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Ranking Aaggregation of Preferences with Common Set of Weights Using Goal Programming Method

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ABSTRACT

In aggregation of preferences system, each decision maker (DM) selects a subset of the alternative and places them in a ranked order. The key issue of the aggregation preference is how to determine the weights associated with different ranking places. To avoid the subjectivity in determining the weights, data envelopment analysis (DEA) is used in Cook and Kress to determine the most favorable weights for each alternative. With respect to DEA-based models, two main criticisms appear in the literature: multiple top-ties and overly diverse weights. DEA models use assignments of the same aggregate value (equal to unity) to evaluate multiple alternatives as efficient. There is no criterion to discriminate among these alternatives in order to construct a ranking of alternatives. furthermore, overly diverse weights can appear, given that each alternative can have its own vector of weights (i.e., the one that maximizes its aggregate value). Thus, the efficiencies of different alternatives obtained by different sets of weights may be unable to be compared and ranked on the same basis In order to solve these two problems above, In order to rank all the alternatives on the same scale, In this paper we proposed an improvement to Kornbluth's approach by introducing an multiple objective linear programming (MOLP) approach for generating a common set of weights in the DEA framework. In order to solve the MOLP model we use a goal programming (GP) model. solving the GP model gives us a common set of weights and then the efficiency scores of candidate can be obtained by using these common weights and finally we can rank all alternative.

1. Introduction

The manner in which a group or social ranking from a set of individual preferences is achieved is an important aspect of the Decision-Making (DM) context. This class of procedures contains either those in which each DM, selects a subset of the alternatives and places them in a ranked order. Let $A = \{A_1, \dots, A_n\}$ be a set of n (n>3) alternatives that have been evaluated by a group of DMs. Each DM gives preferences by selecting a subset of (k) alternatives (or the complete set A) and ranking them from most to least preferred. Weighting scoring rules operate by computing, for each alternative, a score that depends on the rank position of the alternative in the individual's order of preferences. Subsequently, the alternatives are ranked by the sum of scores received. The value obtained by the ith alternative is computed as $V_i = \sum_{j=1}^{k} w_j\, v_{ij}\,.(i = 1, \dots, n)$. Where w_j is the weighting applied to the rank- position

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votes and v_{ij} represents the number of the jth-rank positions votes obtained by alternative ith. In Borda rule, the winner alternative is the one who receives more votes in the first place. The Borda–Kendall rule is considered the origin of this class of preference-aggregation procedures. In Borda rule, the weight assigned to the first place equals to number of alternatives and to the second place is one less than the first place and so on. In spite of the Borda rule has interesting properties in relation to other scoring rules, but the utilization of a fixed scoring vector has weak point such that an alternative that is not the winner with the scoring vector imposed initially could be so if another scoring is used. Thus, the key issue is how to obtain the weighted associated with different places. Cook and Kress [5] state that models involving an imposed set of weights fail to provide a fair overall assessment. Each evaluation that implies the use of an externally imposed scoring vector is somewhat arbitrary. By using Data Envelopment Analysis (DEA) (Charnes et al. [4]), Cook and Kress [5] have proposed a method for estimating preference scores without imposing any fixed weights from outset. This method determines the most favorable weights for each alternative. The principal drawback of this method is very often leads to more than one alternative to be efficient alternative. To avoid this shortcoming and to choose a real winner among the efficient alternative, Cook and Kress [5], proposed to maximize the gap between consecutive weights of the scoring vector so that only one alternative is left efficient alternative .Green et al. [7] applied the cross-efficiency evaluation technique in DEA to get the best efficient alternative. Noguchi et al. [16] also use the cross-efficiency evaluation to get the best alternative and give a strong ordering constraint condition on weights. In order to discriminate among efficient alternatives, Hashimoto [9] addresses an AR/exclusion model based on the concept of super-efficiency proposed in Andersen and Petersen [1]. Obata and Ishii [17] proved that these methods have a weak point that the order of efficient alternatives may be changed by existence of an inefficient alternative. So, they proposed a new method without to need of any information about inefficient candidates to discriminate efficient alternatives. Their method is subsequently extended to rank non-DEA-efficient alternatives by Foroughi and Tamiz [6]. Wang et al. [19] proposed three new models for preference voting and aggregation. Zerafat Angiz, et al. [23] introduces a mathematical method, inspired by DEA methodology, which determines the importance of rank positions according to decision makers in order to reach a more realistic solution. DEA model, it allows each Decision Making Unite (DMU) to measure its efficiency with the weights that are only most favorable for itself. In other words, each DMU chooses the most favorable weighting schemes in order to pursuit its own maximum efficiency. Thus, the efficiencies of different DMUs obtained by different sets of weights may be unable to be compared and ranked on the same basis [20]. Another problem is that there are always more than one DMU to be evaluated as efficient because of the flexibility in the selection of weights, which would cause the problem that all DMUs cannot be fully discriminated. To derive a common set of weights for DMUs, a number of approaches have been proposed in the DEA literature. For example, Ganley and Cubbin [8] determined the common weights by maximizing the sum of the efficiencies of DMUs. Liu and Peng [15] proposed a Common Weights Analysis (CWA) methodology to search for a common set of weights for DMUs. Kao and Hung [11] based on multiple objective nonlinear programming and by using compromise solution approach, proposed a method to generate a common set of weights. Wang, Luo, and Liang [21] suggested ranking DMUs by imposing a minimum weight restriction, which also produces a common set of weights for the DMUs to be compared.

