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Evolutionary Algorithm for Multi-Objective Multi-Index Transportation Problem Under Fuzziness

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ABSTRACT

An Improved Genetic Algorithm (I-GA) for solving multi-objective Fuzzy Multi–Index Multi-objective Transportation Problem (FM-MOTP) is presented. Firstly, we introduce a new structure for the individual to be able to represent all possible feasible solutions. In addition, in order to keep the feasibility of the chromosome, a criterion of the feasibility was designed. Based on this criterion, the crossover and mutation were modified and implemented to generate feasible chromosomes. Secondly, an external archive of Pareto optimal solutions is used, which best conform a Pareto front. For avoiding an overwhelming number of solutions, the algorithm has a finite-sized archive of non-dominated solutions, which is updated iteratively at the presence of new solutions. Finally, the computational studies using two numerical problems, taken from the literature, demonstrate the effectiveness of the proposed algorithm to solve FM-MOTP Problem under fuzziness.

1. Introduction

The transportation problem is a special category of linear programming problem. It has been widely studied in logistics and operations management where distribution of goods and commodities from sources to destinations is an important issue. The task of distributor's decisions can be optimized by reformulating the distribution problem as generalization of the classical transportation problem [1, 2]. The conventional transportation problem can be represented as a mathematical structure which comprises an objective function subject to certain constraints. In classical approach, transporting costs from m sources or wholesalers to n destinations or consumers are to be minimized. Multi-index transportation problems are the extension of conventional transportation problems and are appropriate for solving transportation problems with multiple supply points, multiple demand points as well as problems using diverse modes of transportation demands or delivering different kinds of merchandises. Thus, the forward problem would be more complicated than conventional transportation problems. Hayley [3] considered the multi-index transportation problem and presented an algorithm to solve multi-index transportation problem. Junginer [4], who proposed a set of logic problems to solve multi-index

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transportation problems, has also conducted a detailed investigation regarding the characteristics of multi-index transportation model. Rautman et al. [5] used multi-index transportation model to solve the shipping scheduling and suggested that the employment of such transportation problems model would not only enhance the entire transportation efficiency but also optimize the integral system. Ahuja and Arora [6] presented an algorithm for identifying the efficient cost–time trade off pairs in a multi-index fixed charge bi-criterion transportation problem. This is an exact method of finding solution to this problem.

Since there is instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, a fuzzy representation of the multi-objective multi-index transportation problem has been defined, where the cost matrix data involve many parameters whose possible values may be assigned by the experts. In practice, it is natural to consider that the possible values of these parameters as fuzzy numerical data which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers. Recently, Sakawa [7] introduced the concept of α-problem based on the α-level sets of the fuzzy. Based on this concept, Fuzzy Multi–Index Multi-Objective Transportation Problem (FM-MOTP) can be transformed to M-MOTP at certain degree of α (α-cut level).

This paper presents an improved genetic algorithm for solving multi-objective FM-MOTP. The transportation problem as a special type of the network optimization problems has the special data structure in solution characterized as transportation graph. In encoding transportation problem, we introduce a novel individual structure which is able to represent all possible feasible solutions alternative. Also, in order to keep the feasibility of the chromosomes, a criterion of the feasibility was designed. Based on this criterion, the crossover and mutation were implemented and they can always generate feasible chromosomes. Since there is instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, a fuzzy representation of the multi-index multi-objective transportation problem has been defined, where the input data involve many parameters whose possible values may be assigned by the experts. To avoid an overwhelming number of solutions, the algorithm maintains a finite-sized archive of non-dominated solutions, which gets iteratively updated in the presence of new solutions based on the concept of Pareto-dominance. Finally, we report numerical results in order to establish the actual computational burden of the proposed algorithm and to assess its performances for solving FM-MOTP.

2. Classical Transportation Problem

A certain class of linear programming problem [8] known as transportation type problems, arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the context of determining optimum shipping pattern. For example, a product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of product that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destinations have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origins and it is required that given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known.

The shipping schedule which minimizes the total cost of shipment is to be determined [9-11]. The single objective transportation problem can be formulated as:

Minimize $\quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij},$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, .., m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, .., n,$$

$$x_{ij} \geq 0 \quad \forall i, j.$$

a_i is the quantity of the product available at origin $i; b_j$ is the quantity of the product required at destination $j; c_{ij}$ is the cost of shipping one unit from origin i to destination j.

The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

3. Multi-Objective Transportation Problem

Multi-objective optimization [12] can be defined as: “A vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. Hence, the term “optimize” means finding such a solution which would give the values of all the objective functions acceptable to the designer”. Kasana and Kumar [13] formulated the multi-objective transportation problem as follows:

Consider m origins and n destinations and also the quantities available at each origin and the quantities to be transported to each destination. The total quantities required at the destinations may differ from the total quantities available at the origins. For such situations, the problem is balanced by introducing fictitious origin or destination, whichever is needed in order to get precisely the same quantities at the origins and the destinations. Specifically, a balanced transportation problem is considered as it amounts to no loss of generality.

Let x_{ij} be the quantity to be transported from origin i to destination j and for each fixed objective function l: $c_{ij}^{l}, i = 1,2,...,m, j = 1,2,....,n$ be the units of the parameter required for transporting one unit of the quantity from origin i to the destination j satisfying l objectives. Let us consider m locations (origins) as $O_1,.......O_m$ and n locations (destination) as $D_1,.......D_n$, respectively. Let $a_i \geq 0; i = 1,...,m$ the amount available at i^{th} plant O_i. Let the amount required at j^{th} shop D_j be $b_j \geq 0; j = 1,...N$. Further, the cost of transporting one unit of commodity form i^{th} plant to j^{th} destination is $C^{l}_{ij} : i = 1,..m, j = 1,...n$. If $x_{ij} \geq 0$ be the amount of commodity to be transported form i^{th} plant to j^{th} destination, then the problem is to determine x_{ij} so as to minimize.

$$F(x)=\begin{Bmatrix} F_1=\sum_{i=1}^{M}\sum_{j=1}^{N}C_{ij}^{1}x_{ij}, \\ F_2=\sum_{i=1}^{M}\sum_{j=1}^{N}C_{ij}^{2}x_{ij}, \\ \dots \\ F_l=\sum_{i=1}^{M}\sum_{j=1}^{N}C_{ij}^{l}x_{ij} \end{Bmatrix}$$

Subject to

$$\sum_{j=1}^{n}x_{ij}=a_i, i=1,2,..,m,$$

$$\sum_{i=1}^{m}x_{ij}=b_j, j=1,2,..,n,$$

$$x_{ij}\geq 0 \quad \forall i,j.$$

The necessary and sufficient condition for the existence of feasible solution to transportation problem is:

$$\sum_{i=1}^{M}a_i=\sum_{j=1}^{N}b_j.$$

The constraint $\sum_{j=1}^{n}x_{ij}=a_i, i=1,2,..,m$ & $\sum_{i=1}^{m}x_{ij}=b_j, j=1,2,..,n$ represent $m+n$ equations in MN non negative variables. Each variable x_{ij} appears in exactly two constraints, one is associated with origin and other with destination. The data in matrix from are shown in *Fig. 1*.

