



Paper Type: Research Paper



Solving a Multi-Objective Mathematical Model for Aggregate Production Planning in a Closed-Loop Supply Chain under Uncertain Conditions

Ommolbanin Yousefi^{1,*} , Saeed Rezaeei Moghadam¹, Neda Hajheidari¹

¹ Department of Industrial Engineering, Faculty of Malek-e-Ashtar University of Technology, Isfahan, Iran; yousefi_1302@yahoo.com; kasra_kk2002@yahoo.com; Nedahajheidari@gmail.com.

Citation:



Yousefi, O., Rezaeei Moghadam, S., & Hajheidari, N. (2023). Solving a multi-objective mathematical model for aggregate production planning in a closed-loop supply chain under uncertain conditions. *Journal of applied research on industrial engineering*, 10(1), 25-44.

Received: 02/03/2021

Reviewed: 10/04/2021

Revised: 20/06/2021

Accepted: 03/08/2021

Abstract

One of the most important decisions taken in a supply chain is the issue of Aggregate Production Planning (APP) where a program-within a medium time-range- is determined for optimum manufacturing of all products using shared equipment and resources. This research presents a multi-objective model that helps the decision makers to make such decisions. The proposed model comprises four main objectives, the first one of which considers minimizing costs (including costs of manufacturing product, supplying, maintenance, inventory stock shortage, and expenditures related to man power). The second objective is defined as maximizing customers' satisfaction. Minimizing suppliers' satisfaction makes up the third objective and maximizing the quality of the manufactured products constitutes the fourth objective. In this model, the demand parameter is investigated under uncertain conditions; hence, other parameters influenced by this parameter are also presented under uncertain conditions occurring within three differing scenarios. This model is solved through LP-metric and the LINGO v14.0.1.55 software. At first the model is solved by means of numerical example; then it is solved by the actual data that are related to a military industry. Finally, process, variables like inventory level, overtime work hours etc, are valued with the help of closed-loop supply chain of the proposed model.

Keywords: Customers and suppliers' satisfaction, Aggregate production planning, Closed-loop supply chain, Multi-objective mathematical planning model.

1 | Introduction

In the industrial world of today, managers follow two strategic objectives: 1) optimizing operational efficiency of their organization, 2) extending and developing relations with other organizations. One of the relationships which bring about integration and unity among organizations is the concept of supply chain management [1]. Hence, the way the supply chain is designed has become a strategic decision in the process of supply chain management. The design plays a crucial role in the proper performance of supply chain. The problem that makes the design of the supply chain procedure all the more significant is the way waste products are managed efficiently. Put it differently, it is required that in the design of the supply chain, particular attention should be paid to the returned products, a problem which demands the creation of reverse supply chain.



The problem that makes the design of the supply chain procedure all the more significant is the way waste products are managed efficiently. Put it differently, it is required that in the design of the supply chain, particular attention should be paid to the returned products, a problem which demands the creation of reverse supply chain [2].

A suitable supply chain is a competitive advantage for companies and plants and help them to survive in competitive market [3]. An investigation made of supply chain models indicates that a huge portion of research is devoted to studying progressive supply chain. However, since 2005, when reverse supply chain was introduced, we have witnessed a plethora of research studies in the field [4]. While the forward supply chain concerns the flow from raw materials to end products and from manufacturer to consumer, the reverse supply chain concerns the reverse flow from consumer to manufacturer. In addition, some of supply chain concern both of reverse and forward flow. The name of this supply chain is closed-loop [5]. Recycling and reconstructing the products which are spending final stage of their life cycle are importantne in a reverse supply chain. In this regard, after gathering and inspecting the returned products, they are partitioned into recyclable and non-recyclable products. The recyclable products are carried to the recycle centers, where - based on their observable qualities - they undergo re-manufacturing (repair) process. Sometimes separation operation is executed on the returned products and reusable parts are utilized in manufacturing operations. The non-recyclable products are transported to extermination centers where they undergo safe elimination procedure [6]. Uncertainty is one of the key factors in the reverse supply chain that must be controlled, thus, the company could optimize the supply chain function [7]. What is obtained through reverse supply chain has great leverage on producers' programming. In fact, through implementing the above supply chain, there would be economic gains in production costs, usage of new facilities, and optimum exploitation of available resources, all playing effective roles in producer's decision making in the design of Aggregate Production Planning (APP). APP is a process which determines the optimal level of production and stock inventory to meet the demands for product on a long-term basis while simultaneously considers the capacity limitation of the means and resources [6]. One of the main decisions for effectiveness and responsiveness of manufacturing and supply chain systems is APP. In addition, it can help to determine the best way to utilize resources to meet forecasted demand [8].

In this research, investigation is carried out on designing and solving a mathematical model for APP in a closed-loop supply chain of a specific military industry. Military products are usually made up of chemical, mechanical, and electronic components. Inspection of the products in the supply chain of the mentioned industry is of a demolition type, that is, in case where the quality of the products is not confirmed by the employer (IRGC military force, IRI-Iranian-Army, law enforcement units, and foreign purchasers), they are end masse retuned to the supplier. The returned products are either demolished in the reconstruction units or delivered to the producer after reconstruction. Also, in case of the non-usage of the products by the customer after technical warranty expiration (10-15 years), they are dispatched to the Repair and Maintenance (RM) unit so that after undergoing correctional jobs, they are re-dispatched to the customer or producer.

The objective of the present research is to demonstrate how decision making on a specified product's manufacturing and supplying process can help producers in the field. To this end, the producer can manufacture the required products on his/her own plant. Accordingly, he/she should make his decision in the light of the capacity of available means and resources, production expenditures, and the quality of the produced commodity, what amount of products to produce at regular working hours and what amount to produce at overtime working hours. In his/her APP, he/she might also decide on outsourcing the production of a portion of his/her required products to outside suppliers. Such planning becomes of utmost importance as he should make his decision based on such requisite indices and criteria as expenditure, quality level, and prioritization- what amount of each product to delegate from each customer. Along this line, in the proposed model, a win-win relation with the suppliers is deemed essential. Hence, in the model offered, the optimization of the customers' satisfaction is taken into account so that - by considering customers' prioritization - the shortage rate of the unmet demands on

the part of the supplier is kept at minimum. It should be noted that most supply chain models consider minimizing costs, while the supply chain of the proposed model encompasses maximizing customers and suppliers' satisfaction, too. Further, the proposed supply chain, unlike other supply chains, comprises two centers-depot and RM.