In this paper we proposed an improvement to Kornbluth's [12] approach by introducing a Multiple Objective Linear Programming (MOLP) for finding common weights in DEA. The proposed model which provides the same results as the Cook and Krees [5] model. We use a Goal Programing (GP) (developed by Charnes and Cooper [3]) method for solving the MOLP model. This model has some advantages compared to foregoing models that will be discussed later. The structure of this paper is organized as follows. In the next section we will review tow voting system model. Multiple Objective

Linear Programming and goal programming will be presented in Section 3. we represent our proposed method in Section 4. In Section 4 By tow numerical example we illustrate our proposed method and finally Conclusions will be presented in Section 6.

2. Review of Voting System Model

In this subsection we are going to review Cook & Kress [5] and Obata & Ishii's [17] models in preferential voting system.

2.1. Cook and Krees's Model

In the preferential voting framework each alternative $i(i = 1, \dots, n)$ receives some number $v_{i1}$of first place votes, $v_{i2}$of second place votes ,…, $v_{ik}$of kth place votes we consider ranked voting data which is obtained when voters select and rank more than one alternative. Here, it is assumed that a voter selects k(>0) alternatives from a set of n(≥ k) alternatives and ranks them from top to kth place. Let v_{ij}denotes the number of jth-place ranks that candidate i occupies $(i = 1, \dots, n, j = 1, \dots, k)$ preference score V_I of the alternative i should be calculated as a weighted sum of votes with certain weights, w_j i.e. $V_i = \sum_{j=1}^{k} w_j v_{ij}$. $(i = 1, \dots, n)$.

By using data envelopment analysis (DEA) (develope by charnes et al. [4]), Cook and Kress [5] have proposed a method for estimating preference scores without imposing any fixed weights from outset. Each alternative's score is calculated with their most favorable weights. Their formulation when alternative **p** is under evaluation as the following:

$$\begin{aligned} & V^*{}_p = \mathrm{Max} \sum_{j=1}^{k} w_j v_{pj} \\ & \text{subject to} \\ & \sum_{j=1}^{n} w_j v_{pj} \le 1 \quad , i = 1, .., n \\ & w_j - w_{j+1} \ge d(j, \varepsilon) \quad , \qquad j = 1, \dots, k-1 , \\ & w_k \ge d(k, \varepsilon) \end{aligned} \tag{1}$$

