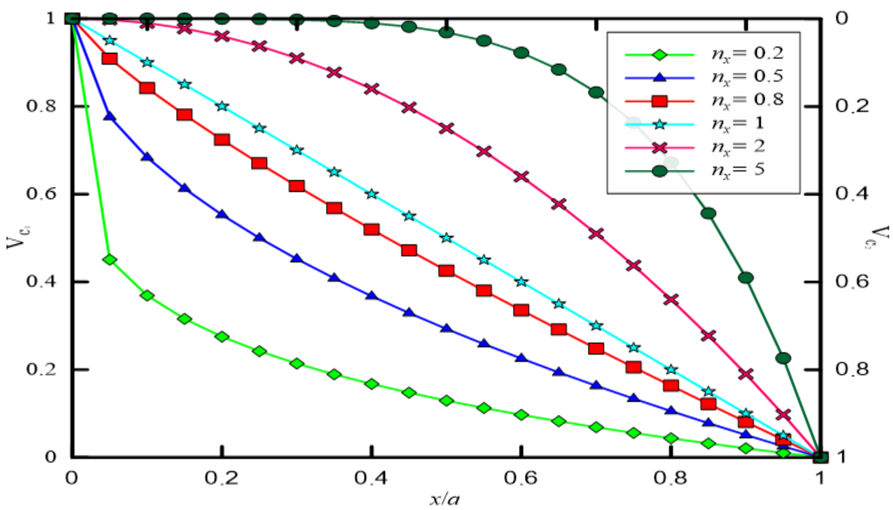

Fig. 2. Through-width volume fraction variation of both c_1 and c_2 material constituents for different values of n_x and a value of zero for n_z.

Material properties such as modulus of elasticity *E* at any arbitrary point (*x*, *z*) in the 2D-FG layer can be determined by a linear combination of volume fractions multiplied by the material property for each constituent material [13].

$$E(x,z) = E_{C_1}V_{C_1} + E_{C_2}V_{C_2} + E_{M_1}V_{M_1} + E_{M_2}V_{M_2} \tag{5}$$

where E_{C_1}, E_{C_2}, E_{M_1} and E_{M_2} are the moduli of elasticity for material constituents C_1, C_2, M_1 and M_2, respectively. In the present study, a combination of FSDT and TSDT will be used to assign the displacement fields for the FGM structure embedded between two isotropic homogenous layers, where the use of higher order displacement fields such TSDT results in a higher DoF comparing to lower order displacement fields such as FSDT. The displacement fields for the three layers of the plate shown in *Fig. 1* can be expressed as

$$\begin{aligned} u^{(1)}(x,y,z) &= u_o + z^{(1)}\theta_x^{(1)}, \\ v^{(1)}(x,y,z) &= v_o + z^{(1)}\theta_y^{(1)}, \\ w^{(1)}(x,y,z) &= w_o. \end{aligned} \tag{6}$$

$$\begin{aligned} u^{(2)}(x,y,z) &= u_o + z^{(2)}\theta_x^{(2)} + \left(z^{(2)}\right)^2 u^* + \left(z^{(2)}\right)^3 \theta_x^*, \\ v^{(2)}(x,y,z) &= v_o + z^{(2)}\theta_y^{(2)} + \left(z^{(2)}\right)^2 v^* + \left(z^{(2)}\right)^3 \theta_y^*, \\ w^{(2)}(x,y,z) &= w_o. \end{aligned} \tag{7}$$

$$\begin{aligned} u^{(3)}(x,y,z) &= u_o + z^{(3)}\theta_x^{(3)}, \\ v^{(3)}(x,y,z) &= v_o + z^{(3)}\theta_y^{(3)}, \\ w^{(3)}(x,y,z) &= w_o. \end{aligned} \tag{8}$$

The TSDT is assigned for the middle 2D-FG layer, *Eq. (7)*, whereas FSDT is assigned for both the top and bottom layers of the plate. In *Eqs. (6-8)*, u_o, v_o and w_o are the displacements of mid-plane along x, y and z-directions, θ_x and θ_y are the rotations of normal to mid-plane about the y and x-axes, respectively, u^*, v^*, θ_x^* and θ_y^* are the coefficients of the higher-order terms in Taylor's series expansion of the displacement fields of the middle layer. Superscripts 1, 2 and 3 inside parentheses indicate the layer's number assigned in ascending order, i.e., the bottom layer is assigned by layer 1 whereas the upper layer is assigned by layer 3. To maintain the continuity conditions at the layer interfaces, the layerwise approach should be included in the displacement field equations for both the upper and lower layers. Thus, the conditions needed to modify the displacement field equations at the interfaces between layers 1-2 and layers 2-3, can be written as

$$\begin{aligned} &u^{(1)}(x,y,-\psi) = u^{(2)}(x,y,-\psi),\ v^{(1)}(x,y,-\psi) = v^{(2)}(x,y,-\psi), \\ &u^{(3)}(x,y,\psi) = u^{(2)}(x,y,\psi),\ v^{(3)}(x,y,\psi) = v^{(2)}(x,y,\psi). \end{aligned} \tag{9}$$

where ψ is the half-thickness of the middle layer, i.e., $\psi = h_2/2$. Using *Eqs. (6)-(9)*, the modified displacement field equations for both the upper and lower layers are obtained as [26]:

$$\begin{aligned} u^{(1)}(x,y,z) &= u^{(2)}(x,y,-\psi) + (z+\psi)\theta_x^{(1)} \\ &= u_o - \psi\theta_x^{(2)} + \psi^2 u^* - \psi^3\theta_x^* + (z+\psi)\theta_x^{(1)}. \\ v^{(1)}(x,y,z) &= v^{(2)}(x,y,-\psi) + (z+\psi)\theta_y^{(1)} \\ &= v_o - \psi\theta_y^{(2)} + \psi^2 v^* - \psi^3\theta_y^* + (z+\psi)\theta_y^{(1)}. \end{aligned} \tag{10}$$

$$\begin{aligned} u^{(3)}(x,y,z) &= u^{(2)}(x,y,\psi) + (z-\psi)\theta_x^{(3)} \\ &= u_o + \psi\theta_x^{(2)} + \psi^2 u^* + \psi^3\theta_x^* + (z-\psi)\theta_x^{(3)}. \\ v^{(3)}(x,y,z) &= v^{(2)}(x,y,\psi) + (z-\psi)\theta_y^{(3)} \\ &= v_o + \psi\theta_y^{(2)} + \psi^2 v^* + \psi^3\theta_y^* + (z-\psi)\theta_y^{(3)}. \end{aligned} \tag{11}$$

Proceeding further deriving the strain-displacement relations including both normal and shear strains (ε and γ). Strains are simply the partial derivatives of displacement fields with respect to x or y or a combination of both which are stated for each layer as:

