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The Voting Linear Assignment Method for Determining Priority and Weights in Solving MADM Problems

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Abstract

Linear Assignment (LAM) is one of the Multi-Attribute Decision Making (MADM) methods that uses integer programming models in the solution process. In this method, only the final priority of the alternatives is determined and the distance between the alternatives is not clear. The purpose of this paper is to modify this method so that instead of the final priority of the alternatives, the final weight of the alternatives is presented. This is done using a linear programming model of Data Envelopment Analysis (DEA). In this paper, we propose a hybrid MADM-DEA method called Linear Assignment Voting (VLAM). The new method is explained with a numerical example. The method will then be implemented on a problem in the real world to demonstrate the application of the method. In this case study, VLAM demonstrates the prioritization of models proposed by experts for the purchase of excavators in a road construction company. Also, based on the results of this method, the weight of the first, second and third priorities are 0.39, 0.35 and 0.26, respectively. These results increase the decision maker's power in making the final decision and choice.

Keywords: Decision support system, Multi-Attribute Decision Making (MADM), Linear Assignment Method (LAM), Data Envelopment Analysis (DEA), Preferential voting.

1 | Introduction

Decision-making is the approved and accurate stating of targets to determine the different and feasible solutions. It also is as to assess their possibilities and consequences, including the results arising from the implementation of each one of the solutions. Finally, managers will have the choice and implementation of these results in the decision-making process. In most cases, decision-making is desirable and is to the satisfaction of the decision-maker, when decision-making is on the basis of investigating multiple criteria. These criteria could be quantitative or qualitative. In Multi–Attribute Decision Making (MADM) methods, which have been under the consideration of researchers in the current decades, instead of employing one optimal measure, a number of criteria are used for appraisal. MADM can be categorized from different perspectives.

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For instance, based on the way of getting preferential information from the Decision Maker (DM), the MADM approaches are classified as aggregation and disaggregation paradigm approaches. Being a direct preference elicitation process, the aggregation approach requires the DM to specify the parameters of the aggregation model. In the disaggregation approach, on the other hand, the DM is asked to make holistic judgments about the decision alternatives. According to another point of view, the MADM approaches can be classified as 1) value functions, 2) outranking relations, 3) "if...then..." decision rules approaches. This classification is based on the aggregation model that is utilized in the methodology. Also as a point of view, MADM consist of methods and models which are divided into two classes, which are compensatory and non-compensatory models. In the non-compensatory models, exchange between the criteria is not permissible; and methods such as, the Mastery Method, Max-Min, Max-Max, the Satisfactory Inclusion Method, Specific Satisfactory Method and the Lexicographic Method and the like, can be designated. Though, exchange between the criteria is permitted in compensatory models. This denotes that a weakness of a criterion could be compensated by the score of another criterion. Models such as, ELECTRE, TOPSIS, Analytical Hierarchy Process (AHP) and Linmap and such examples are models of this classification [54], [18] and [19]. A good number of these models have been developed by researchers in objectives like interval, fuzzy and probabilistic versions [45], [51] and [64]. Many of them can be sought after in [26], [31], [35] and [63]. Alinezhad and Khalili [2] have recently expressed the MADM methods in a volume containing 27 such methods. In many of these methods, the type of information received is such that in some cases, the opinions of experts are not properly reflected. This complex process will reduce the motivation of experts to participate. For example, in the AHP method, information is obtained from experts in the form of matrices of pairwise comparisons. This issue in some cases leads to inconsistencies in judgments and necessitates the study of achieving consistent judgments [37], [1], and [41]. Also with AHP the decision problem is decomposed into a number of subsystems, within which and between which a substantial number of pairwise comparisons need to be completed. This approach has the disadvantage that the number of pairwise comparisons to be made, may become very large, and thus become a lengthy task [43].

One of the methods in MADM literature is Linear Assignment Method (LAM). The various applications of MADM methods such as LAM in industry and society have also led to the revelation of some shortcomings of these methods. In LAM, the size of the difference in the score of the alternatives in each criterion has no role in the final ranking. This feature is considered as a method defect by Keramati et al. [34]. They proposed a modified method with the impact of the distance between the performances of alternatives for alternatives evaluation and ranking. From another point of view, this feature can be considered as the superiority of LAM. Based on this feature, prioritizing alternatives in each criterion is sufficient for decision making. In other words, many costs associated with preparing the decision matrix will be saved. In this paper, we will examine the LAM with this perspective. We will highlight another drawback of LAM. LAM is only a ranking method and does not specify the distance between alternatives with different rankings. In many real-world decision-making issues, this shortcoming can diminish the value of methodological results. The results of MADM methods are in fact available to managers as a tool to support decision making. The fact that a method only prioritizes results and does not specify the distance between alternatives with different priorities reduces the power of the tool. The DM for some reason may not be able to choose the first priority alternative. Now the question is, at what risk can she/he choose the next alternatives? Therefore, MADM methods, which in addition to prioritizing alternatives also determine the weight of each alternative, can be a more powerful tool to support decision making. In this paper, we intend to address this shortcoming by providing a modified version of LAM. Researchers have used the advantages of other methods to cover the weaknesses of Multi-Criteria Decision Making (MCDM) methods. They presented various hybrid methods [5], [6], [7] and [46]. One of the most widely used techniques to cover the weakness of MADM methods is Data Envelopment Analysis (DEA). We will eliminate the mentioned defect for LAM with the help of a DEA model. In fact, in this paper, motivated by LAM modification, a hybrid model called Voting Linear Assignment Method (VLAM) will be presented. VLAM fixes the defect mentioned for LAM. Also,

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instead of an integer programming model to extract the priority of alternatives, a linear programming model will be used to extract the weights of alternatives. In this way, the model solving process will be improved; because there are always more efficient algorithms for solving linear programming than integer programming.