Thus, in the design of the extended applied model proposed in this research study, such cases as determining the contribution of suppliers, reconstruction centers, RM measures, production at regular hours, and overtime manufacture of each of the products as well as the amount of dispatched products to each of the customers are among decisions taken in the proposed model. Moreover, such objectives as minimizing producer's costs including production expenditures, cost of retaining and inventory deficit, costs related to supplying products through outsourcing, maximizing the quality of the manufactured products at regular time, overtime, and production by suppliers or procuring products from repair, maintenance and reconstruction centers, where each one has a distinct quality are among parameters considered in the advanced model. Also, special attention is paid to the assessment of suppliers and customers so that optimum satisfaction of these two groups is provided. Therefore, in view of the particular attention paid by the authorities of the concerned military industry to the issue of APP, the current research (regarded as a proper response to the need of that industry) was conducted in the format of a multi-objective mathematical model built on implications of APP in closed-loop supply chain while paying due attention and regard to each of the ingredients of the closed-loop chain. Furthermore, primary exchanges of views with the authorities in the above-named industry and conducting a survey of available research on theoretical foundations have given new dimensions and extensions to the subject under study. To proceed with the research, we first provide a review of the research background. Then, the proposed mathematical model is introduced. Next, the solution to the model is explained. Finally, the model is solved given the extracted data from the concerned industry.

2 | Literature Review

Hafezalkotob et al. [9] developed the cooperative APP. This planning help to decrease the production costs and workforce and inventory costs. These costs constitute a large fraction of the operating costs of many manufacturing plants. In addition, this research quantifies the cost saving and synergy of different coalitions of production plants. The research accomplished by Masud and Hwang [10], on the issue of APP resulted in a model with multiple objectives where the concept of APP with resource limitation is raised and investigated via genetic meta-heuristic algorithm. In their model, such objectives as maximizing profits, bringing costs to a minimum, minimizing the amount of stock inventory, minimizing goods shortage, maximizing the usage of existing means, and minimum amount of overtime work are among factor taken into consideration. Also, references are made to such parameters as man power working hours for manufacturing each unit of product, time to use machine for manufacturing each unit of product, cost of manufacturing each unit of product, overtime cost of manufacturing each product unit, machine capacity at regular manufacturing hour for each unit of product and so on. Ghahremani-Nahr et al. [3] proposed a mathematical model of a multi-product multi-period multi echelon closed-loop supply chain network design under uncertainty. In this paper, the quantities of products and raw material transported between the supply chain entities in each period by considering different transportation mode, the number and locations of the potential facilities, the shortage of products in each period, and the inventory of products in warehouses and plants with considering discount and uncertainty parameters are determined. In addition, the robust possibility optimization approach was used to control the uncertainty parameter.

Cheraghalikhani et al. [11] conducted a literature review on APP. They accomplished a comprehensive classification of APP from two perspectives. In the first perspective, they considered the level of uncertainty existing in the APP model in addition to the number of objective functions that a model contains, whereas, in the second perspective, besides primary issues in APP models, further issues are considered e.g., multiple product item, labor characteristics, degree of DM satisfaction from solution, product characteristics, setup, multiple manufacturing plant, time value of money, financial concepts, supply chain concepts as well as multiple product market.

Hatefi et al. [12] developed a novel mathematical model. In their model, network design decisions integrate in both forward and reverse flows drawing upon reliability concepts. Reliability concepts confront with resource disruptions. Owing to the importance of the role of hybrid distribution-collection resources, in both forward and reverse flows, the authors assumed that they might be randomly disrupted. Random resource disruptions give rise to risks that might be related to the epistemic uncertainties in the model parameters. The proposed model preserves an integrated forward-reverse logistics network against them. To deal with random resource disruptions, two effectively reliable strategies are considered: 1) locating reliable and unreliable hybrid resources to deal with disruption strikes, 2) Unreliable hybrid resources might lose a percentage of their capacities because they are permitted to be partially disrupted. In the end, several numerical experiments have been proposed along with sensitivity analysis. These experimentations clarify the importance, applicability, and effectiveness of the developed model.

Ghorbani et al. [13] proposed a fuzzy goal programming-based approach. Through their approach, they solve a multi-objective mathematical model of reverse SC design while considering three objective functions. Objective functions minimize the recycling cost of the product and the rate of the waste made through the recycle process. In the end, a numerical example is conducted to illustrate the effectiveness of the model. Rivaz et al. [14] suggested a new model based on fuzzy goal programming. They focused on MOTPs (a special type of multi objective programming problems). Given that, there does not usually exist an optimal solution that would simultaneously satisfy all objectives in multi objective problems, the best way in this situation is seeking suitable compromise solutions for such problems. In addition, they tried to vary the weights in the new model and obtain the different solutions.

Khalifa [15] surveyed Multi-criteria De Novo Linear Programming (F-MDNLP) problems. In this research, the fuzzy goal programming approach has considered as a suitable approach to obtaining α -optimal compromise solution and to achieve satisfactory results for the DM. According to this topic, author tried to use of fuzzy goal programming approach. In this approach the decision maker's role only was the evaluation of the α – efficient solutions to limit the influences of his/ her incomplete knowledge about the problem domain.

Baykasoglu [16] made an investigation of APP with multiple objectives within tabu search meta-heuristic algorithm. In this study, APP is defined as programming for middle-term capacity of a 2 to 18-month planning span. However, in the light of the industry type and the organization products, the timing can change and encompass longer spans. In this model, such decision variables as product inventory in each period, returned products, number of work force at each period, and the profit level are presented and discussed.

Leung et al. [17] developed a decision back-up system for solving multi-objective mixed- integer model of APP - through adopting an ideal planning method. The model addresses overall manufacturing products, manufacturing units, and manufacturing periods, minimizing work force in the factory in the periods under study, bringing inventory shortage to a minimum, thus minimizing the returned product levels etc.

Gholamian et al. [18] produced one research on APP of multi-products, multi-objectives, and a total of seven units in a supply chain under uncertain conditions adopting a phase approach of multi-objective optimization. In their model, phase parameters include cost of each regular and overtime working hours, cost related to suppliers as against each unit of raw material, cost of transportation from presenter, cost of raw material procured by supplier, cost of hiring, firing and training of the personnel, cost of holding product inventory, cost of transporting goods to customers, cost of holding raw material, penalty cost of deficit in product dispatched to customer, sale price of each unit of product to customers as well as the number of requests made by customer. Also definite parameters include maximum product procured from supplier, machine-hour expended for manufacturing each product unit, maximum machine capacity, warehouse space for each product unit, warehouse space for each unit of primary material,

maximum available space in warehouse, available regular and overtime work hour, number of available workforce, required delay time for carrying raw material, and permissible shortage, In this model, decision variables consist of number of products produced at regular and overtime work hours, number of products by suppliers, number of personnel, number of primary materials, quality level of personnel, number of end products sent to customers, inventory of final product as well as deficit in product inventory.