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$$
\begin{aligned}
\varepsilon_{xx}^{(1)} &= \frac{\partial u^{(1)}}{\partial x} = \frac{\partial u_o}{\partial x} - \psi \frac{\partial \theta_x^{(2)}}{\partial x} + \psi^2 \frac{\partial u^*}{\partial x} - \psi^3 \frac{\partial \theta_x^*}{\partial x} + (z+\psi) \frac{\partial \theta_x^{(1)}}{\partial x}, \\
\varepsilon_{yy}^{(1)} &= \frac{\partial v^{(1)}}{\partial y} = \frac{\partial v_o}{\partial y} - \psi \frac{\partial \theta_y^{(2)}}{\partial y} + \psi^2 \frac{\partial v^*}{\partial y} - \psi^3 \frac{\partial \theta_y^*}{\partial y} + (z+\psi) \frac{\partial \theta_y^{(1)}}{\partial y}, \\
\varepsilon_{xy}^{(1)} &= \frac{\partial u^{(1)}}{\partial y} + \frac{\partial v^{(1)}}{\partial x} = \left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \right) - \psi \left(\frac{\partial \theta_x^{(2)}}{\partial y} + \frac{\partial \theta_y^{(2)}}{\partial x} \right) + \psi^2 \left(\frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x} \right) \\
&\qquad - \psi^3 \left(\frac{\partial \theta_x^*}{\partial y} + \frac{\partial \theta_y^*}{\partial x} \right) + (z+\psi) \left(\frac{\partial \theta_x^{(1)}}{\partial y} + \frac{\partial \theta_y^{(1)}}{\partial x} \right). \\
\gamma_{xz}^{(1)} &= \frac{\partial w^{(1)}}{\partial x} + \frac{\partial u^{(1)}}{\partial z} = \left(\frac{\partial w_o}{\partial x} + \theta_x^{(1)} \right), \\
\gamma_{yz}^{(1)} &= \frac{\partial w^{(1)}}{\partial y} + \frac{\partial v^{(1)}}{\partial z} = \left(\frac{\partial w_o}{\partial y} + \theta_y^{(1)} \right).
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\varepsilon_{xx}^{(2)} &= \frac{\partial u^{(2)}}{\partial x} = \frac{\partial u_o}{\partial x} + z \frac{\partial \theta_x^{(2)}}{\partial x} + z^2 \frac{\partial u^*}{\partial x} + z^3 \frac{\partial \theta_x^*}{\partial x}, \\
\varepsilon_{yy}^{(2)} &= \frac{\partial v^{(2)}}{\partial x} = \frac{\partial v_o}{\partial y} + z \frac{\partial \theta_y^{(2)}}{\partial y} + z^2 \frac{\partial v^*}{\partial y} + z^3 \frac{\partial \theta_y^*}{\partial y}, \\
\varepsilon_{xy}^{(2)} &= \frac{\partial u^{(2)}}{\partial y} + \frac{\partial v^{(2)}}{\partial x} = \left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \right) + z \left(\frac{\partial \theta_x^{(2)}}{\partial y} + \frac{\partial \theta_y^{(2)}}{\partial x} \right) \\
&\qquad + z^2 \left(\frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x} \right) + z^3 \left(\frac{\partial \theta_x^*}{\partial y} + \frac{\partial \theta_y^*}{\partial x} \right). \\
\gamma_{xz}^{(2)} &= \frac{\partial w^{(2)}}{\partial x} + \frac{\partial u^{(2)}}{\partial z} = \left(\frac{\partial w_o}{\partial x} + \theta_x^{(2)} + 2zu^* + 3z^2 \theta_x^* \right), \\
\gamma_{yz}^{(2)} &= \frac{\partial w^{(2)}}{\partial y} + \frac{\partial v^{(2)}}{\partial z} = \left(\frac{\partial w_o}{\partial y} + \theta_y^{(2)} + 2zv^* + 3z^2 \theta_y^* \right).
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\varepsilon_{xx}^{(3)} &= \frac{\partial u^{(3)}}{\partial x} = \frac{\partial u_o}{\partial x} + \psi \frac{\partial \theta_x^{(2)}}{\partial x} + \psi^2 \frac{\partial u^*}{\partial x} + \psi^3 \frac{\partial \theta_x^*}{\partial x} + (z-\psi) \frac{\partial \theta_x^{(3)}}{\partial x}, \\
\varepsilon_{yy}^{(3)} &= \frac{\partial v^{(3)}}{\partial y} = \frac{\partial v_o}{\partial y} + \psi \frac{\partial \theta_y^{(2)}}{\partial y} + \psi^2 \frac{\partial v^*}{\partial y} + \psi^3 \frac{\partial \theta_x^*}{\partial y} + (z-\psi) \frac{\partial \theta_y^{(3)}}{\partial y}, \\
\varepsilon_{xy}^{(3)} &= \frac{\partial u^{(3)}}{\partial y} + \frac{\partial v^{(3)}}{\partial x} = \left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \right) + \psi \left(\frac{\partial \theta_x^{(2)}}{\partial y} + \frac{\partial \theta_y^{(2)}}{\partial x} \right) + \psi^2 \left(\frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x} \right) \\
&\qquad + \psi^3 \left(\frac{\partial \theta_x^*}{\partial y} + \frac{\partial \theta_y^*}{\partial x} \right) + (z-\psi) \left(\frac{\partial \theta_x^{(3)}}{\partial y} + \frac{\partial \theta_y^{(3)}}{\partial x} \right). \\
\gamma_{xz}^{(3)} &= \frac{\partial w^{(3)}}{\partial x} + \frac{\partial u^{(3)}}{\partial z} = \left(\frac{\partial w_o}{\partial x} + \theta_x^{(3)} \right), \quad \gamma_{yz}^{(3)} = \frac{\partial w^{(3)}}{\partial y} + \frac{\partial v^{(3)}}{\partial z} = \left(\frac{\partial w_o}{\partial y} + \theta_y^{(3)} \right).
\end{aligned} \tag{14}
$$

These relations for all the layers are quite similar, except for the 2D-FG layer, where the modulus of elasticity E is not constant as in the homogenous top and bottom layers. In 2D-FGM, the modulus of elasticity E is a function of both x and z-coordinates. The following matrix form represents the general form of normal stress-strain relations, which can be used for any layer configuration considering a constant value of Poisson's ratio ν throughout plate's geometry and plain stress assumptions.

$$\begin{bmatrix} \sigma_{xx}^{(i)} \\ \sigma_{yy}^{(i)} \\ \tau_{xy}^{(i)} \end{bmatrix} = \frac{E(x,z)}{\left(1-\nu^2\right)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^{(i)} \\ \varepsilon_{yy}^{(i)} \\ \varepsilon_{xy}^{(i)} \end{bmatrix}. \tag{15}$$

$$\begin{bmatrix} \tau_{xz}^{(i)} \\ \tau_{yz}^{(i)} \end{bmatrix} = \frac{E(x,z)}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{xz}^{(i)} \\ \gamma_{yz}^{(i)} \end{bmatrix}. \tag{16}$$