where $d(., \varepsilon)$ called the discrimination intensity function, monotonic increasing in ε and the parameter ε is called the *discriminating factor*. and satisfies $d(.,0) = 0$. Parameter ε is nonnegative. This is solved for each alternative **p**, $p = 1, .., n$. The resulting score V_p^* is the preference score of the alternative $\boldsymbol{p}$. Here, the alternatives in ranked voting systems are regarded as DMUs (Decision Making Units) in DEA, and each DMU is considered to have k outputs (ranked votes) and only one input with amount unity. This is equivalent to the well-known DEA-AR model (See Thompson et al. [18]) The constraints $w_j - w_{j+1} \ge d(j, \varepsilon)$ represent the assurance region (AR) and those constraints are introduced in order that the vote of the higher place may have a greater importance than that of the lower place. By compute dual of model (1) we have following model:

$$
\begin{aligned}
&\beta^*_p = \text{Min} \sum_{i=1}^{n} x_i - \sum_{j=1}^{k} d(j,\varepsilon) y_j \\
&\text{subject to} \\
&\sum_{i=1}^{n} x_i v_{i1} - y_1 \qquad \geq v_{p1} \\
&\sum_{i=1}^{n} x_i v_{ij} + y_{j-1} - y_j \ \geq v_{pj} \quad , \ j = 2, \dots, k \\
&x_i \ \geq o \ , \qquad i = 1, \dots, n \\
&y_j \ \geq o \ , \qquad j = 1, \dots, k
\end{aligned}
\tag{2}
$$

After the problems are solved for all alternatives, several (not only one) alternatives often achieve the maximum attainable score 1. We call these alternatives efficient alternatives. We can judge that the set of efficient alternatives is the top group of alternatives, but cannot single out only one winner among them. Cook and Kress [5] have proposed to maximize the gap between the weights. In model (1), the choice of form for $d(.,\varepsilon)$ and the value of ε are two existing issues. For the discrimination intensity function $d(.,\varepsilon)$ Cook and Kress [5] investigate three special cases of $d(.,\varepsilon)$: $d(.,\varepsilon) = \varepsilon$, $d(.,\varepsilon) = \varepsilon/j$ and $d(.,\varepsilon) = \varepsilon/j!$. Each of them leads to a different winner.

2.2. Obata and Ishii's Model

Obata and Ishii [17] consider that, in order to compare the maximum score obtained by each alternative, it is fair to use weight vectors of the same size. So, they suggest to normalize the most favorable weight vectors for each candidate. The model proposed by these authors for evaluate alternative **p** is as the following:

$$
\begin{aligned}
&\frac{1}{\hat{V}_p^*} = \text{Min} \ \|w\| \\
&\text{subject to} \\
&\sum_{j=1}^{k} w_j v_{pj} = 1 \\
&\sum_{j=1}^{k} w_j v_{ij} \leq 1 \quad , i = 1, \dots, n \ , \ i \neq p, \qquad \text{(a)} \\
&w_j - w_{j+1} \geq d(j,\varepsilon) \ , \qquad j = 1, \dots, k-1 \ , \\
&w_k \geq d(k,\varepsilon)
\end{aligned}
\tag{3}
$$

The normalized preference score $\hat{V}_p^*$ is obtained as a reciprocal of the optimal value and $\|.\|$ is a certain norm. We note tow statements. First, from the context of DEA, all of constraints (a) need not be considered, or we may use only the constraints about efficient alternatives, i.e., (a) can be changed with ($\sum_{j=1}^{k} w_j v_{ij} \leq 1$ for all efficient (i ≠p). Therefore, our method does not use any information about inefficient alternatives and the problem of changing the order of efficient alternatives does not occur. Second, if **p** is inefficient, this problem has no feasible solution. Furthermore, in this model it is necessary to determine the norm and the discrimination intensity functions to use. because types of norm and discrimination intensity functions change the optimal solution.