Mirzapour et al. [19] made a study of multi-objective robust optimization model of multi-product APP in a supply chain under uncertain conditions. In this research, the supply chain includes numerous suppliers, producers, and customers and a discussion on multi-period, multi-product APP under uncertain conditions is presented. This model proposes a multi-objective non-linear programming scheme for the first time for a new mixed-integer within robust optimization approach while simultaneously considering conflicting objectives in a supply chain under uncertain conditions. The first objective includes minimizing production cost, hiring, firing and training costs, cost related to primary material, cost of holding product inventory as well as transportation and shortage costs. The second objective concentrates on minimizing total maximum shortage rate among customers' place of residence throughout the designated period paying special attention to customers' satisfaction. Also, taken into consideration in this study is the work level, laborers' productivity, over time, contractual work, storage capacity, and time parameters. Eventually, the proposed model is solved as an integer-programming model.

Rahimi et al. [20] proposed a robust optimization model for multi-objective multi-period supply chain planning under uncertainty considering quantity discounts. In this model the current profit and company's expected profit maximize respectively, by making a balance between the total costs of the supply chain and the distributor company's revenues of selling products and by, introducing brands and taking the risk of loss on it.

Zanjani et al. [21] developed a multi-objective Robust Mixed-Integer Linear Programming (RMILP) model. This model is related to Hybrid Flow Shop (HFS) scheduling. They focused on this topic because it has good adaptability with most real-world applications including innumerable cases of uncertainty of parameters that would influence jobs processing when the schedule is executed. The developed model is able to assign a set of jobs to available machines in order to obtain the best trade-off between two objectives including total tardiness and make span under uncertain parameters.

Mirzapour Al-e-hashem et al. [22] conducted a study on an efficient algorithm for solving robust multi-objective APP under uncertainty circumstances. In their research study, they presented a multi-objective model for solving the problem of an APP extended over a few periods of multi-products for a middle-term perspective under uncertain conditions. In this model, the first objective is defined as minimizing expected overall value and cost of inventory quantity, cost of overtime and contractual work, returned orders, machinery and warehouse capacities. The second objective function is expressed as minimizing shortages among all customers' regions. Finally, the third objective function considers maximizing laborers' productivity, weighted mean of productivity level in all factories throughout the designated period. At this stage, the model is solved through a genetic algorithm where the obtained results demonstrate the model's efficiency

Mirzapour Al-e-hashem et al. [23] seek to develop an APP model in a green supply chain for several time periods and multi-products in a green supply chain over a middle term perspective assuming demand uncertainty. The proposed model highlights such features as transportation costs, relations between delay time to delivery and transportation costs, and the discount rate for encouraging manufacturer for higher number of orders. This model for the first time employs a non-linear mixed-integer programming.

Mulvey [24] has introduced a robust optimization framework which includes two robust types. This method consists of robust solution (a solution almost optimal in all scenarios) and robust model (a model having almost plausible answer in all scenarios). In this method, optimization is generally defined as a penalty objective function, which situation is considered both for the robust model and the robust solution.

The objective function is also weighted by parameters of uncertainty and in the objective function and by the afore-mentioned restrictions. The robust optimization method presented by Mulvey is, in fact, a model developed from randomized planning. The method comes about as a result of replacing the expected classic (traditional) minimization of cost function with a penalty objective function having clear references to changeable costs.

In what follows, the robust optimization method is briefly explained [25]. Consider the following linear programming model which includes randomized parameters:

$$\text{Min } c^T x + d^T y. \quad (1)$$

$$\text{Subject to :} \quad (2)$$

$$Ax = b. \quad (3)$$

$$Bx + Cy = e. \quad (4)$$

$$x, y \geq 0. \quad (5)$$

In the above model, the following assumptions are observed:

x: Decision variables to be determined under model's uncertain conditions.

B, C, e: Randomized matrix of technology coefficient and right-hand values.

N: Set of scenario $n \in N \{1, 2, \dots\}$ relative to under randomized model's parameters of uncertainty character, with any scenario, is a subset of $n \in N$. For sets of scenarios $P_n = (n \sum P_n = 1)$ and probability of all scenarios (dn, Bn, Cn, en), values 1 to n are scenario N members.

B, C, e explained above, under uncertain conditions are in Bn, Cn, en forms for any scenario $n \in N$. Also, y defined as control variable whose variables are under concerned scenarios.

Therefore, yn for any n causes n implausibility. If the model is plausible, then σ_n holds true. After all, parameters of uncertainty nature indicate non-plausibility of the model under any scenario n.

If the model is plausible, then σ_n equals zero; otherwise σ_n is of positive value, based on Eq. (7).

Robust optimization model is formulated as follows:

$$\text{Min } \sigma(x, y_1, y_2, \dots, y_n) + w \times p(\sigma_1, \sigma_2, \dots, \sigma_n). \quad (6)$$

$$\text{Subject to :} \quad (7)$$

$$Ax = b. \quad (8)$$

$$B_n x + C_n y_n + \sigma_n = e_n \text{ for all } n \in N. \quad (9)$$

$$x \geq 0, y_n \geq 0, \sigma_n \geq 0 \text{ for all } n \in N. \quad (10)$$

The first part of the model indicates robust solution, the important decision of the decider is not "DISLIKE", what is intended is to reduce costs and risk level. While the second part indicates robust model intending to prevent non-plausibility of the model.

$\Psi_n = f(x, y)$ expresses profit or cost function for scenario n.

High variance for $\Psi_n = f(x, y_n)$ denotes that the solution involves high risk decision. In other words, a minor change in the value of parameters can bring about a major change in the function value [22].

$$\sum_{n \in N} P_n \Psi_n + \lambda \sum_{n \in N} P_n (\Psi_n + \sum_{nl \in N} P_{nl} \Psi_{nl})^2 = \sigma. \quad (11)$$

Table 1 also summarizes these researches. According to this table, all of other mentioned researches parallel with this research. Present study is an APP that is done in a closed-loop supply chain. In addition, this study is considered all of objectives of supply chain and uncertain conditions and, was done at a high-tech industry.

Table 1. A summary of done studies.

Authors	Years	Objectives		Process		Type of Demand		Type of Supply Chain	
		One	Multi	Supply Chain	Producer	Uncertain	Certain	Reverse	Forward
Ghahremani-Nahr et al. [3]	2020	✓		✓		✓		✓	✓
Jang and Chung [8]	2020	✓		✓		✓			✓
Rivaz et al. [14]	2020		✓	✓			✓		✓
Rahimi et al. [20]	2018		✓	✓		✓			✓
Gholamian et al. [18]	2015		✓		✓		✓		
Mirzapour et al. [22]	2012		✓		✓	✓			
Mirzapour et al. [19]	2011		✓		✓	✓			✓
Pan and Nagi [25]	2010		✓	✓		✓			✓
Baykasoglu [16]	2001		✓		✓		✓		
Mulvey [24]	1995	✓		✓		✓			✓
Masud and Hwang [10]			✓		✓		✓		

3 | The Proposed Mathematical Model

This model is a supply chain that has three levels- producer, consumers and a center for reconstruction (depot), RM. In this chain, a producer starts out by sending several merchandises to customers. The manufacturer produces a part of customers' demands at regular and overtime work hours, whilst the remaining demands are delegated to outside suppliers. Eventually the goods delivered to the customers, in case they are defective, are returned by customers to the depot center, where, after undergoing correctional procedures, are sent back to the producer, such that in later cycles, they are re-sent to the customers.