	D_1	D_2	....	D_n	
O_1	C_{11}	C_{12}	...	C_{1n}	a_1
O_2	C_{21}	C_{22}	...	C_{2n}	a_2
..	..	..	...	..	..
O_m	C_{m1}	C_{m2}	...	C_{mn}	a_n
	B_1	B_2	....	Bm	

Fig. 1. *Transportation problem.*

4. Multi-Index Bi-Criterion Transportation Problem

An ordinary transportation problem can be written in the form of a two-dimensional table for *i= 1,2,...,n*; *j=1,2,…,m*, as shown in *Fig. 2*. Each cell of this table represents one of the x_{ij} *'s*. When these x_{ij} *'s* are summed along the rows of the table they must equal *bi* and when they are summed down the columns they must equal a_i .

	1	2	..	m	
1	x_{11}	x_{12}		x_{1m}	b_1
2	x_{21}	x_{22}	..	x_{2m}	b_2
..	..	..	..	..	
N	x_{n1}	x_{n2}		x_{nm}	b_n
	a_1	a_2	.....	a_m	

Fig. 2. *Transportation problem.*

An extension of the transportation type of problem was stated by Haley [14], and may be thought of as a block in which the layers in all directions form restricted transportation problem. The solid problem can be set out as a three dimensional block for *i=1,2,...,n*; *j=1,2,…,m*; *k=1,2,.., p*. Each cell of this block represents one of the x_{ijk}'s .When these are summed along the rows (for constant *j* and *k*) they equal A_{jk} . When they are summed along the columns (for constant *k* and *i*) they equal B_{ki} . When they are summed down the heights (for constant *i* and *j*) they equal C_{ij} . The arrangement of x_{ijk}'s and the boundary conditions are shown in *Fig. 3*.

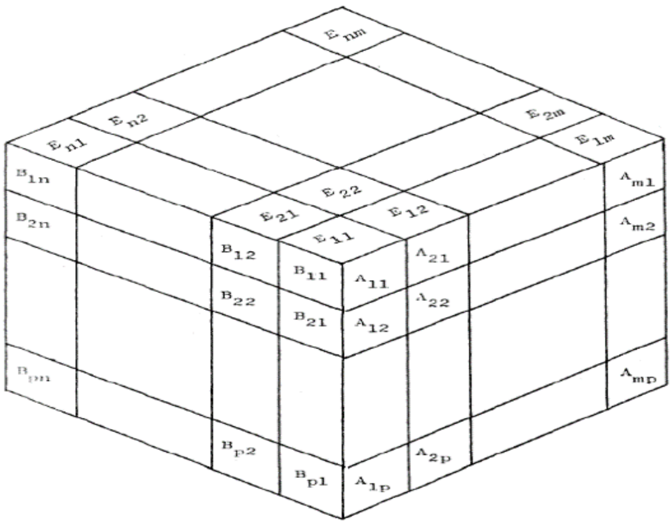


Fig. 3. *The multi-index transportation problem.*

The multi-index problem can be described as minimizing the cost and time of moving a set of *p* different commodities (*k*=*1,2,…,p*) from *n* origins (*i=1,2,…,n*) to *m* destinations (*j*=*1,2,…,m*). The equations then give rise to the conditions on the amount of the various type of combination that is available and required. Alternatively, the same set of restrictions arise when a single commodity has to be moved by different methods e.g. road, rail, sea, canal, air etc. Similarly, the use of intermediate depots may require the use of a multi-index formulation. A special type of problem where the method can be used is the capacitated transportation problem (each variable has an upper bound). Here both the cost and time have equal priorities. If the unit costs of transportation and the associated duration of transportation are given for each supply demand pair of points, then the cost-time trade-off solutions are of interest [15].

5. Formulation of Multi-index Bi-Criterion Transportation Problem

The multi-index transportation problem in which there are *m* origins, *n* destinations and *p* type of commodities to be transported can be formulated as follows. In this problem two objectives one of minimizing the total cost and the other is to minimize the total time of transportation.

$$\text{minimize}\left\{\begin{array}{l} z_1=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}C_{ijk}x_{ijk}, \\ z_2=\max\{t_{ijk}:x_{ijk}>0,(i=1,2,..,m;j=1,2,..,n;k=1,2,..,p)\}\end{array}\right\}$$

Subject to

$$\sum_{i=1}^{m}x_{ijk}=A_{jk},$$

$$\sum_{j=1}^{n}x_{ijk}=B_{ki},$$

$$\sum_{k=1}^{p}x_{ijk}=E_{ij},$$

$$x_{ijk}\geq 0\ ;\ i=1,2,..,m;\ j=1,2,..,n;\ k=1,2,..,p,$$

$$\sum_{j=1}^{n}A_{jk}=\sum_{i=1}^{m}B_{ki},\quad \sum_{k=1}^{p}B_{ki}=\sum_{j=1}^{n}E_{ij},\sum_{j=1}^{m}E_{ij}=\sum_{i=1}^{p}A_{jk},$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p}A_{jk}=\sum_{k=1}^{p}\sum_{i=1}^{m}B_{ki}=\sum_{i=1}^{m}\sum_{j=1}^{n}E_{ij}.$$

This formulation can be transformed to general form as follows:

$$\text{minimize}\left\{z_1=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}C^1_{ijk}x_{ijk},\qquad z_2=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}C^2_{ijk}x_{ijk},\right\}$$