After deriving the detailed constitutive relations, the next step involves FE formulation. In the present model, an 8-noded quadrilateral element will be used for the plate structure with 13-DoF per node that appears in *Eqs. (6-8)*. The shape functions associated with the 8-node element are given as:

$$\begin{aligned} N_1(\xi,\eta) &= \frac{1}{4}(1-\xi)\,(1-\eta)\,(-\xi-\eta-1), & N_5(\xi,\eta) &= \frac{1}{2}(1-\eta)\,(1+\xi)\,(1-\xi), \\ N_2(\xi,\eta) &= \frac{1}{4}(1+\xi)\,(1-\eta)\,(\xi-\eta-1), & N_6(\xi,\eta) &= \frac{1}{2}(1-\eta)\,(1+\eta)\,(1+\xi), \\ N_3(\xi,\eta) &= \frac{1}{4}(1+\xi)\,(1+\eta)\,(\xi+\eta-1), & N_7(\xi,\eta) &= \frac{1}{2}(1+\eta)\,(1+\xi)\,(1-\xi), \\ N_4(\xi,\eta) &= \frac{1}{4}(1-\xi)\,(1+\eta)\,(-\xi+\eta-1), & N_8(\xi,\eta) &= \frac{1}{2}(1-\xi)\,(1+\eta)\,(1-\eta). \end{aligned} \tag{17}$$

Where ξ and η are the natural-coordinate system defined by the element geometry, whereas N_i is the shape function associated with the i^{th} node. The total 13-DoF for the whole element can be written using the shape functions stated earlier as the summation of individual nodal displacement given as:

$$\Delta = \sum_{i=1}^{8} N_i \Delta_i. \tag{18}$$

Where $[\Delta]$ is the nodal unknown vector for the 13-DoF written as:

$$[\Delta] = \begin{bmatrix} u_o & v_o & w_o & \theta_x^{(2)} & \theta_y^{(2)} & u^* & v^* & \theta_x^* & \theta_x^* & \theta_x^{(1)} & \theta_y^{(1)} & \theta_x^{(3)} & \theta_y^{(3)} \end{bmatrix}^T. \tag{19}$$

The generalized strain-displacement relations can be expressed in terms of the nodal unknown vector in matrix form as:

$$\varepsilon^{(i)} = \left\lfloor B^{(i)} \right\rfloor [\Delta]. \tag{20}$$

Where $[B^{(i)}]$ are 13-matrices that relates strains to nodal unknown vector. The next step involves applying the principle of energy conservation, which relates energies associated with the system under consideration, i.e., strain and external work energies. To formulate the static problem, Hamilton's principle is used, which is used by many authors including Belarbi et al. [26]. Hamilton's Principle relates strain and work done by the external load of the system as:

$$\delta \Pi = \delta \int_{t_1}^{t_2} (W - U)\, dt. \tag{21}$$

Where U and W are the strain energy and work done by an external load, respectively. Each of these energies will be derived independently. Strain energy reflects the stiffness of the system; it is the integration of the stress-strain product integrated through plate's volume. For the three-layered plate, the strain energy is expressed as:

$$\delta U=\frac{1}{2}\left(\begin{array}{l}\int\int_{A_1}\int_{-(\psi+h_1)}^{\psi}\left(\sigma_{xx}^{(1)}\ \delta\varepsilon_{xx}^{(1)}+\sigma_{yy}^{(1)}\ \delta\varepsilon_{yy}^{(1)}+\sigma_{xy}^{(1)}\ \delta\varepsilon_{xy}^{(1)}+\tau_{xz}^{(1)}\ \delta\gamma_{xz}^{(1)}+\tau_{yz}^{(1)}\ \delta\gamma_{yz}^{(1)}\right)dV_1\\ +\int\int_{A_2}\int_{-\psi}^{\psi}\left(\sigma_{xx}^{(2)}\ \delta\varepsilon_{xx}^{(2)}+\sigma_{yy}^{(2)}\ \delta\varepsilon_{yy}^{(2)}+\sigma_{xy}^{(2)}\ \delta\varepsilon_{xy}^{(2)}+\tau_{xz}^{(2)}\ \delta\gamma_{xz}^{(2)}+\tau_{yz}^{(2)}\ \delta\gamma_{yz}^{(2)}\right)dV_2\\ +\int\int_{A_3}\int_{\psi}^{(\psi+h_3)}\left(\sigma_{xx}^{(3)}\ \delta\varepsilon_{xx}^{(3)}+\sigma_{yy}^{(3)}\ \delta\varepsilon_{yy}^{(3)}+\sigma_{xy}^{(3)}\ \delta\varepsilon_{xy}^{(3)}+\tau_{xz}^{(3)}\ \delta\gamma_{xz}^{(3)}+\tau_{yz}^{(3)}\ \delta\gamma_{yz}^{(3)}\right)dV_3\end{array}\right). \tag{22}$$

where A_1, A_2, A_3 and V_1, V_2, V_3 are the xy-plane areas and volumes for the lower, middle and upper layers, respectively. Using previously stated relations, the strain energy can be rewritten in the following matrix form

$$\delta U=\frac{1}{2}[\delta\Delta]^T\left([K^{(1)}]+[K^{(2)}]+[K^{(3)}]\right)[\Delta]=\frac{1}{2}[\delta\Delta]^T[K_{tot}]\,[\Delta]. \tag{23}$$

Where the total elemental stiffness matrix $[K_{tot}]$ with a matrix size of *[104×104]* is the sum of the elemental stiffness matrices for the lower, middle and upper layers. External work done by a distributed static load applied on the top surface of the plate is given by:

$$\begin{aligned}&\delta W=\int\int_A[\Delta]^T[N_i]^T[f(x,y)]dA=[\Delta]^T\int\int_A[F(x,y)]_i\,dA,\\&[F(x,y)]_i=[N_i]^T[f(x,y)]^T.\end{aligned} \tag{24}$$

Where *F(x, y)* is the static load applied transversely on the top surface of the plate, whereas *F(x, y)* is the discretized load on each nodal point. Total elemental load vector $[F(x,y)]_{tot}$ of a size [104×1] is calculated by adding the nodal forces for one element as

$$\begin{aligned}[F(x,y)]_{tot}&=\sum_{i=1}^{8}\int\int_A[N_i]^T[f(x,y)]^T dA\\&=\sum_{i=1}^{8}\int\int_A[N_i]^T[0\quad 0\quad F\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]^T\ dA.\end{aligned} \tag{25}$$

The equation of motion for the one element plate structure considering the FE model will lead to the following static equation of motion.

$$[K_{tot}][\Delta]=[F_{tot}]. \tag{26}$$

Where $[K_{tot}]$ and $[F_{tot}]$ are the elemental stiffness matrix and force vector presented in *Eq. (23)* and *Eq. (25)*, respectively, whereas $[\Delta]$ is the unknown vector. Equations of Motion are solved after imposing the boundary conditions applied to the plate structure. Each supported plate edges could be either Free (*F*), Simply (*S*), or Clamped (*C*). For a plate edge with simply supported conditions, the following equations are applied [26].

$$\begin{aligned}&u_o=w_o=\theta_x^{(2)}=u^*=\theta_x^*=\theta_x^{(1)}=\theta_x^{(3)}=0 &&\text{at}\quad y=0,\ b,\\&v_o=w_o=\theta_y^{(2)}=v^*=\theta_y^*=\theta_y^{(1)}=\theta_y^{(3)}=0 &&\text{at}\quad x=0,\ a.\end{aligned} \tag{27}$$

However, for a plate edge with clamped supported conditions, all DoF associated with that edge are set to be be zero [26].

$$u_o=v_o=w_o=\theta_x^{(2)}=\theta_y^{(2)}=u^*=v^*=\theta_x^*=\theta_y^*=\theta_x^{(1)}=\theta_y^{(1)}=\theta_x^{(3)}=\theta_y^{(3)}=0\,. \tag{28}$$

The nomenclature of plate's supports is stated starting from the lower, right, upper and left edges, respectively, i.e., a plate with (SFSC) boundary conditions means that the plate is simply, free, simply and clamped supported on the lower, right, upper and left edges of the plate, respectively. *Fig. 4* demonstrates various boundary conditions to be used in the current study, which are widely in the literature [27].