3. Multiple Objective Linear Programming and Goal Programming

Within the multi-criteria decision aid paradigm, several criteria, objectives or attributes are considered simultaneously. These dimensions are usually conflicting and the Decision-Maker (DM) will look for the solution of the best compromise. Finally, the multi-objective programming problem can be formulated as follows:

$$\begin{aligned} &\text{Max} \ \ \{f_1(x), \dots, f_k(x)\} \\ &\text{subject to} \\ &x \in X \subset \mathbb{R}^n \end{aligned} \tag{4}$$

Where $f_i(x)$ represents the ith objective function and X designates the set of feasible solutions. $X = \{x | g_j(x) \geq 0 \ \ , \ j = 1, \dots, m \ \ \}$.

Definition. 1 $x^* \in X$ is called an efficient solution (or non-dominated solution) iff there does not Exist another $x \in X$ such that $f(x^*) \geq f(x)$.

The Goal Programming (GP) model is one of the well-known multi-objective mathematical programming (MOP) models. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions. More precisely, the GP designed to find a solution that minimizes the deviations between the achievement level of the objectives and the goals set for them. In the case where the goal is surpassed, the deviation will be positive and in the case of the under achievement of the goal, the deviation will be negative. The first GP formulation was developed by Charnes and Cooper [3] and then used by Lee [13] and Lee and Clayton [14]. The popularity of the GP is due to the fact that is a single model and easy to understand and to apply. The standard mathematical formulation of the GP model (see [2]) is as follows:

$$\begin{aligned} &Min \sum_{i=1}^{k} h_i(d_i^- , d_i^+) \\ &\text{subject to} \\ &g_j(x) \geq 0 \qquad , \qquad j = 1, \dots, m \\ &f_i(x) + d_i^- - d_i^+ = b_i \quad , \quad i = 1, \dots, k \\ &d_i^- , d_i^+ \geq 0 \qquad , \qquad i = 1, \dots, k \\ &d_i^- . d_i^+ = 0 \qquad , \qquad i = 1, \dots, k \end{aligned} \tag{5}$$

Where h_i (i=1,..,k) are functions of (d_i^- , d_i^+) , $f_i(x)$ are objectives, b_i are the goals set by the DM for the objectives and d_i^- , d_i^+ are the under-achievement and over-achievement of the jth goal respectively.

4. Our Proposed Method

Kornbluth [12] noticed that the DEA model could be expressed as a multi-objective linear Fractional programming problem. In this section, we present an improvement to Kornbluth's approach [12] by introducing an MOLP. Firstly, the following model is introduced to find efficiency value of alternative **p**.

$$
\begin{aligned}
&V^{*}{}_{p} = \mathrm{Max} \sum_{j=1}^{k} w_j\, v_{pj} - \beta_p^{*} \\
&\text{subject to} \\
&\beta_i^{*} - \sum_{j=1}^{k} w_j\, v_{ij} \geq 0 \quad , \quad i = 1, \ldots, n \\
&w_j - w_{j+1} \geq d(j, \varepsilon) \quad , \quad j = 1, \ldots, k-1 \quad , \\
&w_k \geq d(k, \varepsilon)
\end{aligned} \tag{6}
$$

Where β_i^{*}, $i = 1, \ldots, n$ is the optimum value obtained from model (2) when candidate i is under evaluation?

Theorem 1. The optimum value of the model in (6) is zero and for its optimal solution, say $w^{*} = (w_1^{*}, \ldots, w_k^{*})$**,** we have. $\sum_{j=1}^{k} w^{*}{}_{j}\, v_{pj} = \beta_p^{*}$.

Proof. According to the models (1) and (2) proof is clear. □

Theorem 2. A alternative **p** which is shown to be efficient by model (6), also is efficient in the model (2).