Additionally, when the expiry data of the products' warranty approaches, they are shipped to the depot center by the customers, and if possible, after undergoing necessary repairs and corrections, are re-sent to the customers; otherwise, the products are de-assembled and returned to the producer. So, the supply chain of the proposed model is reverse and forward, simultaneously and it is a closed-loop supply chain. A graphic representation of this chain is provided in Fig. 1.

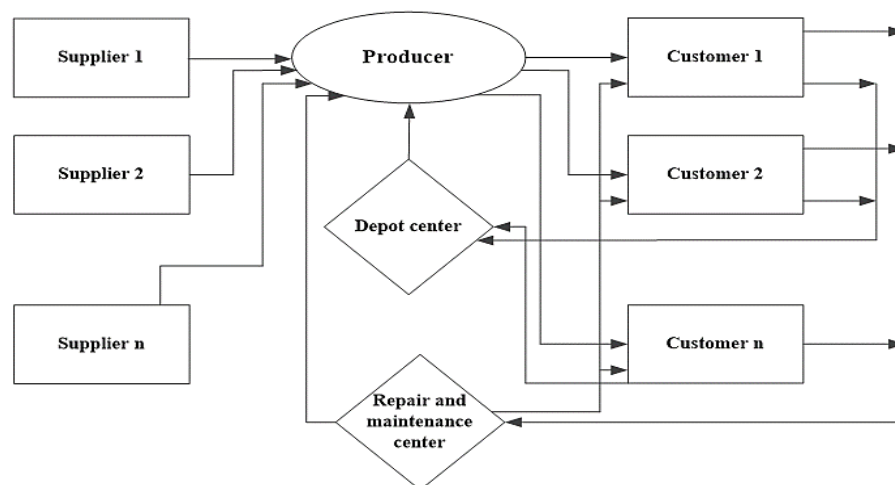


Fig. 1. Network with eight vertices.

3.1 | Assumptions of the Proposed Model

- I. The products are produced and sold in closed loop three- level supply chain. The chain consists of several suppliers, a producer, and several customers as well as centers for depot and RM as well.
- II. In case of non-usage of the products, after a few years, they are transferred to the RM center by the customer.
- III. In the RM center, after rendering repair and correctional work on the products, they are delivered to the customer or de-assembled and shipped to the producer.
- IV. The returned products to the depot center (by the customer) are either demolished or de-assembled and dispatched to the producer.
- V. What is dealt with in the supply chain of the proposed model is production and sale of a product composed of several components by itself.
- VI. Not all customers are equally important and some relative to others enjoy higher importance.
- VII. The cost of RM, the capacity and the quality of production at regular and overtime work hours are different.
- VIII. The suppliers, as regard the price and the delivery time of the product, are different.
- IX. It is intended that a win-win relation between the producer and the supplier is established.
- X. The products are produced and sold in a closed loop three-level supply chain in which several suppliers, a producer and several customers are involved. The chain also includes a center for depot and one for RM.
- XI. The products not used for several years by the customer are returned to the RM center.
- XII. In the RM center, the products are repaired, after which they are returned to the customer or converted into spare parts and shipped to the producer.
- XIII. In the depot center, the products returned by the customer, are exterminated or de-assembled and dispatched to the producer in the form of spare parts.
- XIV. In the supply chain of our model, several products are manufactured and presented for sale.
- XV. The capacity, the cost, and the quality of production at regular and overtime work hours of supplying merchandise by the suppliers, the RM center and the reconstruction center are different.
- XVI. The forecast demand includes uncertainty.
- XVII. The manufacturing cost of one unit of the product at regular work hours includes uncertainty.
- XVIII. The manufacturing cost of one unit of the product at overtime work hours includes uncertainty.
- XIX. The cost of supplying one unit of the product from the suppliers includes uncertainty.
- XX. The cost of one laborer at regular work hours includes uncertainty.
- XXI. The cost of laborer at overtime work hours includes uncertainty.
- XXII. The hiring cost of one instance of work force includes uncertainty.
- XXIII. The firing cost of one instance of work force includes uncertainty.
- XXIV. The holding cost of one unit of merchandise in the depot center warehouse includes uncertainty.
- XXV. The holding cost of one unit of merchandise RM center includes uncertainty.
- XXVI. The holding cost of one unit of merchandise in the warehouse of the producer's center includes uncertainty.
- XXVII. The sale price of the product includes uncertainty.
- XXVIII. The capacity of holding the merchandise in the producer's center includes uncertainty.
- XXIX. The deficit rate of the product includes uncertainty.

3.2 | Indices

i ($i = 1, 2, \dots, I$): denotes i^{th} product.

k ($k = 1, 2, \dots, K$): denotes k^{th} customer.

t ($t = 1, 2, \dots, T$): denotes t^{th} period.

j ($j = 1, 2, \dots, J$): denotes j^{th} supplier.

$n \in N$: denotes n^{th} scenario.

3.3 | The Model Parameters

d_{ikt_n} : Forecast demand of i^{th} product at t^{th} period for k^{th} customer under n^{th} scenario.

α_{ikt_n} : Percentage of returned i^{th} product by k^{th} customer to depot center at t^{th} period under n^{th} scenario.

β_{ikt_n} : Percentage of returned i^{th} product by k^{th} customer to RM center at t^{th} period under n^{th} scenario.

CAPP: Capacity for holding merchandise at producer center.

CAPD: Capacity for holding merchandise at depot center.

CAPM: Capacity for holding merchandise at RM center.

CPR_{in}: Cost of producing one unit of i^{th} product at regular work hours under n^{th} scenario.

CPO_{in}: Cost of producing one unit of i^{th} product at overtime work hours under n^{th} scenario.

CD_{in}: Cost of producing one unit of i^{th} product from depot center under n^{th} scenario.

CM_{in}: Cost of producing one unit of i^{th} product from RM center under n^{th} scenario.

CSC_{jin}: Cost of supplying one unit of i^{th} product from j^{th} supplier under n^{th} scenario.

CLR_{in}: Cost of one laborer at t^{th} period at regular work hours under n^{th} scenario.

CLO_{in}: Cost of one laborer at t^{th} period at overtime work hour under n^{th} scenario.