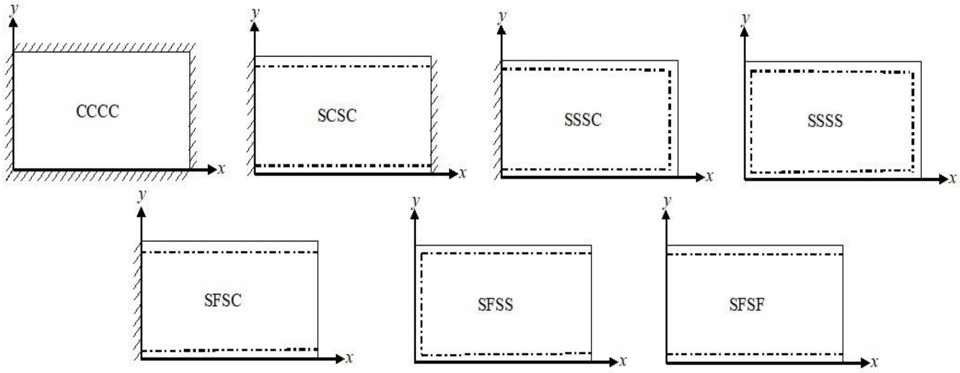

Fig. 4. Boundary conditions nomenclature.

Table 1 summarizes plate's geometry to be investigated in the current study. The aforementioned formulations and boundary conditions will be numerically applied in the following section.

Table 1. Plate's Geometry to be investigated.

Plate's Width to Length Ratio b/a	Plate's Side to Thickness Ratio a/h	Layers Thickness Ratio
1 (Square)	a/h = 5 - 100	1-8-1, 1-3-1,1-2-1, 1-1-1, 0-1-0 and 2-1-2

Due to small variation in Poisson's ratio v for both ceramic and metallic materials, a constant value for all materials is used, i.e., 0.3. Plate's geometry used in the current study includes only square plates ($b/a=1$), with different plate's side to thickness ratios ($a/h=5$-100). Layer thickness ratio is described as three numbers describing the fraction of each layer to the total thickness of the layered plate h. For example, a thickness ratio of (1-8-1) means the lower and upper layers are (0.1 h) each, while the middle layer is (0.8 h). Seven different applied boundary conditions are used in the current study, *Fig 4*. Plane stress assumptions are used in this problem where the structure is considered a thin structure assuming normal stress and strains in z-direction to be zero. Bending behavior will be investigated for the layered plate where a uniformly distributed load is considered.

3 | Results and Discussions

Results obtained in this section are extracted from a computer-based program using Maple software. Results are mainly divided into two categories, two verification problems and parametric studies. The verification problems will be carried out to compare the present formulations with some special case studies presented in the literature. The first verification problem presents the determination of the plate's central deflection of a single 1D-FG layer only. Whereas the second verification problem compares central deflection of the 1D-FG layer embedded between ceramic and metal layers. *Table 2* summarizes verification problems (material properties and types of applied loads) to be verified in subsequent sections. An optimum number of elements considering both the relative approximate error and CPU elapsed time is chosen for a plate mesh grid of (6×6), hence 36 plate elements will be used in the upcoming analysis.

Table 2. Material properties and type of applied loads for verification problems 1 and 2.

Model no.	E_M [GPa]	E_C [GPa]	Poisson's Ratio v	Type of applied load
Verification Problem 1	70	380	0.3	Uniformly distributed static load
Verification Problem 2	70	380	0.3	Bi-sinusoidal static load

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3.1 | Verification Problem 1

In this verification problem by setting $h_1 = h_3 = n_x = 0$, $h_2 = h$, $C_1 = C_2 = C$ and $M_1 = M_2 = M$ where E_C and E_M are given in *Table 2*. The dimensionless form for the plate's central deflection $\bar{w}$ for 1D-FG layer is defined as [28]:

$$\bar{w} = \frac{10\ w\ E_C h^3}{F a^4}. \tag{29}$$

Where h is the thickness of the plate, w is the central deflection of the plate, E_C is the ceramic modulus of elasticity, a is the plate length and F is the intensity of the uniform load applied on the top of the plate. *Table 3* illustrates comparisons with different solutions for various types of theories and the current TSDT for various volume fraction exponents n_z starting from pure ceramic plate ($n_z = 0$), pure metallic plate ($n_z = \infty$) and several FGM combinations ($n_z = 1, 2, 3, 5, 10$). It can be observed from *Table 3* good agreements between present results compared to referenced publications including Reddy's theory, generalized shear deformation theory and with the results obtained using the HSDT approach.

Table 3. Dimensionless central deflection $\bar{w}$ for (ssss) al/al2o3 square plate with plate's a/h of 10.

Reference	Reddy [28]	Zenkour [29]	Bui et al. [30]	Van Do et al. [31]	Present (6×6)
Method	Reddy's theory	GSDT	HSDT	HSDT	TSDT
n_z					
Ceramic	0.4665	0.4665	0.4630	0.4659	0.4659
1	0.9421	0.9287	0.9130	0.9224	0.9273
2	1.2227	1.1940	1.2069	1.1787	1.1928
3	1.3530	1.3200	1.3596	1.2970	1.3189
5	1.4646	1.4356	1.4874	1.4055	1.4341
10	1.6054	1.5876	1.6308	1.5651	1.5856
Metal	2.5328	2.5327	2.5120	2.4968	2.5291

3.2 | Verification Problem 2

In this verification problem, the layerwise approach will be validated for the 3-layered plate structure stated previously. Pandey and Pradyumna [32], Brischetto [33], Carrera et al. [34] and Neves et al. [35] published works will be used for this purpose. The same dimensionless central plate deflection $\bar{w}$ parameter aforementioned in *Eq. (29)* will be used in this verification problem. However, a bi-sinusoidal static load instead of a uniformly distributed load will be applied transversely to the plate structure. The applied load is given in the following equation.

$$F(x,y) = F\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right). \tag{30}$$

Validation will be carried out for a (1-8-1) square (b/a=1) simply supported (SSSS) plate consists of a 1D-FG layer embedded between ceramic and metal layers with various combinations of n_z and plate side to thickness ratios (a/h), *Table 4*. From *Table 4*, it is observed the good agreement between present model results compared to the published ones for various plate theories including TSDT, Equivalent Single Layer method, Carrera's Unified Formulation and Zig-Zag theory. Besides, the effect of increasing the n_z is also observed, where the dimensionless central deflection of the plate increases when increasing n_z. This phenomenon can be explained due to the proportional relation between the n_z and the increment of metallic content in the 1D-FG layer. Therefore, plate's overall stiffness becomes softer compared to the pure ceramic rigid plate.