Proof. According to the first inequalities of model (6) we have: $\sum_{j=1}^{k} w^{*}{}_{j}\, v_{pj} \leq \beta_p^{*} \leq 1$**.** Therefore, if $\sum_{j=1}^{k} w^{*}{}_{j}\, v_{pj} = 1$ then $\beta_p^{*} = 1$ and alternative **p** with model (2) is efficient.

According to the model (6) and the proposed approach by Kornbluth [12], The idea behind the identification of the common weights is formulated as the simultaneously maximizing the ratio of outputs to inputs for all projected DMUs. So, we present the following MOLP problem. (See model (8)).

$$
\begin{aligned}
&\mathit{M}\mathrm{ax}\left(\sum_{j=1}^{k} w_j\, v_{1j} - \beta_1^{*}, \ldots, \sum_{j=1}^{k} w_j\, v_{nj} - \beta_n^{*}\right) \\
&\text{subject to} \\
&\beta_i^{*} - \sum_{j=1}^{n} w_j\, v_{ij} \geq 0 \quad , \quad i = 1, \ldots, n \\
&w_j - w_{j+1} \geq d(j, \varepsilon)\,, j = 1, \ldots, k-1 \quad , \\
&w_k \geq d(k, \varepsilon).
\end{aligned} \tag{7}
$$

In order to solve the above MOLP model we use a goal programming with goals equal to $\beta_i^{*}(i = 1, \ldots, n)$. Then the following model to be obtained.

$$
\begin{aligned}
&\mathrm{Min} \sum_{i=1}^{n} d_i^{-} + d_i^{+} \\
&\text{subject to} \\
&\beta_i^{*} - \sum_{i=1}^{n} w_j v_{ij} \geq 0 \quad , \quad j = 1, \ldots, k
\end{aligned} \tag{8}
$$

$$\sum_{i=1}^{n} w_j v_{ij} + d_i^- - d_i^+ = \beta_i^* \quad , \qquad j = 1, \dots, n$$
$$w_j - w_{j+1} \geq d(j, \varepsilon) \, , j = 1, \dots, k-1 \quad ,$$
$$w_k \geq d(k, \varepsilon)$$
$$d_i^-, d_i^+ \geq 0 \qquad i = 1, \dots, n$$
$$w_j \geq 0 \qquad j = 1, \dots, k$$

The last set of constraints in model (5) does not appear in the above model. Moreover, the first and the second set of constraints in model (8) force d_i^+ to take value zero. However, solving the above GP model gives us a common set of weights and then the efficiency scores of candidate **i ,** i=1,…,n can be obtained by using these common weights as. $\sum_{j=1}^{k} {w^*}_j \, v_{ij}$. if for $w^* = (w_1^*, \dots, w_k^*)$ we have $\sum_{j=1}^{k} {w^*}_j \, v_{pj} = 1$ then candidate **p** called efficient. we can be sure that there exists at least one candidate that has a preference score of 1.0

Theorem 2. There is at last one candidate such as $DMU_i(\, i = 1, \dots, n)$ whit $V_i^* = \sum_{j=1}^{k} {w^*}_j \, v_{ij} = \beta_i^*$.

Proof. There is a candidate p (p=1,..,n) for which the first inequality in (8)is binding. Becausethat is not the case, there exists a sufficiently small value $\varepsilon > 0$ for which $\overline{w} = \left(w^* + [\varepsilon, 0, \dots, 0]_{1\times k}^{T}\right)$ satisfies the set of restrictions in (8). On the other hand, the value of $\overline{d}_i^-$ which is associated with $\overline{w}$ and the second restriction will tend to decrease which runs contrary to the optimality of $\overline{d}_i^-$ Therefore, there is a candidate **p** (p=1,..,n) for which we have: $\sum_{j=1}^{k} w_j \, v_{pj} - \beta_p^* = 0$. We know that,$\left(\beta^*, v_p\right)$ is efficient. Therefore,$(1, w)$ is associated with the gradient vector of a supporting hyperplane. Furthermore, this supporting hyperplane must support the PPS at some extreme efficient candidates. Therefore, such a candidates is indicated to be efficient by the model (8).