HC_{in}: Cost of hiring manpower at t^{th} period under n^{th} scenario.

FC_{in}: Cost of firing manpower at t^{th} period under n^{th} scenario.

HIP_{in}: Cost of holding one unit of i^{th} product at t^{th} period in producer's warehouse under n^{th} scenario.

HID_{in}: Cost of holding one unit of i^{th} product in depot center at t^{th} period under n^{th} scenario.

HIM_{in}: Cost of holding one unit of i^{th} product at t^{th} period in RM center warehouse under n^{th} scenario.

π_{ikt_n} : Cost of shortage of one unit of i^{th} product for k^{th} customer at t^{th} period under n^{th} scenario.

QR_{it}: Production Quality Coefficient (QC) of i^{th} product at t^{th} period at regular work hours.

QO_{it}: Production quality coefficient of i^{th} product at t^{th} period at overtime work hours.

QSC_{jit}: Production quality coefficient of i^{th} product by j^{th} supplier at t^{th} period.

QD_{it}: Production quality coefficient of i^{th} product at t^{th} period by the depot center.

QM_{it}: Production quality coefficient of i^{th} product at t^{th} period by the RM center.

WC_k: Worth coefficient of k^{th} customer.

WSC_j: Worth coefficient of j^{th} supplier.

MW_t : Maximum work force available at t^{th} period.

MOT_t : Maximum overtime work hour available at t^{th} period.

TW : Maximum work hour needed.

TP_i : Total person-hour rate needed for producing i^{th} product (at regular and overtime work hours).

γ : Percentage of permissible change in human work force at t^{th} period.

MSC_{ijt} : Maximum permissible supply of i^{th} product from j^{th} supplier at t^{th} period.

P_{ikt} : Sale price of i^{th} product to k^{th} customer at t^{th} period under n^{th} scenario.

$CPRD_{in}$: Cost of producing one unit of i^{th} product at regular work hours in the depot center under n^{th} scenario.

$CPOD_{in}$: Cost of producing one unit of i^{th} product overtime work hours in the depot center under n^{th} scenario.

$CPRM_{in}$: Cost of producing one unit of i^{th} product at regular work hours in the RM center under n^{th} scenario.

$CPOM_{in}$: Cost of producing one unit of i^{th} product overtime work hours in the RM center under n^{th} scenario.

P_n : Probability of any scenario.

A_n : Designed scenario for parameters with uncertainty in the first objective function.

θ_n : Variable used for costs variation linearization.

λ_n : Weight on solution's variance.

3.4 | Decision Variables

B_{ikt} : Deficit amount (back order) of i^{th} product at t^{th} period for the k^{th} customer.

X_{it} : Amount of producing i^{th} family products at regular work hour production at t^{th} period.

Y_{it} : Amount of producing i^{th} family products at overtime work hour production at t^{th} period.

ZD_{it} : Amount of supplying i^{th} family products by the depot center at t^{th} period.

ZM_{it} : Amount of supplying i^{th} family products by the RM center at t^{th} period.

F_{ikt} : Amount of i^{th} family shipped product for the k^{th} customer at t^{th} period.

SC_{ijt} : Amount of i^{th} family products that are procured by j^{th} supplier at t^{th} period.

OT_t : Overtime work hours needed at t^{th} period.

IP_{it} : Inventory level of i^{th} family product at the end of t^{th} period at the producer's site.

WL_t : Number of work laborers needed at t^{th} period.

HL_t : Number of hired laborers at t^{th} period.

FL_t : Number of fired laborers at t^{th} period.

ZC_{ikt} : Number of i^{th} family product shipped for k^{th} customer at t^{th} period from the RM center.

IM_{it} : Inventory level of i^{th} family product at the end of t^{th} period in the RM center.

ID_{it} : Inventory level of i^{th} family product at the end of t^{th} period in the depot center.

XD_{it} : Amount of producing i^{th} family product at regular work hours at t^{th} period in the depot center.

YD_{it} : Amount of producing i^{th} family product at overtime work hours at t^{th} period in the depot center.

XM_{it} : Amount of producing i^{th} family product at regular work hours at t^{th} period in the RM center.

YM_{it} : Amount of producing i^{th} family product at overtime work hours at t^{th} period in the RM center.

3.5 | Mathematical Model

$$\text{Min}z_1 = E + \lambda_1 [(P_{n1} \times A_{n1}) - E + 2\theta_{n1}] + \lambda_2 [(P_{n2} \times A_{n2}) - E + 2\theta_{n2}] + \lambda_3 [(P_{n3} \times A_{n3}) - E + 2\theta_{n3}]. \quad (10)$$

$$\text{Max}z_2 = \sum_t \sum_i (X_{it} \times QR_{it} + Y_{it} \times QO_{it}) + \sum_t \sum_i \sum_j (SC_{ijt} \times QSC_{ijt}) + \sum_t \sum_i (ZD_{it} \times QD_{it}) + \sum_t (ZM_{it} \times QM_{it}). \quad (11)$$

$$\text{Min}z_3 = \sum_{t_k} \max(WC_k \times \sum_i B_{ikt}). \quad (12)$$

$$\text{Max}z_4 = \sum_{t_j} \min(WCS_j \times \sum_i SC_{ijt}). \quad (13)$$

Subject to:

$$IP_{i(t-1)} + X_{it} + Y_{it} + \sum_j SC_{ijt} + ZD_{it} + ZM_{it} + \sum_k B_{ik(t-1)} = \sum_k B_{ik(t)} + \sum_k F_{ikt} + IP_{it} \quad \text{for all } i, t, \quad (14)$$

$$ID_{it} = ID_{i(t-1)} + \sum_k \alpha_{iktn} \times F_{ikt} - ZM_{it} - ZD_{it} \quad \text{for all } i, t, n, \quad (15)$$

$$IM_{it} = IM_{i(t-1)} + \sum_k \beta_{iktn} \times F_{ikt} - ZM_{it} - ZC_{ikt} \quad \text{for all } i, t, n, \quad (16)$$

$$\sum_i IP_{it} \leq \text{CAPP} \quad \text{for all } t, \quad (17)$$

$$\sum_i ID_{it} \leq \text{CAPD} \quad \text{for all } t, \quad (18)$$

$$\sum_i IM_{it} \leq \text{CAPM} \quad \text{for all } t, \quad (19)$$

$$WL_t \leq MW_t \quad \text{for all } t, \quad (20)$$

$$WL_t = WL_{(t-1)} + HL_t + FL_t \quad \text{for all } t, \quad (21)$$

$$HL_t \times FL_t = 0, \quad (22)$$

$$(IP_{it} + IM_{it}) \times \sum_k B_{ikt} = 0 \quad \text{for all } i, t, \quad (23)$$