Subject to

$$\sum_{i=1}^{m}x_{ijk}=A_{jk},$$

$$\sum_{j=1}^{n}x_{ijk}=B_{ki},$$

$$\sum_{j=1}^{p}x_{ijk}=E_{ij},$$

$$x_{ijk}\geq 0\ ;\ i=1,2,..,m;\ j=1,2,..,n;\ k=1,2,..,p,$$

$$\sum_{j=1}^{n}A_{jk}=\sum_{i=1}^{m}B_{ki},\quad \sum_{k=1}^{p}B_{ki}=\sum_{j=1}^{n}E_{ij},\sum_{j=1}^{m}E_{ij}=\sum_{i=1}^{p}A_{jk},$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p}A_{jk}=\sum_{k=1}^{p}\sum_{i=1}^{m}B_{ki}=\sum_{i=1}^{m}\sum_{j=1}^{n}E_{ij}.$$

Where

$$C^2_{ijk}=\delta_{ijk}t_{ijk},$$

$$\delta_{ijk}=\begin{cases}1 & \max\{t_{ijk}:x_{ijk}>0,(i=1,2,..,m;j=1,2,..,n;k=1,2,..,p)\} \\ 0 & \text{otherwise}\end{cases}$$

The general multi-index multi-objective transportation problem in which there are *m* origins, *n* destinations and *l* types of objectives and *p* type of commodities to be transported can be formulated as follows:

$$\text{minimize}\begin{Bmatrix} F_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{1} x_{ijk}, \\ F_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{2} x_{ijk}, \\ ...., \\ F_l = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{l} x_{ijk} \end{Bmatrix}$$

Subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk},$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki},$$

$$\sum_{j=1}^{p} x_{ijk} = E_{ij},$$

$$x_{ijk} \geq 0 \; ; \; i = 1,2,..,m; \; j = 1,2,..,n; \; k = 1,2,..,p,$$

$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}, \quad \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{n} E_{ij}, \sum_{j=1}^{m} E_{ij} = \sum_{i=1}^{p} A_{jk},$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m}\sum_{j=1}^{n} E_{ij}.$$

6. Relaxed Multi-Index Multi-Objective Transportation Problem

The author considers some modification of the usual multi-index multi-objective transportation problem, by not allowing upper bounds for the admissible capacity for the total quantity to be sent from i^{th} origin to the j^{th} destination.

$$\text{minimize}\begin{Bmatrix} F_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{1} x_{ijk}, \\ F_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{2} x_{ijk}, \\ ...., \\ F_l = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{l} x_{ijk} \end{Bmatrix}$$

Subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk},$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki},$$

$$x_{ijk} \geq 0 \; ; \; i = 1,2,..,m; \; j = 1,2,..,n; \; k = 1,2,..,p,$$

$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}, \quad \sum_{j=1}^{n}\sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\sum_{i=1}^{m} B_{ki}.$$

Here $i = 1,2,..,m$ are the origins; $j = 1,2,..,n$ are the destinations; $k = 1,2,..,p$ are the various types of commodities; x_{ijk} = the amount of k^{th} type of commodity transported from the i^{th} origin to the j^{th} destination; C^{l}_{ijk} = the l objective per unit amount of k^{th} type of commodity from the i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long as $x_{ijk} > 0$; A_{jk} = the total quantity of k^{th} type of commodity to be sent to the j^{th} destination; B_{ki} = the total quantity of k^{th} type of commodity available at the i^{th} origin.

However, in real life situations, the information available is of imprecise nature and there is an inherent degree of vagueness or uncertainty present in the problem under consideration. In order to tackle this uncertainty the concept of fuzzy sets [16] can be used as an important decision making tool. Imprecision here is meant in the sense of vagueness rather than the lack of knowledge about parameters present in the system. Fuzzy set theory thus provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied. In this work, we have concentrated on the relaxed multi-objective multi-index transportation problem under fuzzy (uncertainties) [17].

7. Relaxed Multi-Index Multi-Objective Transportation Problem under Fuzziness

Naturally, the objective functions and constraints involve many parameters whose possible values may be assigned by the experts. In the traditional approaches, such parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters. In practice, however, it is natural to consider that the possible values of these parameters are often only ambiguously known to experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers. A FM-MOTP is defined as follows:

$$\text{minimize}\left\{\begin{array}{l} F_1=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{C}^1_{ijk}x_{ijk}, \\ F_2=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{C}^2_{ijk}x_{ijk}, \\ ...., \\ F_l=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{C}^l_{ijk}x_{ijk} \end{array}\right\}$$

Subject to

$$\sum_{i=1}^{m}x_{ijk}=A_{jk},$$

$$\sum_{j=1}^{n}x_{ijk}=B_{ki}$$

$$x_{ijk}\geq 0\ ;\ i=1,2,..,m;\ j=1,2,..,n;\ k=1,2,..,p$$

$$\sum_{j=1}^{n}A_{jk}=\sum_{i=1}^{m}B_{ki},\quad \sum_{j=1}^{n}\sum_{k=1}^{p}A_{jk}=\sum_{k=1}^{p}\sum_{i=1}^{m}B_{ki}.$$

Where $F=(F_1,F_2,...,F_l)$ is a vector of *l* objective functions, and *m* and *n* are the number of sources and destinations, respectively. Fuzzy parameters $\tilde{C}^l_{ijk}$ are assumed to be characterized as the fuzzy numbers. The real fuzzy numbers $\tilde{c}$ form a convex continuous fuzzy subset of the real line whose membership function $\mu_{\tilde{c}}(c)$ is defined by:

- A continuous mapping from R^1 to the closed interval [0,1].
- $\mu_{\tilde{c}}(c)=0$ for all $c\in(-\infty,c_1]$.
- Strictly increasing on $[c_1,c_2]$.
- $\mu_{\tilde{c}}(c)=1$ for all $c\in[c_2,c_3]$.

- Strictly decreasing on $[c_3, c_4]$.
- $\mu_{\tilde{c}}(c) = 0$ for all $c \in [c_4, +\infty)$.

Assume that $\tilde{c}$ in the FMOTP are fuzzy numbers whose membership functions are $\mu_{\tilde{c}}(c)$.