Table 4. Dimensionless central deflection $\bar{w}$ of a (1-8-1) (ssss) al/al2o3 square plate embedded between ceramic and metal layers.

a/h	Reference	Mesh	Method	Volume Fraction Exponent n_z			
				1	4	5	10
4	Pandey and Pradyumna [32]	8×8	TSDT	0.7636	-	-	1.2235
	Brischetto [33]	N/A	ESL	0.7629	-	1.1327	1.2232
	Carrera et al. [34]	N/A	CUF	0.7735	1.0977	-	1.2240
	Neves et al. [35]	15×15	Zig-Zag	0.7746	1.0833	1.1236	1.2183
	Present	6×6	TSDT	0.7723	1.0947	1.1348	1.2273
10	Pandey and Pradyumna [32]	8×8	TSDT	0.6323	-	-	0.8697
	Brischetto [33]	N/A	ESL	-	-	-	-
	Carrera et al. [34]	N/A	CUF	0.6337	0.8308	-	0.8743
	Neves et al. [35]	15×15	Zig-Zag	0.6357	0.8273	0.8415	0.8712
	Present	6×6	TSDT	0.6320	0.8285	0.8430	0.8734
100	Pandey and Pradyumna [32]	8×8	TSDT	0.6074	-	-	0.8076
	Brischetto [33]	N/A	ESL	0.6073	-	0.7892	0.8077
	Carrera et al. [34]	N/A	CUF	0.6072	0.7797	-	0.8077
	Neves et al. [35]	15×15	Zig-Zag	0.6087	0.7779	0.7870	0.8045
	Present	6×6	TSDT	0.5953	0.7654	0.7749	0.7938

3.3 | Parametric Studies

After verifying the current model formulations, in this section, parametric studies will be investigated for various parameters including applied boundary conditions, volume fraction exponents in both x and z-directions (n_x and n_z) and plate's side to thickness ratio a/h. These parameters will be used to study their effect on bending behavior on the 2D-FG layered plate. The same non-dimensional plate's central deflection stated earlier in *Eq. (29)* will be used in the upcoming analysis. Whereas non-dimensional normal and shear stresses terms that will be used are summarized in *Eq. (31)*.

$$\bar{\sigma}_{xx} = \left(\frac{h}{a\ F}\right)\sigma_{xx},\ \bar{\sigma}_{yy} = \left(\frac{h}{a\ F}\right)\sigma_{yy},\ \bar{\tau}_{xy} = \left(\frac{h}{a\ F}\right)\tau_{xy},\ \bar{\tau}_{xz} = \left(\frac{h}{a\ F}\right)\tau_{xz},\ \bar{\tau}_{yz} = \left(\frac{h}{a\ F}\right)\tau_{yz}. \tag{31}$$

Where $\bar{\sigma}, \bar{\tau}$ are the non-dimensional normal and shear stresses parameters, respectively. The plate under investigation is made of a 2D-FG layer that consists of two ceramics (Silicon carbide [SiC], Aluminum oxide [Al_2O_3]) and two metals (Titanium Alloy [Ti-6Al-4V], Aluminum [Al]) embedded between Titanium Alloy and Silicon carbide in both lower and upper faces of the 2D-FG layer, respectively. *Table 5* illustrates material constituent's properties considering a constant value of 0.3 for Poisson's ratio ν for all materials.

Table 5. The properties of material constituents for the 2d-fg layer.

Constituents	Material	Modulus of Elasticity E [GPa]	Poisson's Ratio ν
M_1	Ti-6Al-4V	115	0.3
M_2	Al 1100	69	0.3
C_1	SiC	440	0.3
C_2	Al_2O_3	300	0.3

Initially, the effect of varying n_x and n_z on the plate's central deflection will be investigated. *Fig. 5* illustrates the dimensionless central deflection $\bar{w}$ of a simply supported (SSSS) plate with a side to thickness ratio a/h of five for various combinations of n_x and n_z.

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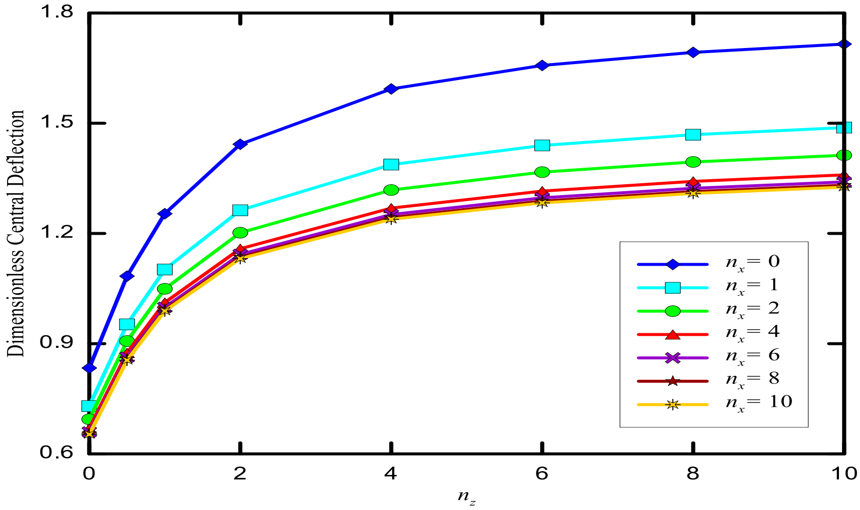

Fig. 5. Variation of non-dimensional central deflection $\overline{w}$ for a simply supported layered plate for various volume fraction exponents n_x and n_z.

It can be observed from *Fig. 5* the effect of both exponents on the plate's deflection. It is found that as the n_z increases, the central deflection of the plate increases. This increment in plate's deflection is associated with the increment of the metallic content as n_z increases, thus reducing the middle layer overall stiffness. On the other hand, as n_x increases, the deflection of the plate decreases, i.e., 1D-FGM ($n_x = 0$) deflects more easily compared to 2D-FGM ($n_x \neq 0$). This phenomenon is associated with high mechanical properties (modulus of elasticity) of materials C_1 and M_1 comparing to C_2 and M_2 material constituents. Therefore, as n_x increases, the middle 2D-FG layer will have a higher portion of material constituents C_1 and M_1 and less portion of materials C_2 and M_2, thus a more rigid middle layer will be attained, which is hard to deflect comparing to a 2D-FG layer with lower n_x values. *Table 6* demonstrates the numerical data illustrated in *Fig. 5*.

Table 6. The variation of non-dimensional central deflection $\overline{w}$ for a simply supported layered plate for various volume fraction exponents n_x and n_z.

n_x / n_z	0	1	2	4	6	8	10
0	0.8338	0.7302	0.6942	0.6686	0.6596	0.6553	0.6530
0.5	1.0838	0.9527	0.9072	0.8747	0.8633	0.8579	0.8549
1	1.2537	1.1018	1.0493	1.0119	0.9987	0.9924	0.9890
2	1.4428	1.2633	1.2020	1.1584	1.1430	1.1357	1.1316
4	1.5934	1.3874	1.3182	1.2690	1.2515	1.2432	1.2386
6	1.6574	1.4395	1.3669	1.3153	1.2969	1.2881	1.2833
8	1.6929	1.4689	1.3946	1.3418	1.3229	1.3140	1.3089
10	1.7155	1.4881	1.4129	1.3594	1.3403	1.3312	1.3261

The next parametric study will investigate the effect of the applied boundary conditions on the same dimensionless deflection parameter. *Fig. 6* illustrates the variation of the non-dimensional central deflection parameter for a (1-8-1) square plate structure with exponent $n_x = 1$ and different combinations of n_z and applied boundary conditions.