The rest of the paper is organized as follows. In Section 2, the background and literature on the subject of LAM and preferential voting will be presented. In Section 3, we present the hybrid VLAM method. The new method presented is compared with the previous method in Section 4. Section 5 is devoted to providing a numerical example and then Section 6 will discuss the limitations and implications of the study. To demonstrate the applicability of VLAM, this method is implemented in Section 7 on a real-world problem. Conclusions are presented in Section 8.

2 | Background and Subject Literature

In this section, we present the background and subject literatures of one of the MADM methods called LAM. We will also present the background and subject literature of one of the DEA models called the Preferential Voting Model. These methods are to be combined in Section 3 to form the hybrid VLAM.

2.1 | Linear Assignment Method (LAM)

The LAM is not a very prevalent aspect in respect to the constituents of MADM policies in the respective literature. LAM, chiefly relies on an accordance perception and an integer programming technique to determine the ranking order of alternatives. The decisive rendering of LAM was initially introduced by Bernardo and Blin [11], which offered a selection for consumers amongst the multi-attributed categories by utilizing a set of attribute weights as well as astute attribute rankings. In LAM, a ranking order of preference which satiates a rendered concordance measure in the best manner possible can be created. In addition, LAM also presents a process which is compensatory in the way of attribution, combination and interaction. Moreover, it utilizes only ordinal and not fundamental data as inputs in the decision-making procedure [31] and [68]. Thence, there is no requirement to gauge the qualitative attributes. Due to this, the required performance rankings by LAM are more easily acquired than those rankings needed by other MADM methods available in the relative literature [33]. Though, LAM was practical and simple to apply in varied problems, it has been carried out by relatively a few researchers in this literature till date. A decisive version of LAM was utilized by Jahan et al. [32], in a factual selection procedure so as to select and rank the most competent materials for a given engineering module. *Table 1* lists some of the applications of the LAM method by researchers in the MADM literature. This method has also been used in fuzzy and interval versions [8], [9], [10], [12], [13], [29], [40] and [66].

Table 1. Some applications of LAM.

Reference	Field of Application
Emami Saleh et al. [22]	Local Parking Positioning
Shirouyehzad et al. [57]	Ranking of Hotels with Service Quality Approach
Sadeghravesh et al. [55]	Assessment of Combating-Desertification Alternatives
Xian-Ying [67]	Human Resource Planning
Bernardo and Blin [11]	Brand Selection by the Customer
Amiri et al. [3]	Firms Competence Evaluations
Komijan and Koupaei [36]	Education Management
Foroughi and Esfahani [23]	Ranking of Risk Factors in Construction Management
Nourozi and Shariati [49]	Urban Service Management; Locating Fire Stations
Mianabadi and Afshar, [44]	Urban Service Management; Projects of Municipal Water Preparation
LeBlanc and Farhangian [38]	Urban Service Management; Traffic Management
Cao et al. [47]	Optimizing the Marriage Market
Danchick [15]	Bayesian Classification
Dessouky and Kijowski [17]	Production Scheduling
Janani et al. [20]; Ehsanifar et al. [21]	Stock Portfolio and Finance Management

LAM is a sub-set of the compensatory models, whose output, is in a form of a set of rankings; in such a manner that, it gains the essential coordination in the most appropriate mode. In this model, weak criteria must be compensated by other strengthened criteria. Hence, even if the criteria are not independently assessed or evaluated, it is essential that they are involved in the evaluation process. This is called for, as there is a possibility that the correlation between the two criteria in the first instance is in a desirable situation, from the viewpoint of an alternative, whereas, in the second criterion, conditions are undesirable. It is natural that under such conditions, by taking into consideration one of these two criteria shall lead to the fact that, a complete depiction of the functional status of an alternative shall not be provided for [31].

Specifications of this method can be indicated to in the cases hereunder:

- *In this method, by utilizing a simple ranking method for alternatives, causes an exchange between the criteria and evades complex computations.*
- *This method does not require homogeneous scaling of measurement and the criteria can be of any scale.*
- *LAM does not require extensive information, but has the condition of being compensated.*

On the basis of the simple property of response space in the LAM, in addition to considering the entire combination in an implicit manner, the optimal response is extracted in a simple space of convexity. Also, the criterion weight is based on the experts' opinion and is taken from the Delphi method [52]. The steps involved in utilizing this model, in brief, are as follows:

Step 1. The selection of criteria and effective strategies: the selection of criteria and strategies from a wide-range of criteria and strategies in discussion can be based on expert experience, information resources or field studies. In this regard, scientific methods such as the Delphi method can also be used.

Step 2. The estimation of the relative weight of the criteria: strategies and the formation of a Group-Paired Matrix: a questionnaire has been prepared to achieve relative weights. Experts are then asked to rate the effective criteria and strategies obtained from the previous step on a scale of 1 to 9 (e.g., *Table 2*). After formulating the paired matrix of experts (e.g., *Table 3*), by utilizing the geometric mean method; and in assuming that the views of the experts are uniform in degree of importance, the integration of judgments and the Group-Paired Matrix is formed.