$$OT_t \leq MOT_t \quad \text{for all } t, \quad (24)$$

$$\sum_i TP_i \times X_{it} \leq TW \quad \text{for all } t, \quad (25)$$

$$\sum_i TP_i \times Y_{it} \leq OT \quad \text{for all } t, \quad (26)$$

$$FL_t + HL_t \leq \gamma_{(t-1)} \times WL_{(t-1)} \quad \text{for all } t, \quad (27)$$

$$SC_{ijt} \leq MSC_{ijt} \quad \text{for all } i, j, t, \quad (28)$$

$$SC_{ijt} \leq SC_{(i-1)jt} \quad \text{for all } i, j, t, \quad (29)$$

$$B_{ikt} = B_{ik(t-1)} + d_{iktn} - F_{ikt} - ZC_{ikt} \quad \text{for all } i, k, t, n, \quad (30)$$

$$ZD_{it} \leq CAPD \quad \text{for all } i, t, \quad (31)$$

$$ZM_{it} \leq CAPM \quad \text{for all } i, t, \quad (32)$$

$$ZC_{ikt} \leq CAPM \quad \text{for all } i, k, t, \quad (33)$$

$$F_{ikt} + ZC_{ikt} \leq d_{iktn} \quad \text{for all } i, k, t, n, \quad (34)$$

$$A_{n1} - [(P_{n1} \times A_{n1}) + (P_{n2} \times A_{n2}) + (P_{n3} \times A_{n3})] + \theta_{n1} \geq 0, \quad (35)$$

$$A_{n2} - [(P_{n1} \times A_{n1}) + (P_{n2} \times A_{n2}) + (P_{n3} \times A_{n3})] + \theta_{n2} \geq 0, \quad (36)$$

$$A_{n3} - [(P_{n1} \times A_{n1}) + (P_{n2} \times A_{n2}) + (P_{n3} \times A_{n3})] + \theta_{n3} \geq 0, \quad (37)$$

$$Y_{it}, ZD_{it}, ZM_{it}, F_{ikt}, SC_{ijt}, OT_t, IP_{it}, WL_t, HL_t, FL_t, ZC_{ikt}, IM_{it}, XD_{it}, YD_{it}, XM_{it}, YM_{it}, ID_{it} \geq 0, \quad (38)$$

$$i = 1, 2, 3, \quad k = 1, 2, 3, \quad j = 1, 2, 3, \quad t = 1, 2, 3, \quad n = 1, 2, 3.$$

Eq. (10) shows the first objective function of the problem defined for minimizing the costs. The costs relate to the following cases: producing a unit of product at regular and overtime work hours, supplying a unit of product by the suppliers, the RM and the depot centers, an individual laborer at regular work hour, an individual laborer at overtime work hour, hiring and firing human work force, holding a unit of product in the producer's place, RM, and depot warehouses, shortage in unit of product for customer, and the cost of forecast demand. The function is written in the robust form according to the Mulvey's method. Eq. (11) represents the model's second objective function defined for maximizing the QC. QC embraces the following instances: sum of production QC at regular work hours, production QC at overtime work hours, QC of received product from suppliers, QC of received product from depot center, and QC of received product from RM center. Eq. (12) shows the third objective function of the problem which includes minimizing maximum shortage among customer and customers' importance coefficient. Eq. (13) is a display of the model's fourth objective function whose purpose is maximizing minimum rate of supplying product from suppliers. Eq. (14) expresses the producer's inventory balance. Eq. (15) denotes the inventory balance at depot center. Eq. (16) denotes the inventory balance at RM center. The capacity for holding the product at the producer's center is indicated by Eq. (17). Eq. (18) shows the capacity for holding the product at depot center. Eq. (19) represents the capacity for holding the product at RM center. Eq. (20) represents the limitations of maximum work force available. Eq. (21) is an indication of balance in the producer's human work force. In Eq. (22) demonstrates the hiring or firing of personnel at each period. Eq. (23) shows the inventory or shortage of each product at each period. Eq. (24) represents limitations in overtime work ceiling. Eq. (25) displays the time for manufacturing the product is less at each period of available regular time. Eq. (26) indicates that the time for manufacturing the product is less at overtime work hours. The percentage of permissible changes in human work force at each period is shown in Eq. (27). Maximum purchase of the producer from supplier's product is indicated in Eq. (28). Eq. (29) indicates Maximum purchase of the product from suppliers at each period. Eq. (30) shows the balance in the shortage of the producer's product in relation to the shortage of the previous period, the rate of the product dispatched from the producer to the customer and the RM at each period. Eq. (31) and Eq. (32) points to maximum product supplied from RM and depot centers. Eq. (33) and Eq. (34) demonstrates maximum product shipped from RM center and depot center to the customer at each period. Eqs. (35)-(37) show the robust limitations of the model. Lastly, Eq. (38) represents non-negativity of the decision variables. The linearized form of *Limitations*

(20) and (21) are indicated in Eq. (39) and Eq. (40), respectively. The variables XHL_t and XFL_t are binary. If the new human work forces' hire or fire occurs, the values of XHL_t and XFL_t will be equal to 1, respectively:

$$HL_t \leq MW_t \times XHL_t. \quad (39)$$

$$XHL_t + XFL_t = 1. \quad (40)$$

The linearized form of *Limitations* (17) and (23) are indicated in Eqs. (41)-(43).

$$\sum_k B_{ikt} \leq [\sum_{t=1} \sum_k D_{ikt}] \times \theta_{it} \text{ for all } i, t. \quad (41)$$

$$IP_{it} \leq CAPP \times (1 - \theta_{it}) \text{ for all } t. \quad (42)$$

$$\theta = \begin{cases} 1, & \sum_k B_{ikt} > 0, \\ 0, & IP_{it} > 0. \end{cases} \quad (43)$$

Eqs. (44)-(46) show the designed scenarios in the Mulvey method.