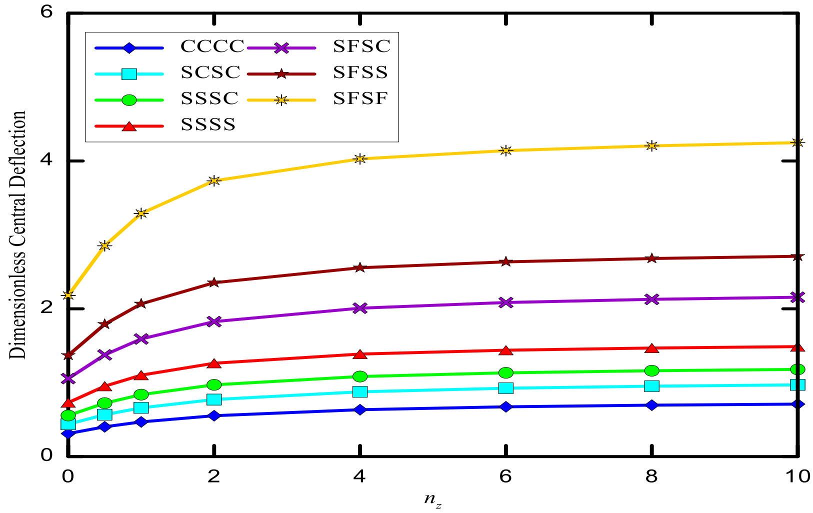

Fig. 6. Variation of non-dimensional central deflection $\overline{w}$ for (1-8-1) layered plate with $n_x = 1$ for different values of n_z and applied boundary conditions.

From *Fig. 6*, the effect of n_z is observed again, whereas in this figure seven different boundary conditions were applied on the plate structure. It can be observed that the central deflection for a (CCCC) plate is less than any other applied boundary condition. Moreover, the highest plate deflection is observed for (SFSF) plate for all values n_z. Furthermore, for (SSSS) plate, the plate's non-dimensional central deflection is between the two extreme cases, i.e., (SFSF) and (CCCC), where it can be concluded that the more constrained the plate is, less deflection is observed.

Similar procedures have been carried out to study the behavior of the plate's central deflection for various applied boundary conditions; however, different values of nx will be investigated instead of n_z. *Fig. 7* illustrates the effect of varying exponent n_x for the seven different applied boundary conditions while maintaining a unity value for n_z. It is observed from *Fig. 7* the effect of increasing nx on the plate's central deflection, however, this effect is hardly observed for n_x values above four. Furthermore, the effect of the applied boundary conditions is also observed for either constrained or unconstrained plate structures.

Fig. 8 illustrates the effect of the plate's side to plate thickness ratio a/h on the plate's central deflection. It can be observed from *Fig. 8* that higher a/h ratios, i.e., thin plates, less deflection on the center of the plate are observed, i.e., the thinner the plate, less deflection is observed. However, for high a/h ratios greater than twenty, plate's deflection remains almost constant, where the transition from thick to thin plates occurs at this ratio.

The last parameter to be investigated is the effect of the plate's layer thickness ratios while maintaining a constant plate total thickness for all plate's layer configurations. Plate configurations that will be examined includes ratios (1-3-1), (1-2-1), (1-1-1), (0-1-0), (2-1-2) and the previously studied plate configuration (1-8-1). The plate under investigation will be fully clamped (CCCC) plate subjected to uniformly distributed load. The middle 2D-FG layer will have a unity value for n_x and a value of five for a/h ratio, *Fig. 9*.

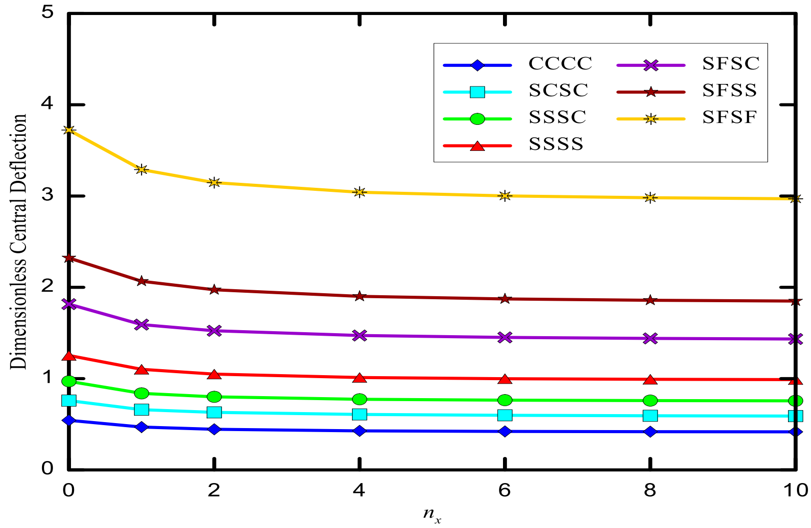

Fig. 7. Variation of non-dimensional central deflection $\overline{w}$ for (1-8-1) layered plate with $n_z = 1$ for different values of n_x and applied boundary conditions.

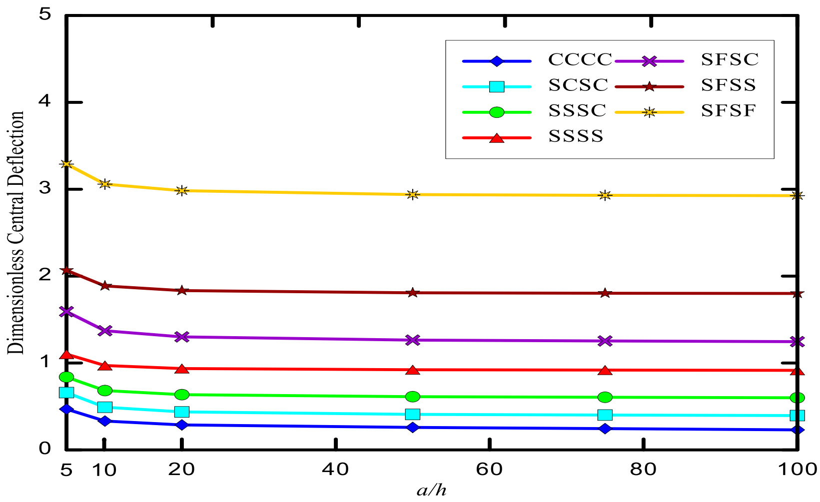

Fig. 8. Variation of non-dimensional central deflection $\overline{w}$ for (1-8-1) layered plate with $n_x = n_z = 1$ for different values of a/h and applied boundary conditions.