Table 2. Preference values for paired comparisons.

Numerical Value	Degree of Importance
9	Extremely Preferred
7	Very Strongly Preferred
5	Strongly Preferred
3	Moderately Preferred
1	Equally Preferred
2,4,6,8	Preference of bits in the above-mentioned

Table 3. Comparison of paired matrix.

Criteria	1	2	…	j	…	n
1	a_{11}	a_{12}	…	a_{1j}	…	a_{1n}
2	a_{21}	a_{22}	…	a_{2j}	…	a_{2n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	a_{i1}	a_{i2}	…	a_{ij}	…	a_{in}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	a_{n1}	a_{n2}	…	a_{nj}	…	a_{nn}

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Step 3. The extraction of priorities based on the principle of the comparison of the Group-Paired Matrix: in this step, *Table 3* is first normalized using *Eq. (1)*. Then, based on the pairwise comparison of the criteria performed by the group experts, the weights of the criteria are extracted. This important issue does have the possibility of being performed by other methods such as, LINMAP, AHP and others alike these.

$$r_{ij} = \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}}. \tag{1}$$

Table 4. Normalized Decision Matrix (NDM).

NDM	Criterion			
	C_1	C_2	…	C_n
Weights	W_1	W_2	…	W_n
A_1	P_{11}	P_{12}	…	P_{1n}
A_1	P_{21}	P_{22}	…	P_{2n}
⋮	⋮	⋮	⋮	⋮
A_m	P_{1m}	P_{2m}	…	P_{mn}

n= number of criteria, m= number of alternative.

Table 5. Ranking matrix of each alternative in each criteria.

Rank↓	Criterion			
	C_1	C_2	…	C_n
1th	A_{11}	A_{12}	…	A_{1n}
2th	A_{21}	A_{22}	…	A_{2n}
⋮	⋮	⋮	⋮	⋮
***m*th**	A_{m1}	A_{m2}	…	A_{mn}

Step 4. Formation of a decision making matrix: in this step: the weighted weights that bear importance in concern with criteria (W_j) and the priorities of strategies (P_{ij}) based on the principle of each criterion, within the framework of the entire Decision Making Matrix (e.g., *Table 4*) is taken into consideration.

Step 5. The ranking of each alternative in lieu of each criterion: after the formation of the Decision Making Matrix: the ranking of strategies $\left(A_i\right)$ is performed in lieu of each criterion $f\left(C_i\right)$, in respect to the incremental or decremented desirability; and in the framework of the matrix $\left(n \times m\right)$, we express the ranking of strategies (e.g., *Table 5*).

Step 6. Formation of the allocation matrix γ: the allocation matrix γ takes into consideration the weight of the estimated criteria, derived from the group-paired comparison and is a square matrix which has *i* row for alternatives and its *k* columns are for ranking. The compilations of the matrix are components of the sum of weights of *i* alternatives that have gained the *k* ranking.

Step 7. Calculating the final rating of each alternative: in this step the final ranking of alternatives is obtained by solving the binary programming *Eq. (2)*. After solving this model and with due attention to the optimal variables which equate to "1", the ranking allocation to the alternative is made.

$$
\begin{aligned}
&\text{Max} \sum_{i=1}^{m}\sum_{k=1}^{m} \gamma_{ik} h_{ik}, \\
&\text{s.t.} \\
&\sum_{k=1}^{m} h_{ik} = 1,\ i = 1,2,\dots,m, \\
&\sum_{i=1}^{m} h_{ik} = 1,\ k = 1,2,\dots,m, \\
&h_{ik} \in \{0,1\},\ i = 1,2,\dots,m;\ k = 1,2,\dots,m.
\end{aligned}
\tag{2}
$$

2.2 | Preferential Voting

Cook and Kress [14] proposed the evaluation of each candidate with the most positive scoring vector in concern with the candidate. Having this intention, and with such a perspective, they introduced the DEA. The DEA/Assurance Region (DEA/AR) model proposed by the above mentioned is as given in *Eq. (4)*.

$$
\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_{or} y_{or}, \\
&\text{s.t.} \\
&\sum_{r=1}^{s} u_{or} y_{jr} \le 1,\quad j = 1,\dots,n, \\
&u_{or} - u_{or+1} \ge d(r,\varepsilon), \quad r = 1,\dots,s-1, \\
&u_{os} \ge d(s,\varepsilon).
\end{aligned}
\tag{3}
$$