Definition 1. (α-level set). The *α*-level set or *α*-cut of the fuzzy numbers $\tilde{c}$ is defined as the ordinary set $L_\alpha(\tilde{c})$ for which the degree of their membership functions exceeds the level $\alpha \in [0,1]$:

$$L_\alpha(\tilde{c}) = \{a \mid \mu_{\tilde{c}}(c) \geq \alpha\}.$$

For a certain degree *α*, the FM-MOTP can be represented as a non-fuzzy αM-MOTP as follows:

$$\text{minimize}\left\{\begin{array}{l} z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{1} x_{ijk}, \\ z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{2} x_{ijk}, \\ \ldots., \\ z_l = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}^{l} x_{ijk} \end{array}\right\}$$

Subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk},$$
$$\sum_{j=1}^{n} x_{ijk} = B_{ki},$$
$$x_{ijk} \geq 0 \;; i = 1,2,..,m; j = 1,2,..,n; k = 1,2,..,p,$$
$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}, \quad \sum_{j=1}^{n}\sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\sum_{i=1}^{m} B_{ki},$$
$$C_{ijk}^{L} \leq C_{ijk}^{l} \leq C_{ijk}^{U} \quad \forall i, j, k, l.$$

By using α -cut level, these fuzzy parameters can be transformed to a crisp one having upper and lower bounds $[c_{ijk}^{L}, c_{ijk}^{U}]$, Where $C_{ijk}^{L} \leq C_{ijk} \leq C_{ijk}^{U}$ gives the lower and upper bound for the parameters C_{ijk} .

Definition 2. (α–Pareto optimal solution). $x^* \in X$ is said to be an *α*–Pareto optimal solution to the (*α*-FM-MOTP), if and only if there does not exist another $x \in X$, $c \in L_\alpha(\tilde{c})$ such that $f_i(x, c_i) \geq f_i(x^*, c_i^*), i = 1,2,..,k,$ with strictly inequality holding for at least one *i*, where the corresponding values of parameters a_i^* are called *α*-level optimal parameters. In real world application problems, input data or related parameters are frequently imprecise/fuzzy owing to incomplete or unobtainable information, so the concept of Pareto stability is introduced for the Pareto optimal solutions of a vector valued problem of the allocation of resources to activities. Naturally, these data (the average profit and cost) involve many controlled parameters whose possible values are vague and uncertain. Consequently, each numerical value in the domain can be assigned a specific "grade of membership" where 0 represents the smallest possible grade of membership, and 1 is the largest possible grade of membership. Thus, fuzzy parameters can be represented by its membership grade ranging between 0 and 1.

The fuzzy numbers shown in *Fig. 4* have been obtained from interviewing decision maker DM or from observing the instabilities in the global market and rate of prices fluctuations. The idea is to transform a problem with these fuzzy parameters to a crisp version using α -cut level. This membership function can be rewritten as follows:

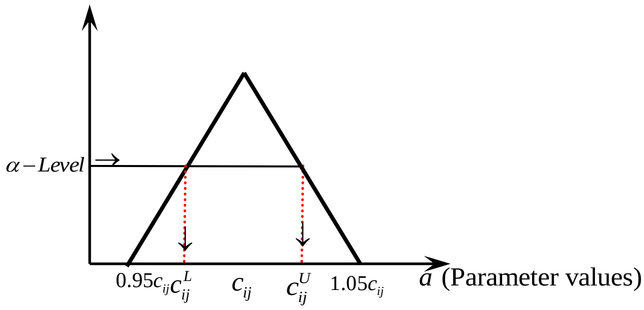


Fig. 4. *Fuzzy numbers of the effectiveness of resource.*

So, every fuzzy parameter $\tilde{a}_{ij}$ can be represented using the membership function in the following equation:

$$\mu(c_{ij}) = \begin{cases} 1, & c = c_{ij} \\ \dfrac{20c}{c_{ij}} - 19 & 0.95c_{ij} \le c \le c_{ij} \\ 21 - \dfrac{20c}{c_{ij}} & c_{ij} \le c \le 1.05c_{ij} \\ 0 & c < 0.95c_{ij} \text{ or } c > 1.05c_{ij} \end{cases}.$$

By using α -cut level, these fuzzy parameters can be transformed to a crisp one having upper and lower bounds $[c_{ij}^L, c_{ij}^U]$, which declared in *Fig. 4*. Consequently, each α -cut level can be represented by the two end points of the alpha level. For example, given the parameter c_{11}^1 has a value of 790, by taking $\alpha = 1$, its value remains as it is as in *Fig. 5(a)*. But for $\alpha = 0$ its value is changed to become $a_{11}^1 \in [750.5, 828.5]$ as in *Fig. 5(b)*. Also, for $\alpha = 0.6$ we get other bounds for the parameter $a_{11}^1 \in [774.2, 805.8]$ as in *Fig. 5(c)*.

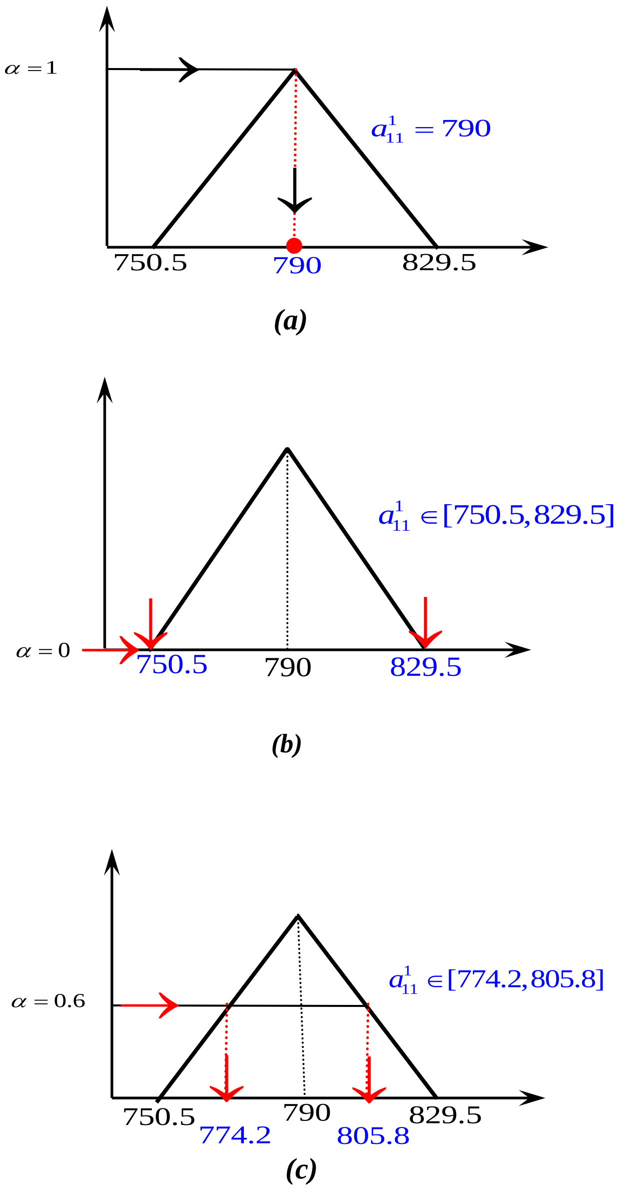


Fig. 5. *Alpha level representation. (a). Alpha=1, (b). Alpha=0.0. (c). Alpha=0.6.*

8. The Proposed Algorithm

Genetic algorithms [18-23] are such a class of population-based algorithms that start with a population of randomly generated candidates and "evolve" toward better solutions by applying genetic operators, modeled on the genetic processes occurring in nature. In the following subsections, we present an Improved Genetic Algorithm (I-GA) for solving the FM-MOTP.