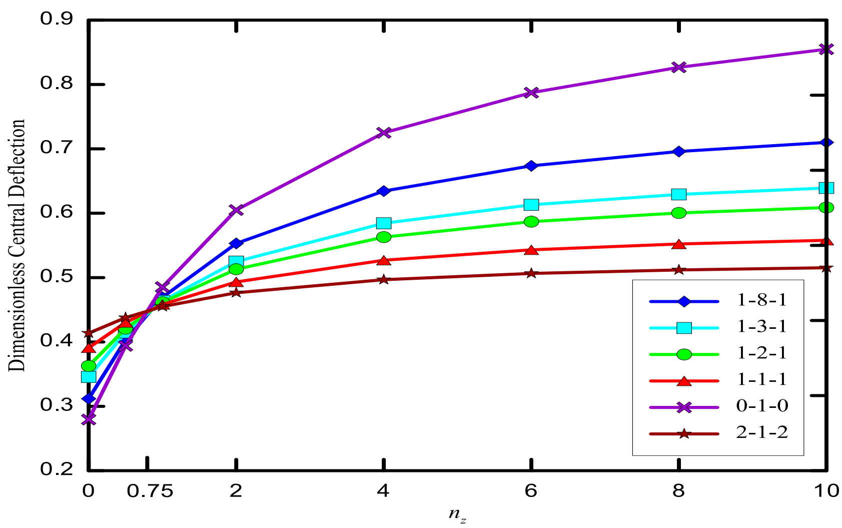

Fig. 9. Variation of non-dimensional central deflection $\overline{w}$ for (cccc) plate with $n_x = 1$, a/h = 5 for different plate configurations and n_z.

It is observed from *Fig. 9* that plate's central deflection also relies on the layer's thickness configurations; this relation is divided into two intervals, i.e., high ceramic and high metallic material contribution as indicated from the intersection point for all plate configurations. The first interval is related to values of n_z less than 0.75, i.e., high ceramic content, where it can be concluded that the higher the ratio of the middle 2D-FG layer compared to the upper and lower isotropic layers, less deflection is observed. This phenomenon can be explained from the high contribution of ceramic content in the whole plate structure in comparison with metallic content that is mainly located in the lower metal layer. Furthermore, as the middle 2D-FG layer ratio increases, such as (0-1-0) plate configuration, the contribution of the ceramic content in the whole plate structure will be maximum, thus a more rigid plate structure is obtained compared to low middle 2D-FG layer ratio configurations, such as (2-1-2) plate configuration. On the other hand, the second interval is related to values of n_z greater than 0.75, i.e., high metallic content, where opposite conclusions comparing to the first interval is obtained. For high values of n_z, the contribution of metallic materials inside the middle 2D-FG layer will be higher. Furthermore, the highest metallic contribution in the whole plate structure will be for high middle 2D-FG layer ratios such as (0-1-0) plate configuration. Consequently, this increment in metallic content in the overall plate structure is reflected in a softer plate structure that tends to deflect more easily compared to low middle 2D-FG layer ratios, such as (2-1-2) plate configuration.

Other than plate's deflection, it is necessary to study the effect of the volume fraction exponents n_x and n_z associated with the middle 2D-FG layer on the through-thickness stress profile for the 3-layered plate. *Fig. 10* through *Fig. 14* illustrate the through-thickness normal and shear stresses at the center of the plate ($x=a/2, y=b/2$). The same plate configurations used earlier will be used, a square (1-8-1) simply supported (SSSS) plate with a/h ratio of five. A unity value for the volume fraction exponent in x-direction ($n_x = 1$) and different combinations of n_z ranging from zero to six, in addition to an isotropic middle layer case ($n_x = n_z = 0$) will be also examined.

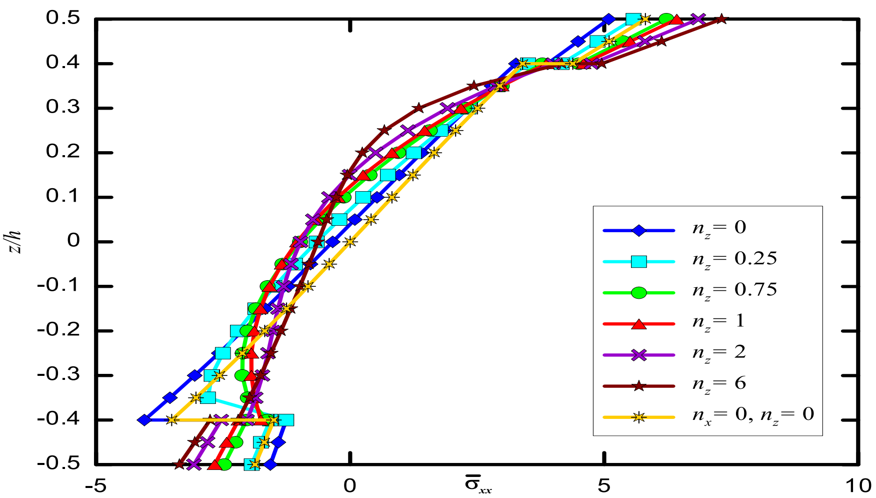

Fig. 10. Variation of non-dimensional normal stress in x-direction σ̄xx for (1-8-1) simply supported plate with nx = 1 and different values of nz.

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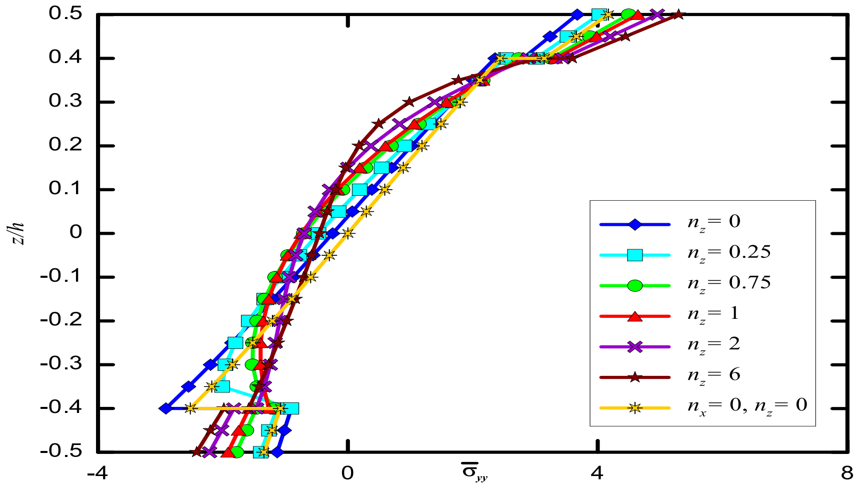

Fig. 11. Variation of non-dimensional normal stress in y-direction $\bar{\sigma}_{yy}$ for (1-8-1) simply supported plate with $n_x = 1$ and different values of n_z.