Where $\varepsilon \ge 0$ and the functions $d(r,\varepsilon)$ which are known as the discrimination intensity functions are non-negative and non-decreasing in ε and also, $d(r,0) = 0$ for all $r \in \{1,\dots,s\}$. They illustrate that in a special case where $d(r,\varepsilon) = \varepsilon$, their model corresponds to the Consensus model of Borda [16]. When the problems are solved for all the candidates, many candidates and not only a single candidate frequently attain the optimal score possible i.e. "1". This category of candidates is known as "efficient candidates". Though, this group of so-called efficient "candidates" top the group, a solitary fore-runner amongst them cannot be identified for ranking purposes. In order to eliminate this flaw or weakness, Cook and Kress [14] proposed incrementing the gaps between the weights, so that one candidate is denoted as being DEA efficient. This means that a common set of weights be enforced on all the candidates. Hence, Green et al. [28] recommended the utilization of the cross-efficiency evaluation method in DEA for the selection of the victor. Noguchi et al. [48] utilize the cross-efficiency evaluation to achieve the optimum candidate and issue a strengthened constraint condition on the weights. Andersen and Peterson [4] with $d(r,\varepsilon) = \varepsilon$ that ε is a positive non-Archimedean minuscule; developed a super-efficiency model to rank efficient candidates by removing them from Production Possibility Set (PPS). Hashimoto [30] also proposes the utilization of the DEA exclusion model. Wang and Chin [65] suggest a model where the total scores are measured within an interlude. Soltanifar [58] introduces an interval efficiency comprising of efficiencies gained from the optimistic and pessimistic aspects. In this method, a minimax regret-based methodology is utilized for the ranking and comparison of the efficiency intervals of candidates. Obata and Ishii [50] recommend the exclusion of non-efficient candidates and the utilization of normalized weights to distinguish the efficient candidates of DEA. Their method is then prolonged to rank the non-efficient candidates in Foroughi and Tamiz [25] (see also Foroughi et al. [24]). Soltanifar [59] suggests a new method to rank the common weight models of the indexes of only the efficient DMU's of DEA on the basis of *Eq. (4)*. Soltanifar et al. [61] also suggest a new method in order to rate the ranking models for the performance of indexes only for the efficient DEA candidates, based on *Eq. (4)*. Soltanifar and Hosseinzadeh Lotfi [62] utilized *Eq. (4)* to recommend a new Voting Analytic

Hierarchy Process (VAHP) method for the ranking of efficient DMU's. Llamazares and Pena [39] have studied the principle methods recommended in the literature to distinguish the efficient candidates. Their study illustrates the chief conclusion that none of the suggested measures are absolutely substantial. Soltanifar [60], in a scientific and research paper, suggested a voting model for groups having affiliates with disproportionate power and competence. Supplementary applications of the classical voting model can be referred to in Post [53], Galanis et al. [27], Ma et al. [42], Sharafi et al. [56] and like them. In the next section, by utilizing *Eq. (4)*, we shall render a new model for ranking alternatives in MADM, based on LAM.

3 | The Voting Linear Assignment Method (VLAM)

The LAM method is just a prioritization method for alternatives. This means that this method can only determine the rank of each alternative and is not able to provide the appropriate weight to the alternatives. It is very important for the DM to know the weight of the alternatives in order to determine the extent of their differences, despite recognizing the ranking. The weakness of LAM in not determining the weight of alternatives is due to the binary *Eq. (2)* in determining the priority of alternatives. This model can determine the merit or unworthiness of gaining a position by an alternative and is not able to determine the amount of this merit. In this section, we change the Step 7 of the LAM algorithm to achieve a method that, despite prioritizing alternatives, also determines the appropriate weight for them. To achieve this aim, we must consider matrix γ as a voting matrix. So that the data of this matrix as the votes obtained by the alternative located in each row in the position attributed to each column. The difference is that the sum of the votes obtained by an alternative at a polling station is usually an integer; but in this process, since the elements of matrix γ are the sum of the weights of the criteria, this number will be non-integer. Finally, using *Eq. (4)*, which is a preferential voting model based on DEA policy, instead of *Eq. (2)*, which is an integer programming model, despite the priority of alternatives, appropriate weights for the alternatives are also achieved.

$$\begin{aligned}
&\text{Max} \sum_{k=1}^{m} w_{ok}\gamma_{ok}, \\
&\text{s.t.} \\
&\sum_{k=1}^{m} w_{ok}\gamma_{jk} \le 1, \quad j=1,\dots,m, \\
&w_{ok} - w_{ok+1} \ge d(k,\varepsilon), \quad k=1,\dots,m-1, \\
&w_{om} \ge d(m,\varepsilon).
\end{aligned} \tag{4}$$

Determining the discrimination intensity function in *Eq. (4)* is very important; because this function determines the distance between different ranks and has a direct effect on the final weight of each alternative. Inaccuracy in determining this function may also make *Eq. (4)* impossible. If we assume that γ is to be considered as a voting matrix, it should be noted that the proper selection of discrimination intensity function is one of the most important steps in implementing this method. It is also in a well-selected position for this factor that ε proves to be very effectual also. Hence, any deviation in these two can attain diverse results. Cook and Kress [14] suggested an increment in the gap between successive weights of the scoring vector, thereby, causing a reduction in the feasible set of *Eq. (4)* in this manner. Thence, the model brought under consideration by these authors is as rendered in *Eq. (5)*.

$$\begin{aligned}
&\varepsilon^* = \text{Max}\,\varepsilon, \\
&\text{s.t.} \\
&\sum_{k=1}^{m} w_{ok}\gamma_{jk} \le 1, \quad j=1,\dots,m, \\
&w_{ok} - w_{ok+1} \ge d(k,\varepsilon), \quad k=1,\dots,m-1, \\
&w_{om} \ge d(m,\varepsilon), \\
&\varepsilon \ge 0.
\end{aligned} \tag{5}$$

In this paper with due attention to Cook and Kress [14], the discrimination intensity function is utilized in the form of $d(k,\varepsilon)=\varepsilon/k$ and this value is computed for $\varepsilon=\varepsilon^*$. Thus, by considering the maximum power of distinction between the alternatives based on a constant discrimination intensity function, the feasibility of the model is also guaranteed. It is evident that one can also use other appropriate discrimination intensity functions in this particular case.