8.1. Initialization Stage

The genetic representation is a kind of data structure which represents the candidate solution of the problem in coding space. In order to form the appropriate design of chromosome using E-GA, first consider each chromosome consists of a sequence of p layers (each layer has a size of $m \times n$, where m is the number of sources and n is the number of destinations). Each individual (*Fig. 6*) consists of *p* layers (*p* is the number of commodities). We generate each layer randomly such that:

$$\sum_{i=1}^{m} x_{ijk} = A_{jk}, \qquad \sum_{j=1}^{n} x_{ijk} = B_{ki} \quad \text{for layer k.}$$

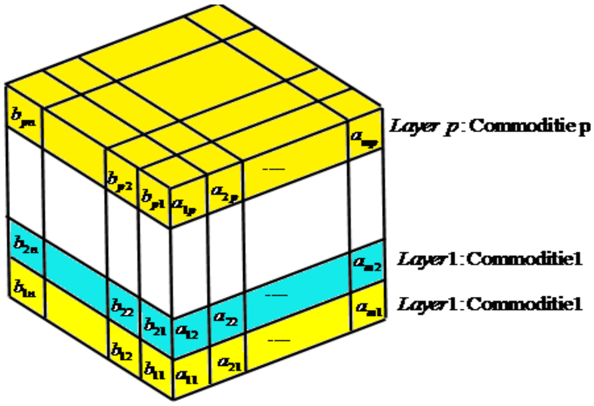
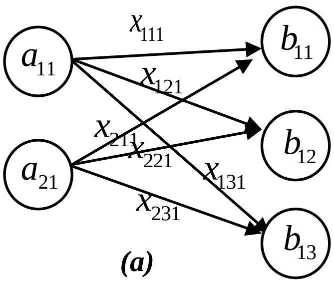


Fig. 6. *Structure of individual for M_MOTP with m sources and n destinations and p commodities.*

In the example problem in *Fig. 7*, we have two supplies (*m=2*) and three demands *(n=3)*, and *(p=2)* commodities. In order to design the appropriate structure of chromosome using I-GA, first consider each individual consists of two layers (i.e. number of commodities *p=2*).

We generate the 1st layer (i.e. 1st commodity) randomly such that, $x_{111} + x_{121} + x_{131} = a_{11}$, $x_{211} + x_{221} + x_{231} = a_{21}$, $x_{111} + x_{211} = b_{11}, x_{121} + x_{221} = b_{12}, x_{131} + x_{231} = b_{13}$.

For the 2nd layer (i.e. 2st commodity) randomly such that, $x_{112} + x_{122} + x_{132} = a_{12}$, $x_{212} + x_{222} + x_{232} = a_{22}$, $x_{112} + x_{212} = b_{21}, x_{122} + x_{222} = b_{22}, x_{132} + x_{232} = b_{23}$.



(a)

	b11	b12	b13
a11	x111	x121	x131
a21	x211	x221	x231

(b)

Fig. 7. *Illustration of layer 1's representation. (a). Transportation graph, (b). Chromosome structure.*

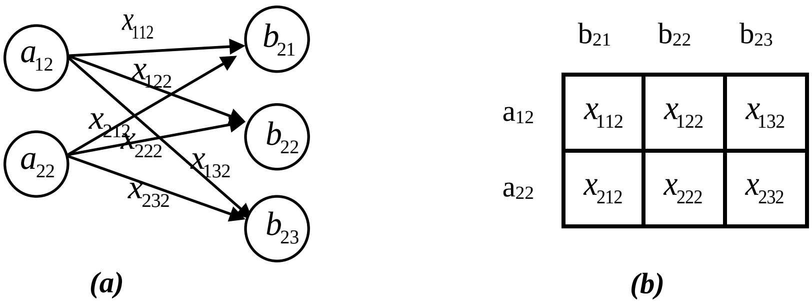


Fig. 8. *Illustration of layer 2's representation. (a) Transportation graph, (b) Chromosome structure.*

It is interesting here to note that all generated chromosome has the following characteristics. All of generated individual are feasible.

The individual length is only $m \times n \times p$, that is $\sum_{i=1}^{m} x_{ijk} = a_{jk} \forall j = 1,2,..n,\ \sum_{j=1}^{n} x_{ijk} = b_{ik} \forall i = 1,2,..m$, for each k layer, where the problem has m sources and n destinations and p commodities.

8.2. Evaluation of Non-Dominated Solutions

A population of size *NIND* can be evaluated according to non-domination concept. Consider a set of population members, having *K (K>1)* objective function values. The following procedure explains the algorithm by which the non-dominated set of solutions can be found.

Step 0: Begin with $i=1$.
Step 1: For all $j = 1,2,..,N$ and $j \neq i$, compare solutions x^i and x^j for domination.
Step 2: If for any j. x^i, is dominated by x^j, mark x^i as "dominated".
Step 3: If all solutions (that is, when $i = N$ is reached) in the set are considered, Go to Step 4, else increment i by one and Go to Step 1.
Step 4: All solutions that are not marked "dominated" are non-dominated solutions.

The algorithm initially locates an externally finite size archive of observed non-dominated solutions.

8.3. Selection Stage

Selection (reproduction) operator is intended to improve the average quality of the population by giving the high-quality chromosomes a better chance to get copied into the next generation. The principle behind GAs is essentially Darwinian natural selection. The selection directs GA search towards promising regions in the search space. We propose a random-weight approach [24, 25] to obtain a variable search direction towards the Pareto frontier. Suppose that we are going to maximize k objective function. The weighted-sum objective is given as follows:

$$f(x) = w_1 f_1(x) + ... + w_k f_k(x) = \sum_{i=1}^{k} w_i f_i(x).$$

Where x is a string (i.e. individual), $f(x)$ is a combined fitness function, $f_i(x)$ is the i^{th} objective function and $\left\{ w_i \mid \sum_{i=1}^{k} w_i = 1 \right\}$ is a constant weight for $f_i(x)$.