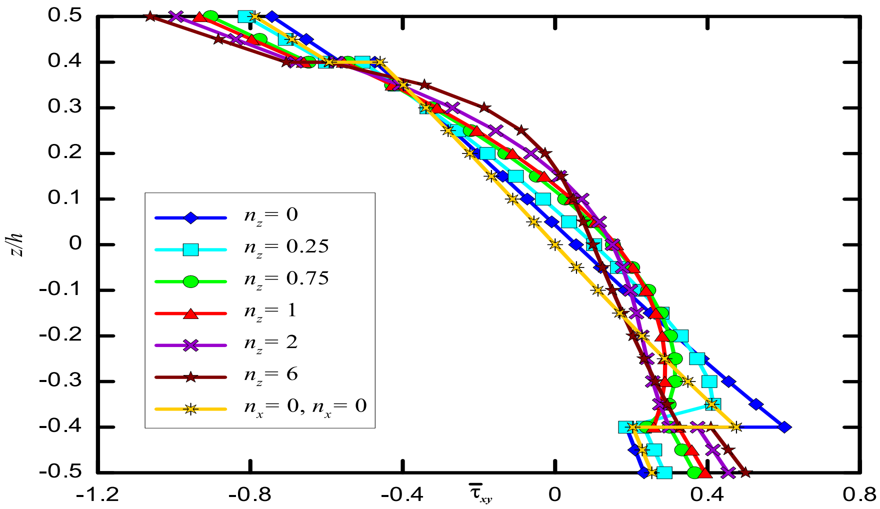

Fig. 12. Variation of non-dimensional shear stress in xy-direction $\bar{\tau}_{xy}$ for (1-8-1) simply supported plate with nx = 1 and different values of nz.

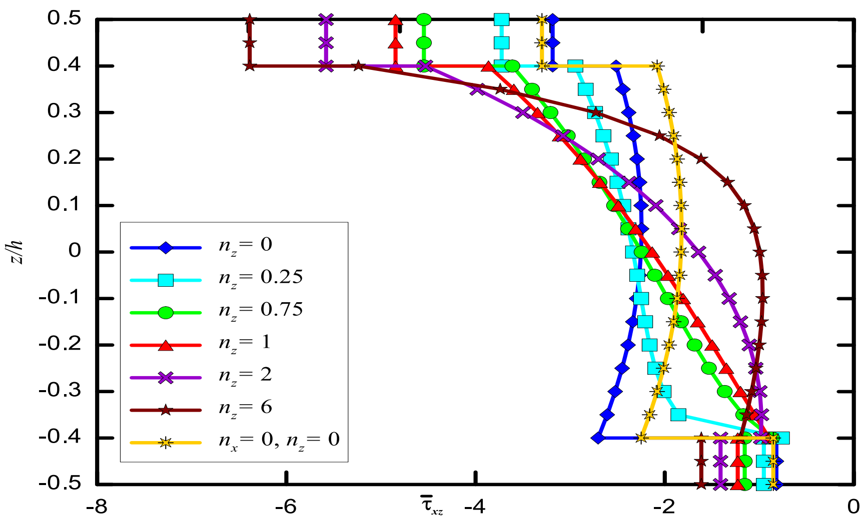

Fig. 13. Variation of non-dimensional shear stress in xz-direction $\bar{\tau}_{xz}$ for (1-8-1) simply supported plate with nx = 1 and different values of nz.

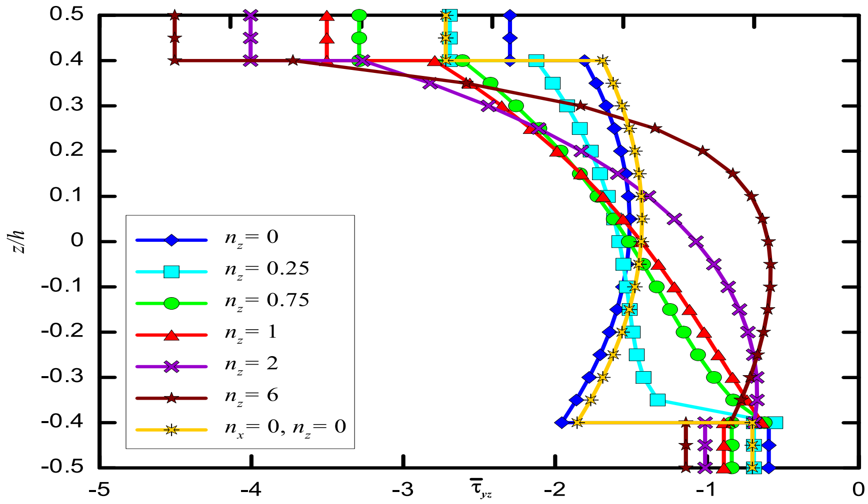

Fig. 14. Variation of non-dimensional shear stress in yz-direction $\bar{\tau}_{yz}$ for (1-8-1) simply supported plate with $n_x = 1$ and different values of n_z.

The effect of volume fraction exponents on the through-thickness normal and shear stresses are clearly shown in *Fig. 10* through *Fig. 14*. Sharp and sudden variation of stresses across layer's interfaces is observed for the isotropic case ($n_x = n_z = 0$), where this effect is due to sudden change in material properties at layer's interface. Sharp peaks at the upper layer interface become smoother as n_z decreases, on the other hand, for lower layer interfaces; sharp peaks become smoother when the value of n_z increases. As n_z decreases, material properties C_1 and C_2 will contribute more than the material properties of M_1 and M_2, where the difference between material properties at the upper layer interface becomes narrower leading to smooth stresses at the upper interface. However, as the value of n_z increases, the material properties M_1 and M_2 will contribute more than the material properties C_1 and C_2. Thus, narrowing the gap in material properties at lower layer interface, consequently leading to smooth stresses at the lower interface.

4 | Conclusions

In this paper, the bending behavior of a rare plate model with an advanced type of material grading using an accurate enough representative element with two shear deformation theories were investigated. In addition, the current study investigated many parameters that affects the bending behavior of the layered plate structure. A layerwise FE approach for the bending analyses of a 3-layered plate consists of a 2D-FGM embedded between isotropic ceramic and metal layers were investigated. The FE approach was carried out using both FSDT for isotropic layers and TSDT for the 2D-FG layer while maintaining a continuous displacement field across the plate's thickness using the layerwise approach.

Initially, the bending analysis verification procedure was carried out using an 8-noded quadrilateral element with 13-DoF per node. Two plate models were compared to approve the accuracy of the present formulation, i.e., single and 3-layered 1D-FGM plates. The bending analysis was investigated mainly for the plate's central deflection. Various parametric studies were carried out to study the effect of the volume fraction exponents in longitudinal and thickness directions. Furthermore, plate's side to thickness ratio, plate's layer configuration and various applied boundary conditions were investigated. Formulations were executed by constructing a computer-based program using Maple Software. From the obtained results, several important conclusions can be summarized as follows:

- *The volume fraction exponent in the thickness direction is proportional to the plate's central deflection. On the other hand, as the exponent in the longitudinal direction increases the deflection of the plate decreases.*
- *Less deflection is observed for high a/h ratios up to 20, where above this value the deflection remains almost constant due to the transition from thick to thin plates.*

- *For the values of volume fraction exponents in thickness direction less than 0.75, less deflection is observed for the high middle layer ratio compared to both the upper and lower layers. On the other hand, for values greater than 0.75 opposite conclusions are obtained.*

Several possible problems appear worth of further investigations, for instance, it is recommended to study the behavior of the present 2D-FGM plate structure with a dynamic load instead of static load, thus examining a plate model, which reflects realistic types of applications. Besides, this work can be also reproduced and validated experimentally while investigating the feasibility of using such materials commercially.

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