Theorem 1. Assuming $d(k,\varepsilon)=\varepsilon^*/k; k=1,2,\dots,m$, *Eq. (4)* is always feasible.

Proof. In *Eq. (5)*, assuming $d(k,\varepsilon)=\varepsilon/k; k=1,2,\dots,m$, it is clear that *Eq. (5)* is always feasible by setting all variables to zero. The feasibility of *Eq. (5)* is also a direct result of the feasibility of *Eq. (5)* and this completes the proof.

Based on this explanation, the algorithm of the new VLAM method is as follows:

Step 1. The selection of criteria and effective strategies.

Step 2. The estimation of the relative weight of the criteria, strategies and the formation of a Group-Paired Matrix.

Step 3. The extraction of priorities based on the principle of the comparison of the Group-Paired Matrix.

Step 4. Formation of a Decision Making Matrix.

Step 5. The ranking of each alternative in lieu of each criterion.

Step 6. Formation of the voting matrix γ: voting matrix γ values equal to the sum of the weights are criteria that confirm the suitability of the alternative located in each row in the rank assigned to each column.

Step 7. Determining the final weight of each alternative: in this step, by considering matrix γ as a voting matrix and using *Eq. (4)* to determine the voting results, the final weights of the alternatives are determined.

Step 8. Ranking of alternatives: in this step, the alternatives are ranked based on the weight obtained from Step 7. If several alternatives in Step 7 have a maximum weight of "1", the final weight can be corrected from different ranking models, such as the cross efficiency method, and then the final rank of the alternative can be obtained.

As you can see, the algorithm of this method in Steps 1 to 6 is similar to the LAM algorithm mentioned in Section 2. After the formation of the allocation matrix γ, this matrix known as a voting matrix and signifies the fact that, the matrix is considered as a set of votes of voters in polling stations for each alternative or option that is present in it. But there is this difference here, that, in the usual voting matrixes and within the layers of the matrix, the sum of votes attained from the voters are in varied voting positions. Thereby, the figure is correct; but in the present matrix there are a total of index weights, the digits of which are within an interval of (0, 1]. By inserting the layers of this matrix in *Eq. (4)*, weights of the alternatives shall be extracted; and this shall be a digit within an interval of (0, 1]. Ultimately, the various alternatives are ranked based upon the weights that have been obtained.

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4 | Comparison of the LAM and VLAM

The phases in the LAM and VLAM are summarized in *Table 6*.

Table 6. Compares the phases of LAM and VLAM.

LAM	VLAM
The selection of criteria and effective strategies.	The selection of criteria and effective strategies.
The estimation of the relative weight of the criteria, strategies and the formation of a Group-Paired Matrix for comparison.	The estimation of the relative weight of the criteria, strategies and the formation of a Group-Paired Matrix for comparison.
The extraction of priorities based on the principle of the comparison of the Group-Paired table.	The extraction of priorities based on the principle of the comparison of the Group-Paired table.
Formation of a decision making matrix.	Formation of a decision making matrix.
The ranking of each alternative in lieu of each criterion.	The ranking of each alternative in lieu of each criterion.
Formation of the allocation matrix γ.	Formation of the voting matrix γ.
Calculating the final rating of each alternative by utilizing the binary *Method (2)*.	Calculating the final weight of each alternative by utilizing the linear programming *Model (4)*.

With due attention to *Table 6*, the VLAM differs in its ultimate and final phase from that of LAM. In LAM, the ranking is determined through a binary model; and according to the constraints of this model, the final result appears to be a complete rating of alternatives. But it cannot be claimed that LAM performs the final ranking of alternatives without nodules. Hence, there is a possibility that the binary *Eq. (2)* is confronted with an optimum multi-response trait, in which case, the ranking illustrated by the LAM shall not be exceptional. In the VLAM, initially, the final weight of each alternative is specified by Linear Programming and consequently by taking this weight into consideration, the final ranking is determined. In utilizing a Linear Programming instead of a binary model to determine the final rating of alternatives, is an advantageous feature of the VLAM in respect to LAM. It could be possible that this error is inserted by the VLAM in comparison with LAM. The Linear Programming Model used in this method may lead to a maximum value of "1" for several alternatives, so in determining the ranking of these alternatives we shall face nodules. In answer to these errors, as mentioned above, when the LAM was also utilized, the multiple optimal solutions for the binary model in this method, also illustrated such complexities. Thereby, it cannot be claimed that, the LAM in comparison to that of the VLAM denotes advantages. Similarly, it is also worth noting that, when the VLAM is confronted with the abovementioned problem, it has the flexibility to take advantage of and utilize other ranking models, such as, Super Efficiency and/or Cross-Efficiency Methods etc., and take measures to eliminate this problems. But this flexibility is not present for the LAM and this aspect displays another beneficial characteristic of VLAM in respect to the LAM. Likewise, in the VLAM approach, based on the optimistic policies of DEA, it shall be possible for each alternative to attain the best result in order to obtain the highest possible ranking. This policy shall cause the securing of better final results by the alternatives; this is in a manner where the LAM is deprived of such advantages. The author is hopeful that by utilizing VLAM in real life problems of the world, greater benefits are specified by this method and this method could be utilized as one of the MADM by researchers in industry and society.