We employ roulette wheel selection as selection mechanism in this study. Where, the individuals on each generation are selected for survival into the next generation according to a probability value proportional to the ratio of individual fitness over total population fitness. This means that on average the next generation will receive copies of an individual in proportion to the importance of its fitness value. The probability of variable selection is proportional to its fitness value in the population, according to the formula given by:

$$p(x) = \frac{f(x) - f_{Min}(\psi)}{\sum_{x \in \psi}\{f(x) - f_{Min}(\psi)\}}.$$

Where $p(x)$, selection probability of a string x in a population ψ and

$$f_{Min}(\psi) = Min\{f(x) \mid x \in \psi\}.$$

8.4. Crossover Operators

The goal of crossover is to exchange information between two parent's chromosomes in order to produce two new offspring for the next population. we present a modified uniform crossover, where the crossover is done on each layers, in such a way that offspring is constructed by choosing every layer with a probability P_c from either parent in such a way that $\sum_{i=1}^{m} x_{ijk} = a_{jk} \forall j = 1,2,..n$, $\sum_{j=1}^{n} x_{ijk} = b_{ik} \forall i = 1,2,..m$, for each *k* layer, where the problem has *m* sources and *n* destinations and *p* commodities as shown in *Fig. 8*. It is interesting here to note that all offspring's chromosome are feasible.

4	2	4
2	5	1

6	1	3
0	6	2

Fig. 9. *Graphs visualizing the parents.*

6	0	4
0	7	1

4	3	3
2	4	2

Fig. 10. *Graphs visualizing the crossover offspring for selected layer.*

8.5. Mutation Operators

A mutation operator is a random process where one genotype is replaced by another to generate a new chromosome. For each layer, mutation operator is implemented as follows:

First select two columns randomly from i^{th} individual and then generate two new columns such that $\sum_{i=1}^{m} x_{ijk} = a_{jk} \forall j = 1,2,..n$, $\sum_{j=1}^{n} x_{ijk} = b_{ik} \forall i = 1,2,..m$, for each *k* layer, as shown in *Figs. 7 & 8*.

	*	*
4	2	4
2	5	1

	*	*
4	3	3
2	4	2

Fig. 11. *Graphs visualizing the mutation operators.*

In the example problem (selected layer) in *Fig. 11*, we have four supplies, $a_{12}=10, a_{22}=8$ and $b_{21}=6, b_{22}=7, b_{23}=5$. The first and second column in the determined individual are mutated in such a way that $a_{12}=10, a_{22}=8$ and $b_{21}=6, b_{22}=7, b_{23}=5$. Through this mutation operator, the population's feasibility was preserved.

8.6. Update Function for the Archive $A^{(t)}$

In order to ensure convergence to the Pareto-optimal solutions, we concentrated on how elitist strategy could be introduced in the algorithm. So, we propose an archiving/selection (*Fig. 12*) strategy that guarantees at the same time progress towards the Pareto-optimal set and a covering of the whole range of the non-dominated solutions. This can be done using update function where it gets the new population $P^{(t)}$ and the old archive set $A^{(t-1)}$ and determines the updated one, namely $A^{(t)}$.

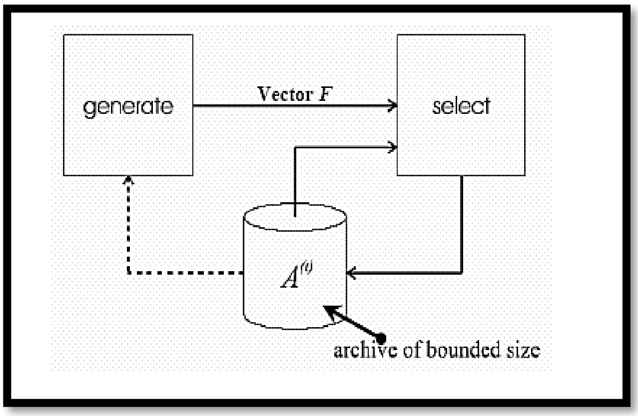


Fig. 12. *Block diagram of archive/selection strategy.*

9. Experimental Results and Discussions

The proposed algorithm was implemented using MATLAB 7.6. To confirm the effectiveness of the algorithm on the transportation problem, two numerical problems were used in the computational studies. *Table 1* lists the parameter setting used in the algorithm for all runs.

Table 1. *Parameters values used by the I-GA for all runs.*

Parameter	Value Problem 1	Problem2
Problem size	4×3	10×5
Number of objectives	*2*	*2*
Population size	*100*	*100*
Mutation rate	*0.02*	*0.02*
Crossover rate	*0.98*	*0.98*
Maximum generation	*200*	*200*

Problem 1. The concept and the algorithm developed will be illustrated numerical example for which $m = 4, n = 3, P = 2$. The 1st objective matrix $C^1(i, j, k)$ is given as follows.

$C^1(i, j, 1)$	4	3	5
	8	6	2
	7	4	1
	9	10	12

$C^1(i, j, 2)$	8	6	3
	5	4	1
	9	2	6
	4	9	3

The 2nd objective matrix $C^1(i, j, k)$ is given as follows.

$C^2(i, j, 1)$	5	6	7
	4	5	2
	7	4	1
	9	10	12

$C^2(i, j, 2)$	10	9	9
	7	9	2
	8	7	9
	8	4	5

The corresponding capacity of the ith plant is $S(i, p)$ and the requirement of the jth warehouse is $D(j, p)$ for various commodity is given below.

$S(i, p)$	9	6
	14	7
	6	5
	7	6

$D(i, p)$	14	12	10
	5	8	11

Fig. 13 shows that the results obtained by proposed approaches, for $\alpha = 1.0$.