$$\begin{aligned} A_{n1} = & \sum_t \sum_i (CPR_{in1} \times X_{it} + CPO_{in1} \times Y_{it} + CPRD_{in1} \times XD_{it} + \\ & CPOD_{in1} \times YD_{it} + CPRM_{in1} \times XM_{it} + CPOM_{in1} \times YM_{it}) + \\ & \sum_t \sum_j \sum_i (CSC_{ijn1} \times SC_{ijt}) + \sum_t \sum_i (CD_{in1} \times ZD_{it} + CM_{in1} \times ZM_{it}) + \\ & \sum_t (CLR_{tn1} \times WL_t + CLO_{tn1} \times OT_t) + \sum_t (HL_t \times HC_{tn1} + FL_t \times FC_{tn1}) + \\ & \sum_t \sum_i (IP_{it} \times HIP_{itn1} + ID_{it} \times HID_{itn1} + IM_{it} \times HIM_{itn1}) + \sum_t \sum_i \sum_k (B_{ikt} \times \pi_{ikt1}) - \\ & \sum_t \sum_i \sum_k (F_{ikt} \times P_{ikt1}). \end{aligned} \quad (44)$$

$$\begin{aligned} A_{n2} = & \sum_t \sum_i (CPR_{in2} \times X_{it} + CPO_{in2} \times Y_{it} + CPRD_{in2} \times XD_{it} + \\ & CPOD_{in2} \times YD_{it} + CPRM_{in2} \times XM_{it} + CPOM_{in2} \times YM_{it}) + \\ & \sum_t \sum_j \sum_i (CSC_{ijn2} \times SC_{ijt}) + \sum_t \sum_i (CD_{in2} \times ZD_{it} + CM_{in2} \times ZM_{it}) + \\ & \sum_t (CLR_{tn2} \times WL_t + CLO_{tn2} \times OT_t) + \sum_t (HL_t \times HC_{tn2} + FL_t \times FC_{tn2}) + \\ & \sum_t \sum_i (IP_{it} \times HIP_{itn2} + ID_{it} \times HID_{itn2} + IM_{it} \times HIM_{itn2}) + \sum_t \sum_i \sum_k (B_{ikt} \times \pi_{ikt2}) - \\ & \sum_t \sum_i \sum_k (F_{ikt} \times P_{ikt2}). \end{aligned} \quad (45)$$

$$\begin{aligned} A_{n3} = & \sum_t \sum_i (CPR_{in3} \times X_{it} + CPO_{in3} \times Y_{it} + CPRD_{in3} \times XD_{it} + \\ & CPOD_{in3} \times YD_{it} + CPRM_{in3} \times XM_{it} + CPOM_{in3} \times YM_{it}) + \\ & \sum_t \sum_j \sum_i (CSC_{ijn3} \times SC_{ijt}) + \sum_t \sum_i (CD_{in3} \times ZD_{it} + CM_{in3} \times ZM_{it}) + \\ & \sum_t (CLR_{tn3} \times WL_t + CLO_{tn3} \times OT_t) + \sum_t (HL_t \times HC_{tn3} + FL_t \times FC_{tn3}) + \\ & \sum_t \sum_i (IP_{it} \times HIP_{itn3} + ID_{it} \times HID_{itn3} + IM_{it} \times HIM_{itn3}) + \sum_t \sum_i \sum_k (B_{ikt} \times \pi_{ikt3}) - \\ & \sum_t \sum_i \sum_k (F_{ikt} \times P_{ikt3}). \end{aligned} \quad (46)$$

Eqs. (47)-(49) present the mathematical expectations of designed scenarios in the suggested model and Eq. (50) presents the mathematical expectations of total scenarios.

$$E_1 = P_{n1} \times A_{n1}. \quad (47)$$

$$E_2 = P_{n2} \times A_{n2}. \quad (48)$$

$$E_3 = P_{n3} \times A_{n3}. \quad (49)$$

$$E = E_1 + E_2 + E_3. \quad (50)$$

4 | Solution for Proposed Model

Considering the multi-objectivity of the proposed model in the present research, attempt is made to find a Pareto optimal solution to the model. That is to say, a Pareto answer represents a decisive and effective solution from among available responses. One of the most common methods for solving multi-objective problems in arriving at Pareto optimal responses is invoking the LP metric method. In this method whose

relevant calculations are readily observable in *Eq. (51)*, the model - in a single objective form - is first solved for each of the objective functions; then the set of obtained answers considering the type of objective function in terms of minimization or maximization are placed in *Eq. (51)* as illustrated below:

$$LP : \{w_1 \left(\frac{Z_1 - Z_1^*}{Z_1^{nadir} - Z_1^*} \right)^p + w_2 \left(\frac{Z_2^* - Z_2}{Z_2^* - Z_2^{nadir}} \right)^p + w_3 \left(\frac{Z_3 - Z_3^*}{Z_3^{nadir} - Z_3^*} \right)^p + w_4 \left(\frac{Z_4^* - Z_4}{Z_4^* - Z_4^{nadir}} \right)^p \}^{\frac{1}{p}}. \quad (51)$$

The assumptions related to the above relation are summarized below:

- $1 \leq P \leq \infty$, whose value determines the degree of emphasis towards existing deviations, as the bigger the latter value, the more the emphasis placed on the biggest deviation.
- W_i : Weight considered for i^{th} objective function ($i=1, 2, 3, 4$).
- Z_i : i^{th} objective function of the problem ($i=1, 2, 3, 4$).
- f_i : Optimal answer obtained through solving the model as against i^{th} objective function.
- Z_{inadir} : Anti-ideal answer as against i^{th} objective function.

5 | The Proposed Model Results

The model proposed was solved in LP metric method making use of software v14.0.1.55 on a Windows-7 system with the specifications RAM 300 HZ 2, 20, GB. The model was solved given real data from the industry in question.

5.1 | Solving the Model in the Case under Study

As mentioned above, the problem was solved using real data taken from the industry under study. In the supply chain of the latter industry, five different products are manufactured.

At the first level and the last level of the chain, four suppliers are placed and the product is dispatched to four customers. In this regard, some research studies consider a three-month period. The problem parameters - in view of the obtained information from the above-mentioned industry - are presented in *Table 2* through *Table 15*. *Table 16* provides some of the obtained Pareto optimal answers. In order to acquire the answers related to each row in this table, the following steps are taken:

- I. Optimize each of the objective functions separately -taking into account the model's constraints - once as maximizing and second as minimizing in the LINGO software.
- II. Write LP metric relation connected with *Eq. (51)* utilizing the results from previous stage.
- III. Optimize the obtained function from the previous stage, taking constraints into consideration by means of the LINGO software.
- IV. Extract optimal values of decision variables using the obtained solution from previous stage.
- V. Compute the value of each of objective functions as against optimal decision variables obtained from previous stage.

The values acquired from the last stage are variable Pareto optimal answers presented in columns 2 through 4 of *Table 16*. As can be seen, these values are related to the objective functions. Values connected with decision variables of each row, are those same values obtained from the fourth stage in the above-mentioned stages. In practice, after selecting one of Pareto optimal responses - by the decision maker/s in the industry in question - the values related to the decision variables can easily be provided.