5 | Illustration Example

In this section, we wish to implement LAM and VLAM on a simple numerical example. Let us assume that after an accurate implementation of Steps 1 to 4 in the LAM and VLAM proceedings which are mutual, the decision matrix given below shall come to hand:

Table 7. Decision matrix

Criteria	C_1	C_2	C_3	C_4
Weights of criteria	0.2	0.3	0.1	0.4
Alternatives				
A_1	2	5	1100	452
A_2	3	4	1200	458
A_3	2.5	3	1300	460

A comparison of alternatives in terms of different criteria is given in *Fig. 1*. In *Fig. 1*, it is clear that the distance between the scores of the alternatives in each criterion does not differ much. For example, the difference between the alternatives scores according to the C_2 is "1". Also, this difference is 100 according to the C_3. We have already said that the difference between alternative scores in each criterion has no effect on the ranking of alternatives. According to *Fig. 1*, the lack of difference between the alternative scores in each criterion justifies the use of the LAM for this example.

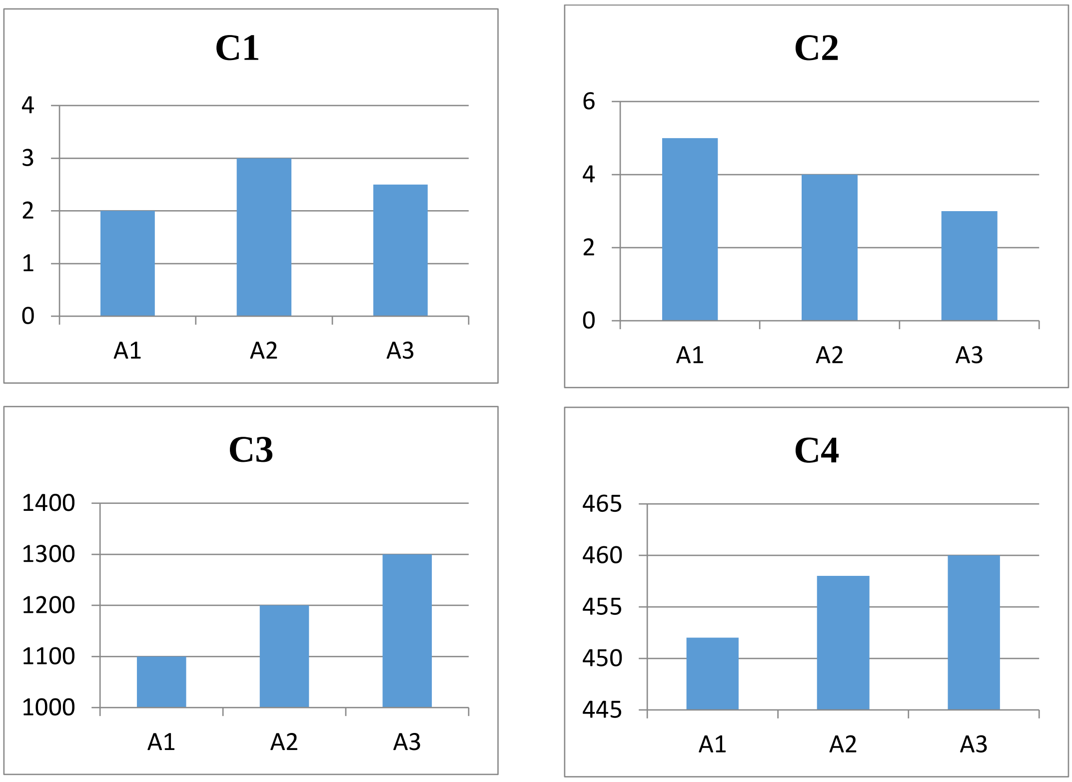

Fig. 1. A comparison of alternatives in terms of different criteria.

Then, in order to execute Step 5, the priorities of each alternative in relative to each of the criteria is determined and illustrated in the matrix hereunder. It must be noted that the entire criteria are of a beneficial kind.

Table 8. Matrix of priorities of each alternative in relative to each of the criteria.

Criteria	C_1	C_2	C_3	C_4
Rank				
1th	A_2	A_1	A_3	A_3
2th	A_3	A_2	A_2	A_2
3th	A_1	A_3	A_1	A_1

After implementing Step 6, the *Table 9* shall be attained.

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Table 9. Matrix γ.

Rank	Rank 1	Rank 2	Rank 3
Alternatives			
A_1	0.3	0	0.7
A_2	0.2	0.8	0
A_3	0.5	0.2	0.3

If *Table 9* is to be considered as the allocation matrix γ, by the utilization of LAM the results in *Table 10* are gained.

Table 10. Final LAM ranking.

Rank 1	Rank 2	Rank 3
A_3	A_2	A_1

Now, we assume that γ is to be taken under consideration as a voting matrix. Thus the final weight of the alternatives is the same as shown in *Fig. 2*. Alternatives can also be easily ranked based on these weights as in *Table 11*.

Table 11. Final VLAM ranking.