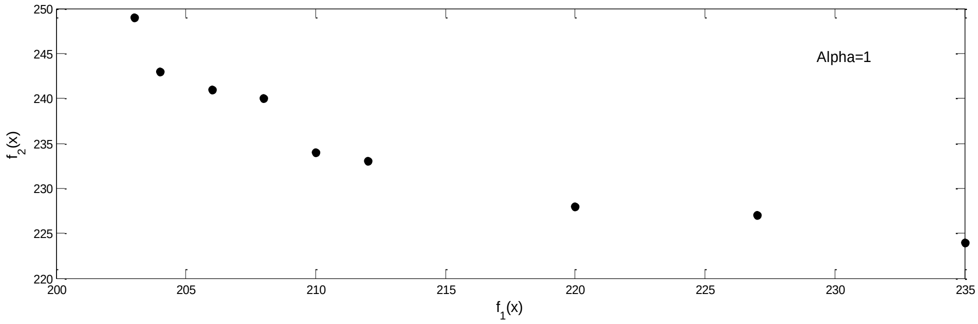


Fig. 13. *Pareto optimal solution for problem 1 in case of alpha=1.*

Using Alpha cut level for $\alpha = 0.8$, *Fig. 14*, shows the obtained Pareto optimal solutions.

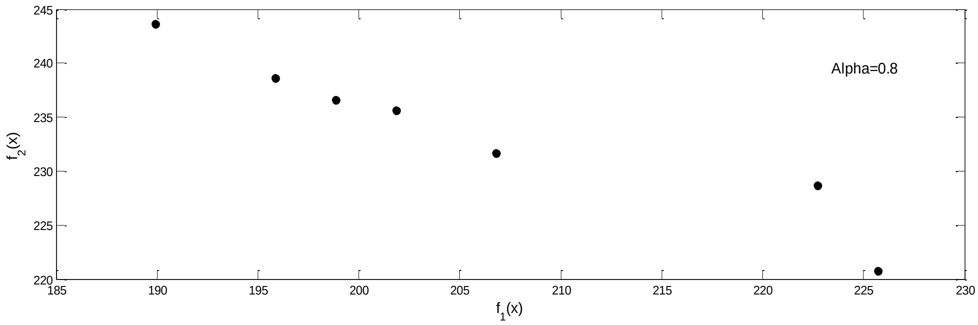


Fig. 14. *Pareto optimal solution for problem 1 in case of alpha=0.8.*

Using Alpha cut level for $\alpha = 0.4$, *Fig. 15* shows the obtained Pareto optimal solutions.

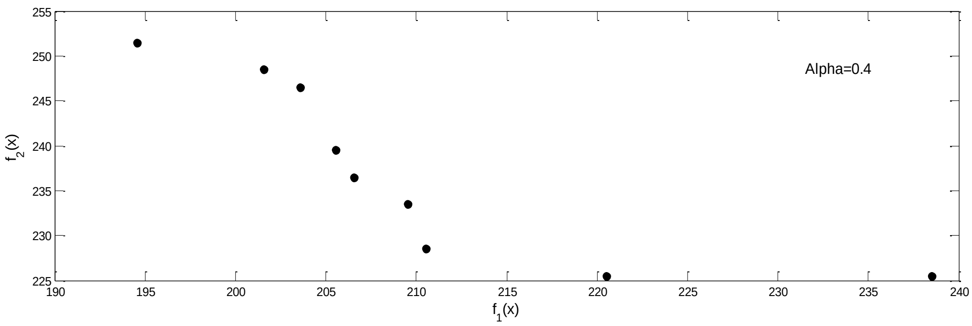


Fig. 15. *Pareto optimal solution for problem 1 in case of alpha=0.4.*

Using Alpha cut level for $\alpha = 0$, *Fig. 16* shows the obtained Pareto optimal solutions.

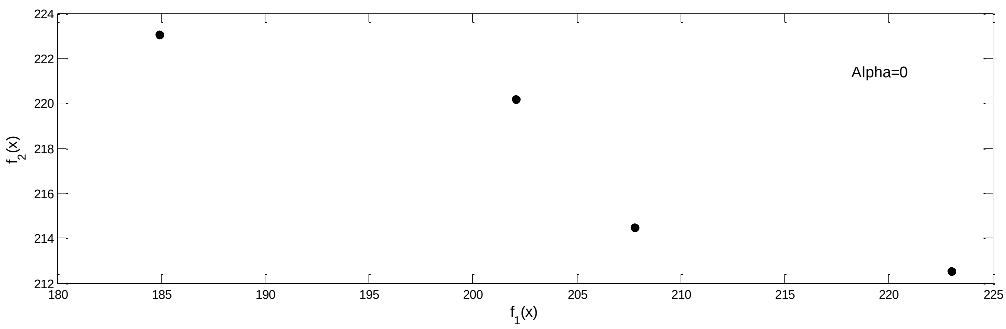


Fig. 16. *Pareto optimal solution for problem 1 in case of alpha=0.*

Problem 2. Concept and the algorithm developed will be illustrated numerical example for which $m = 10, n = 5, P = 2$. The 1st objective matrix $C^1(i, j, k)$ is given as follows.

$C^1(i, j, 1)$	4	5	2	8	3
	8	9	6	2	7
	7	4	1	5	5
	9	10	12	12	14
	1	2	3	4	5
	4	5	2	8	3
	8	9	6	2	7
	7	4	1	5	5
	9	10	12	12	14
	1	2	3	4	5

$C^1(i, j, 2)$	4	4	2	8	3
	8	6	6	2	7
	7	3	1	5	5
	9	10	12	12	14
	1	2	3	4	5
	4	5	2	8	3
	8	9	16	2	6
	7	4	4	5	5
	9	1	12	12	1
	1	2	3	4	5

The 2nd objective matrix $C^2(i, j, k)$ is given as follows.

$C^2(i, j, 1)$	4	3	2	17	3
	8	5	6	2	8
	7	4	0	5	5
	9	0	12	1	14
	1	21	3	4	51
	4	5	2	18	3
	8	9	6	2	7
	7	4	1	5	5
	9	4	12	12	14
	1	5	3	4	5

$C^2(i, j, 2)$	4	5	2	8	3
	8	9	6	2	7
	7	4	1	5	7
	9	10	2	12	14
	1	2	3	4	5
	4	5	2	8	3
	8	9	6	2	7
	7	4	1	5	5
	9	10	2	12	4
	1	2	3	4	5

The corresponding capacity of the i^{th} plant is $S(i, p)$ and the requirement of the j^{th} warehouse is $D(j, p)$ for various commodity is given below.

$S(i, p)$	9	14
	14	15
	6	12
	7	12
	6	10
	8	10
	19	10
	40	13
	45	14
	55	15

$D(i, p)$	50	50	50	50	9
	20	20	20	20	45

Fig. 17 shows that the results obtained by proposed approaches, for $\alpha = 1.0$.