We need dual of model (8) for a better analysis. Here, we use $\{x_i | i = 1, \dots, n\}$, $\{z_i | i = 1, \dots, n\}$ and $\{y_j | j = 1, \dots, k-1\}$ as the standard dual variable associated with the constraints of the model (8). With compute the dual of model (9) we obtained model (10).

$$Min \sum_{i=1}^{n} \beta_i^* x_i + \sum_{i=1}^{n} \beta_i^* z_i - \sum_{j=1}^{k} d(j, \varepsilon) y_j$$
$$\text{subject to} \sum_{i=1}^{n} x_i v_{i1} + \sum_{i=1}^{n} z_i v_{i1} - y_1 \qquad \leq 0$$
$$\sum_{i=1}^{n} x_i v_{ij} + \sum_{i=1}^{n} z_i v_{i1} + y_{j-1} - y_j \; \leq 0 \quad , \; j = 2, \dots, k$$
$$z_1 \qquad \leq 1 \qquad\qquad (9)$$
$$\ddots$$
$$z_n \leq 1$$
$$-z_1 \qquad \leq 1$$
$$\ddots$$
$$-z_n \leq 1$$

$x_i \leq o$, $i = 1, \dots, n$

$y_j \geq o$, $j = 1, \dots, k$

z_i free

Now first we solve model (9) and (10) the results $w^* = (w_1^*, \dots, w_k^*)$, $d_i^{*-} = (d_1^{*-}, \dots, d_n^{*-})$ and $x_i^* = (x_1^*, \dots, x_n^*)$ are obtained. which are the optimal solutions. Then, we calculate the preference score of the candidate under evaluation (candidate **p**). That is, we have $V_p^* = \sum_{j=1}^{k} w^*{}_j\, v_{pj}$. By the value of $V^*{}_i (i = 1, \dots, n)$ we can rank the voting data.

Definition 2. The preference score of candidate **p** is better than that of candidate **q** if $V_p^* > V_q^*$.

Definition 3. If $V_p^* = V_q^* < 1$ then the preference score of candidate **p** is better than that of candidate **q**, if $d_p^{*-} < d_q^{*-}$

Definition 4. If $V_p^* = V_q^* = 1$ then the preference score of candidate **p** is better than that of candidate **q**, if $x_p^{*-} < x_q^{*-}$.

`4.1. Numerical Example 1

Consider the following example, taken from Jahanshahloo et al. [10] that the case of 10 voters, each of whom is asked to rank 3 out of 10 candidates on a ballot. Let the outcome from the vote be as shown in Table 1.

Table 1. *Votes received in each position.*

Candidate	v_{i1}	v_{i2}	v_{i3}
1	*3*	*0*	*1*
2	*1*	*3*	*2*
3	*1*	*1*	*2*
4	*0*	*1*	*0*
5	*2*	*1*	*1*
6	*1*	*1*	*0*
7	*0*	*1*	*2*
8	*0*	*0*	*0*
9	*1*	*1*	*0*
10	*1*	*1*	*1*

As was mention in this paper, v_{ij} denotes the number of the jth place votes of candidate (i=1,...,m, j=1,...,k). Also we define $DMU_i = (1, v_{i1}, \dots, v_{ik})$ Therefore, we have 6 DMUs with 4 outputs and a single input with one value. We consider $d(j, \varepsilon) = 0$ as Cook and Kress [5] did After solving models (8), $w^* = (w_1^*, w_2^*, w_3^*,) = (0.2858, 0.1430, 0.1426)$. According to the definitions presented and the result obtain of solve model (8) and (9) we can rank candidate. Result Summarized in Table 2.