Rank 1	Rank 2	Rank 3
A_3	A_2	A_1

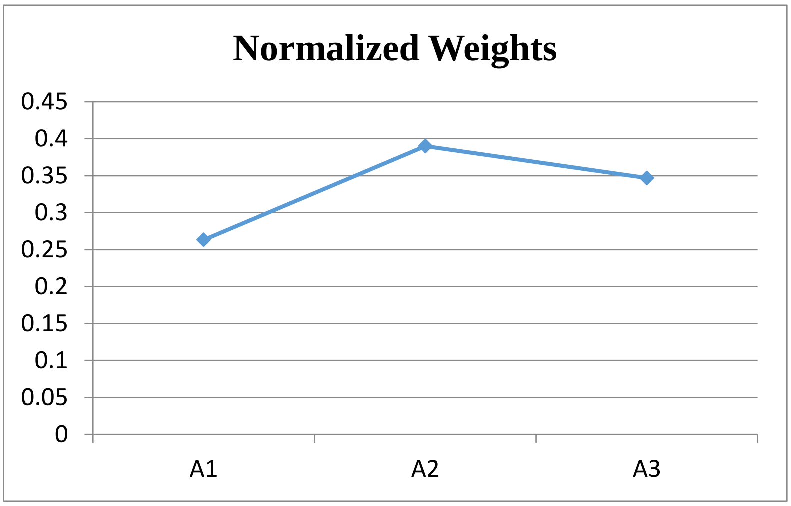

Fig. 2. Final VLAM results by ε=ε*=0.8451.

Fig. 2 shows the main difference between VLAM and LAM. Although the alternatives were ranked in the LAM, no differences were presented between them. For example, if for some reason the DM does not want to choose the alternative with the first rank, with what degree of confidence should he choose the alternative with the second or third rank? The answer to this question is clear in *Fig. 2*, which shows the results of VLAM implementation. In this figure, the weights of A_2, A_3 and A_1 are approximately equal to 0.39, 0.35 and 0.26, respectively. This increases the power of the DM in making better decisions.

The matter which has been mentioned in this section is only a simple example to state the VLAM functional aspects and compare it with LAM. Better results from this proposed method can be secured by utilizing this method for solving real-world problems.

6 | Discussion

VLAM is a modified version of LAM, which is actually a hybrid MADM-DEA method. In this method, one of the shortcomings of LAM has been eliminated by changing the methodology of the method. The main advantage of VLAM over LAM is that it provides weight for the alternatives instead of ranking them. In this way, the DM, while knowing the final ranking of the alternatives, also has the distance of the alternatives. In fact, he runs the risk of not choosing the higher-ranked alternative by weighing the alternatives from each other. This additional information makes them more powerful in decision making. The LAM uses integer programming to prioritize alternatives in the solution process, while VLAM uses a linear programming. Since the complexity of integer programming algorithms is greater than the complexity of linear programming algorithms, VLAM has less computational complexity than LAM. Also, because the VLAM uses a DEA model to determine the final weight of alternatives, if there are equal weight alternatives, there is flexibility to extract the alternatives prioritization process from the ranking models presented in the DEA literature.

An important advantage of VLAM as well as LAM over other MADM methods is the way the decision matrix is formed in them. In these methods, the distance between the scores of each alternative in each criterion has no effect on the final ranking of alternatives. This feature limits the information required from the decision matrix to determining the priority of each alternative in each criterion. This feature saves on the cost of data collection to form a decision matrix. From another point of view, this feature can be considered as one of the limitations of VLAM as well as LAM. If the distance between the different alternatives in each criterion is significantly different, the use of these methods is not allowed. For example, if in the problem presented in Section 5 the alternative score in C_3 was 400, 1299 and 1300, respectively, instead of 1100, 1200 and 1300; the same result was achieved again. This is because VLAM as well as LAM do not use the alternative scoring distance in each criterion for the final ranking. However, each method has advantages and limitations, and choosing the suitable method in each problem can lead to acceptable results, and in fact, choosing the proper method is an art.

7 | Case Study

In this section, to demonstrate the applicability of the new method, the VLAM is implemented on a real-world problem and compared with the results of the LAM.

A road construction company plans to purchase an excavator among A_1, A_2 and A_3 models proposed by experts. Experts provided attributes such as annual maintenance cost (C_1), price (C_2), working weight (C_3), fuel consumption rate (C_4), the complexity level of working with excavator by the operator (C_5), and bucket capacity (C_6).To obtain the weights of the criteria, we ask the expert to present the results of the pairwise comparisons of the criteria, based on a 9-point scale, as shown in *Table 12*. Then we extract the relative weight of the criteria from Expert Choice software (which corresponds to the implementation of the AHP method algorithm) as in *Fig. 3*.

Table 12. Pairwise comparisons of the criteria.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	1/3	1/2	1/5	3	4
C_2	3	1	2	1/4	4	5
C_3	2	1/2	1	1/4	3	4
C_4	5	4	4	1	7	9
C_5	1/3	1/4	1/3	1/7	1	3
C_6	1/4	1/5	1/4	1/9	1/3	9

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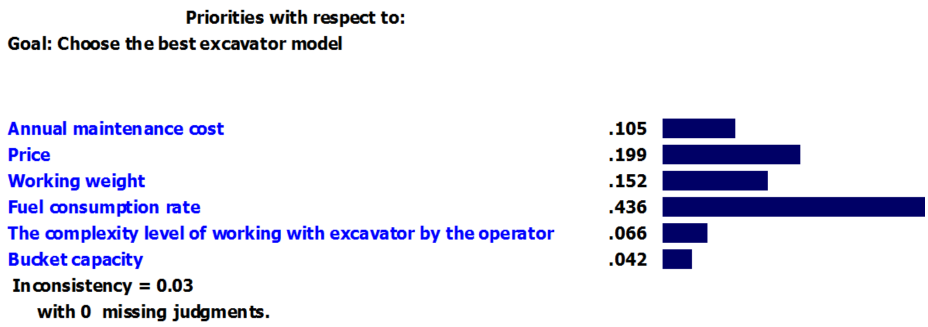

Fig. 3. Results of expert choice software to calculate the relative weights of criteria.