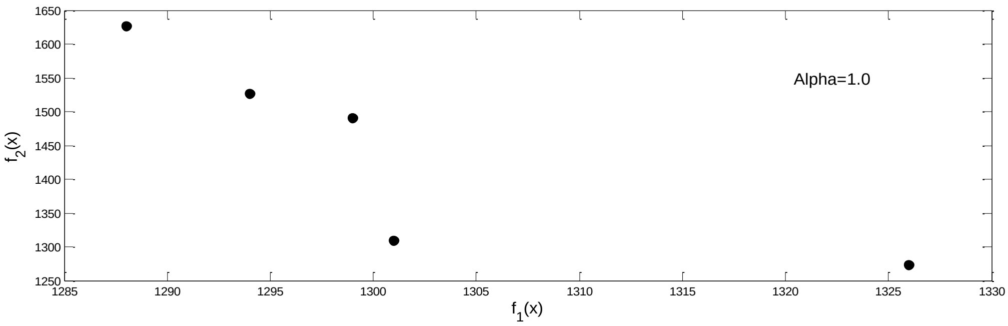


Fig. 17. *Pareto optimal solution for problem 2 in case of alpha=1.*

Using Alpha cut level for $\alpha = 0.8$, *Fig. 18* shows the obtained Pareto optimal solutions.

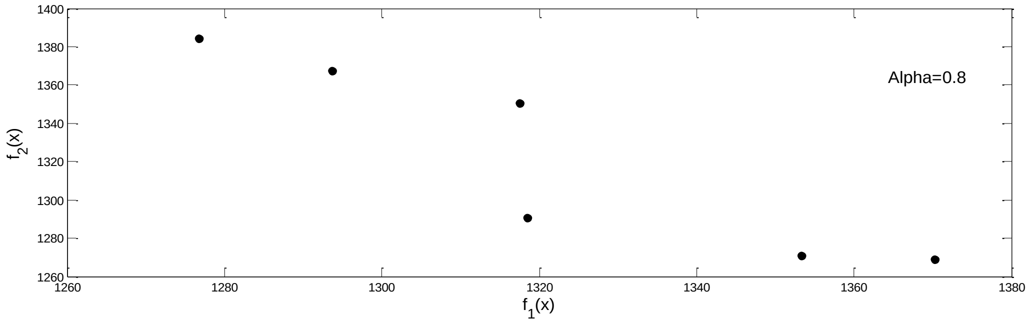


Fig. 18. *Pareto optimal solution for problem 2 in case of alpha=0.8.*

Using Alpha cut level for $\alpha = 0.8$, *Fig. 19* shows the obtained Pareto optimal solutions.

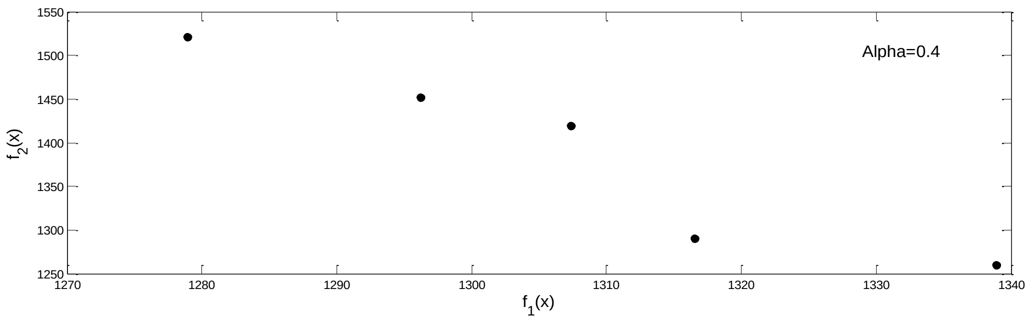


Fig. 19. *Pareto optimal solution for problem 2 in case of alpha=0.4.*

Using Alpha cut level for $\alpha = 0.8$, *Fig. 20* shows the obtained Pareto optimal solutions.

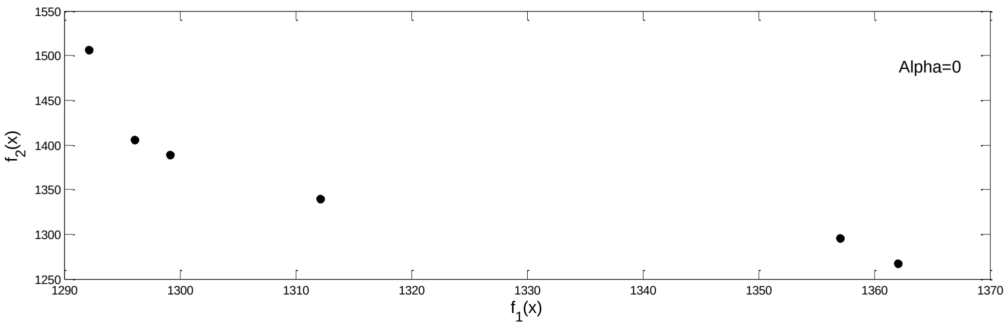


Fig. 20. *Pareto optimal solution for problem 2 in case of alpha=0.*

10. Conclusions

In this paper, we proposed an I-GA for solving fuzzy multi-objective multi-index transportation problem FM-MOTP. Our approach has two characteristic features. Firstly, we introduce a new structure for the individual to be able to represent all possible feasible solutions. Also, in order to keep the feasibility of the chromosome, a criterion of the feasibility was designed. Based on this criterion the crossover and mutation were modified and implemented to generate feasible chromosomes. Secondly, the algorithm is an iterative multi-objective genetic algorithm with an external archive of Pareto optimal solutions that best conform a Pareto frontier. Since there is instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, for this reasons a fuzzy representation of the multi-objective transportation problem has been defined, where the input data involve many parameters whose possible values may be assigned by the experts. To avoid an overwhelming number of solutions, the algorithm maintains a finite-sized archive of non-dominated solutions, which gets iteratively updated in the presence of new solutions based on the concept of Pareto-dominance. Finally, we report numerical results in order to establish the actual computational burden of the proposed algorithm and to assess its performances. The main features of the proposed algorithm could be summarized as follows:

- The proposed approach has been effectively applied to solve the FM-MOTP, with no limitation in handing higher dimensional problems.
- The non-dominated solutions in the obtained Pareto-optimal set are well distributed and have satisfactory diversity characteristics.
- Simulation results verified the validity and the advantages of the proposed approach.

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