Table 2. *Result obtained.*

Candidate	V_i^*	d_i^{-*}	x_i^*	Rank
1	1	0	-1	2
2	1	0	-2	1
3	0.714	0.0003	0	4
4	0.1430	0.1070	0	8
5	0.8572	0.0317	0	3
6	0.4288	0.1268	0	6
7	0.4282	0.0718	0	7
8	0.0000	0.0000	0	9
9	0.4288	0.1268	0	6
10	0.5714	0	0	5

4.2. Numerical Example 2 (comparison whit other model)

In this section, we intend to compare proposed model to the mentioned ones. Therefore we apply given data in Obata et al. [17] in Table 3.

Table 3. *Votes received in each position.*

Candidate	First	Second
a	32	10
b	28	20
c	13	36
d	20	27
e	27	19
f	30	8
g	0	30

We consider $d(j, \varepsilon) = 0$ as Cook and Kress [5]. Table 3 contains the results obtained from solving models (8) and (9). Furthermore, did after solving models (8), $w^* = (w_1^*, w_2^*,) = (0.0214, 0.0201)$. In Table 4, results of comparison to other models are summarized.

Candidate	V_i^*	d_i^{-*}	x_i^*	Rank
1	0.8858	0.1150	0	5
2	1	0	-3.6123	1
3	1	0	-0.6043	2
4	0.9707	0	0	3
5	0.9597	0.0025	0	4
6	0.8028	0.1354	0	6
7	0.603	0.0106	1	7

Table 4. *Results of comparison whit other models.*

Candidate	V^* $d(.,\varepsilon)=0$		$d(.,\varepsilon)=\varepsilon$		DEA/AR Exclusion		Ishii & Obata model		Proposed method
	Score	**Order**	**Score**	**Order**	**Score**	**Order**	**Score**	**Order**	**Order**
a	*1*	*1*	*0.9740*	*2*	*1.0745*	*1*	*32*	*1*	*5*
b	*1*	*1*	*1*	*1*	*1.0428*	*2*	*25.714*	*2*	*1*
c	*1*	*1*	*0.8151*	*6*	*1.0208*	*3*	*24.5*	*3*	*2*
d	*0.9693*	*2*	*0.8813*	*5*	*0.9693*	*4*	*Infeasible*	-	*3*
e	*0.9611*	*3*	*0.9605*	*3*	*0.9611*	*5*	*Infeasible*	-	*4*
f	*0.9375*	*4*	*0.8951*	*4*	*0.9375*	*6*	*Infeasible*	-	*6*
g	*0.6122*	*5*	*0.3940*	*7*	*0.6122*	*7*	*Infeasible*	-	*7*

5. Conclusion

Aggregation of preference orders has wide applications in social choice and voting systems. Some of the most widely used methods are based on the determination of an aggregated value for each alternative. How to determine the weights associated with rank positions is an important issue since it will determine the group's solution. Researchers have developed certain procedures in which the weights associated with the votes become variables in the model. Data Envelopment Analysis (DEA) represents one class of such models. The DEA models, allows to each DMU to measure its efficiency with the weights that are only most favorable for itself. In other words, each DMU chooses the most favorable weighting schemes in order to pursuit its own maximum efficiency. Thus, the efficiencies of different DMUs obtained by different sets of weights may be unable to be compared and ranked on the same basis. Another problem is that there are always more than one DMU to be evaluated as efficient because of the flexibility in the selection of weights, which would cause the problem that all DMUs cannot be fully discriminated. In order to solve these two problems this paper proposes the application of goal programming approach for generating common set of weights. Solving linear problems is an advantage of the proposed approach against general approaches in the literature which are based on solving nonlinear problems. Compared to the original DEA model, this approach discriminates in a better way among DMU's in order to yield the less efficient ones. As in the conventional DEA model, it does not require the formulation of n models. In fact, the efficiencies of all DMU's can be calculated by solving a single model, enabling one to evaluate the relative efficiency of every DMU on a common weight basis. Considering that the place of each candidate is of great importance from an economic and managerial point of view, various organizations use voting systems and their main objective is to rank candidates. The ranking approach in this paper can be applied in various real-world settings, especially in the business and managerial section.

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