After determining the weights of the criteria, we determine the priority of each alternative in each criterion in interaction with the expert. *Table 13* summarizes the result of this action. The advantage of VLAM is that instead of forming a decision matrix in which the scores of alternatives in the criteria are determined by different methods of data collection, *Table 13* is presented. In this table, only the priority of alternatives in each criterion was obtained from experts. Thus, the required data were prepared at a lower cost and in a shorter time.

Table 13. Priority of each alternative in each criterion.

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
Weights of criteria	0.105	0.199	0.152	0.436	0.066	0.042
Alternatives						
1th	A_1	A_1	A_3	A_2	A_3	A_3
2th	A_2	A_3	A_2	A_3	A_2	A_2
3th	A_3	A_2	A_1	A_1	A_1	A_1

According to priority of the alternatives in each criterion and also the weights of the criteria that were specified in *Table 13*, we form the matrix γ as in *Table 14*. The values of this matrix, considering the weights of the criteria, show the degree of suitability of each alternative in each rank.

Table 14. Matrix γ.

Rank	Rank 1	Rank 2	Rank 3
Alternatives			
A_1	0.304	0.000	0.696
A_2	0.436	0.365	0.199
A_3	0.260	0.635	0.105

To know the LAM results, it is necessary to run *Eq. (2)* for the data in *Table 14*. This only specifies the priority of the alternatives as shown in *Table 15*.

Table 15. Final LAM results for the case study.

Rank 1	Rank 2	Rank 3
A_2	A_3	A_1

But using VLAM, the results of *Fig. 4* will be obtained. It should be noted that the discrimination intensity function is considered as $d(k,\varepsilon)=\varepsilon^*/k$ in which ε^* is obtained by solving *Eq. (5)* equal to 0.8548. In these results, while determining the rank of each alternative, the weight of each alternative was also obtained. A_2, A_3 and A_1 with the weights of 0.389954765, 0.346942755 and 0.263102480, respectively, occupied the first, second and third ranks. This weight determines the difference between each alternative and the other alternatives, which can also be seen in *Fig. 4*. For example, if the DM wants to know the difference between the first alternative and the second or third alternative? If he/she did not choose the first alternative, is the second or third alternative recommended considering other aspects? In other words, the DM will have more information about the status of alternatives. It should be noted that the results of MCDM methods are in fact decision support tools. These tools increase decision-making power when they give more information to the DM. Therefore, VLAM will be useful if selected as a decision support system tool.

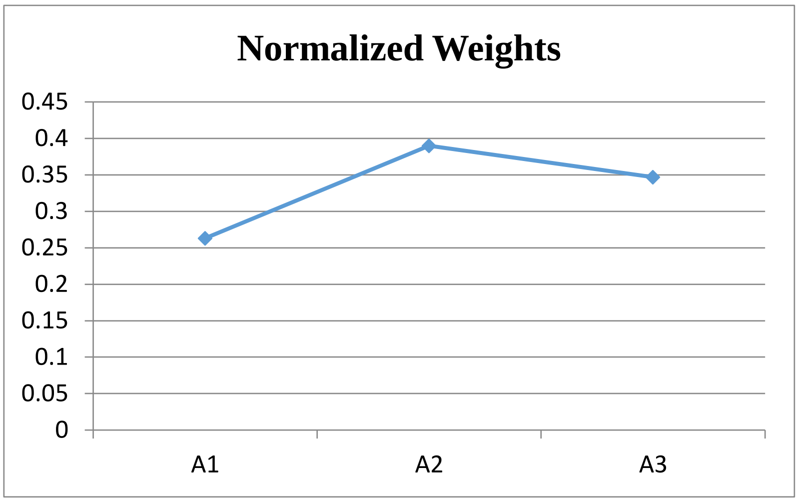

Fig. 4. Final VLAM results for the case study by ε=ε*=0.8548.

8 | Conclusion

In this paper, one of the MADM methods called LAM was examined. In this method, only the priority of each alternative in each criterion is needed, and this results in a significant saving in data collection. But this method only gives the final priority of the alternatives and does not specify the distance between the different alternatives. To correct this defect, we proposed the VLAM, which is a hybrid MADM-DEA method. Implementing this method in the purchase of different models of excavators, in addition to determining the priority of alternatives, also gives the weight of each alternative. This additional information increases the DM's power in the final decision. The application of this method also had limitations, which were discussed in Section 6. One of these limitations is the need to be careful in choosing the method; because this method does not use the distance between the alternatives in each criterion in the final ranking. Therefore, if there is a significant difference between the scores of each alternative in each criterion, the use of the method is not recommended. As a suggestion for future research, it is recommended to modify the method in order to overcome this limitation. The use of uncertainty concepts can also lead to fuzzy and interval versions of this method.

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