Table 2. Sales price, forecast demand, and cost of shortage of one unit of product in each scenario.

k	i	$\pi_{iktn}/$ t=1	$\pi_{iktn}/$ t=2	$\pi_{iktn}/$ t=3	$d_{iktn}/$ t=1	$d_{iktn}/$ t=2	$d_{iktn}/$ t=3	$p_{iktn}/$ t=1	$p_{iktn}/$ t=2	$p_{iktn}/$ t=3
1	1	3	2	1	370	290	100	610	700	460
		2	5	7	339	280	230	500	555	420
		3	6	6	275	240	150	450	500	400
	2	3	2	1	370	290	100	610	700	460
		2	5	7	339	280	230	500	555	420
		3	6	6	275	240	150	450	500	400
	3	3	2	1	370	290	100	610	700	460
		2	5	7	339	280	230	500	555	420
		3	6	6	275	240	150	450	500	400
2	1	3	2	1	395	300	130	620	710	470
		2	5	7	349	290	240	510	570	423
		3	6	6	295	250	190	465	512	411
	2	3	2	1	395	300	130	620	710	470
		2	5	7	349	290	240	510	570	423
		3	6	6	295	250	190	465	512	411
	3	3	2	1	395	300	130	620	710	470
		2	5	7	349	290	240	510	570	423
		3	6	6	295	250	190	465	512	411
3	1	3	2	7	445	350	180	670	760	520
		2	5	6	399	345	290	570	620	480
		3	6	6	335	304	243	510	580	460
	2	3	2	7	445	350	180	670	760	520
		2	5	6	399	345	290	570	620	480
		3	6	6	335	304	243	510	580	460
	3	3	2	7	445	350	180	670	760	520
		2	5	6	399	345	290	570	620	480
		3	6	6	335	304	243	510	580	460

Table 3. Percentage of returned product by customer to depot and rapiar and maintenance center in each scenario.

Scenario	i	k	$\alpha_{iktn}/$ t= 1	$\alpha_{iktn}/$ t= 2	$\alpha_{iktn}/$ t= 3	$\beta_{iktn}/$ t= 1	$\beta_{iktn}/$ t= 2	$\beta_{iktn}/$ t= 3
1	1	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	2	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	3	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
2	1	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	2	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	3	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
3	1	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	2	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07
	3	1	0.07	0.06	0.07	0.07	0.06	0.07
		2	0.01	0.05	0.19	0.03	0.07	0.19
		3	0.08	0.01	0.07	0.08	0.02	0.07

Table 4. Cost of producing at regular and overtime hours from depot, and RM centers.

Scenario	i	CPO _{in}	CPR _{in}	CD _{in}	CM _{in}
1	1	90	90	80	70
	2	110	90	80	90
	3	120	100	90	90
2	1	100	100	90	80
	2	120	100	90	100
	3	130	150	100	100
3	1	150	150	140	130
	2	160	150	140	150
	3	170	200	150	150

Table 5. Capacity for holding merchandise at producer, depot, and RM centers and maximum work hour needed.

CAPP	CAPD	CAPM	TW
15000	10000	10000	60

Table 6. Total person-hour rate needed for goods (at regular and overtime work hours).

i	Ti
1	200
2	200
3	200

Table 7. Cost of holding goods at producer's warehouse, depot, and RM centers in each scenario.

Scenario	i	HIP _{itn} /t= 1	HIP _{itn} /t= 2	HIP _{itn} /t= 3	HID _{itn} /t= 1	HID _{itn} /t= 2	HID _{itn} /t= 3	HIM _{it} n/t= 1	HIM _{it} n/t= 2	HIM _{it} n/t= 3
1	1	85	90	90	25	28	26	20	23	22
	2	90	92	93	27	29	27	22	23	25
	3	92	98	100	29	31	31	25	24	24
2	1	85	90	90	25	28	26	20	23	22
	2	90	92	93	27	29	27	22	23	25
	3	92	98	100	29	31	31	25	24	24
3	1	85	90	90	25	28	26	20	23	22
	2	90	92	93	27	29	27	22	23	25
	3	92	98	100	29	31	31	25	24	24

Table 8. Production quality coefficient at regular and overtime work hours in depot, and RM centers.

i	Qd _{it} / t= 1	Qd _{it} / t= 2	Qd _{it} / t= 3	Qm _{it} / t= 1	Qm _{it} / t= 2	Qm _{it} / t= 3	Qr _{it} / t= 1	Qr _{it} / t= 2	Qr _{it} / t= 3	QO _{it} / t= 1	QO _{it} / t= 2	QO _{it} / t= 3
1	0.97	0.92	0.96	0.92	0.96	0.95	0.98	0.97	0.98	0.97	0.97	0.97
2	0.97	0.98	0.95	0.96	0.98	0.95	0.97	0.97	0.98	0.97	0.97	0.97
3	0.98	0.93	0.96	0.9	0.97	0.98	0.97	0.98	0.97	0.97	0.98	0.97

Table 9. Maximum allowable supply of goods from supplier.

j	i	MSC _{ijt} / t= 1	MSC _{ijt} / t= 2	MSC _{ijt} / t= 3
1	1	136	110	147
	2	167	83	64
	3	170	90	70
2	1	123	53	83
	2	141	41	65
	3	133	70	85
3	1	95	70	110
	2	125	100	96
	3	90	90	90

Table 10. Worth coefficient of suppliers and customers.

$WSC_i / j=1$	$WSC_i / j=2$	$WSC_i / j=3$	$WC_k / k=1$	$WC_k / k=2$	$WC_k / k=3$
0.8	0.6	0.5	0.9	0.6	0.8

Table 11. Production cost at regular and overtime work hours in depot and RM centers in each scenario.

Scenario	i	$CPOM_{in}$	$CPRM_{in}$	$CPOD_{in}$	$CPRD_{in}$
1	1	100	90	105	80
	2	100	90	105	80
	3	100	90	105	80
2	1	110	100	120	90
	2	110	100	120	90
	3	160	170	120	90
3	1	120	200	170	140
	2	120	200	170	140

Table 12. Cost of supplying a unit of product from supplier in each scenario.

Scenario	i	$CSC_{ijn} / j=1$	$CSC_{ijn} / j=2$	$CSC_{ijn} / j=3$
1	1	460	700	610
	2	420	555	500
	3	400	500	450
2	1	470	710	620
	2	423	570	510
	3	411	512	465
3	1	520	760	670
	2	480	620	570
	3	460	580	510

Table 13. Cost of manpower at regular and overtime work hours, and hiring and firing cost of one instance of human work force in each scenario.

Scenario	t	CLR_{tn}	CLO_{tn}	HC_{tn}	FC_{tn}
1	1	150	190	50	70
	2	120	195	50	80
	3	135	190	50	90
2	1	170	249	60	70
	2	190	250	60	80
	3	210	280	60	90
3	1	210	290	80	80
	2	232	270	90	90
	3	240	295	95	100

Table 14. Maximum work force available and overtime work hour and percentage of allowable change in human work force in each period.

t	MW_t	MOT_t	γ_t
1	50	55	0.2
2	50	56	0.2
3	50	57	0.2

Table 15. Production quality coefficient of product by supplier.

j	i	$QSC_{ijt} / t=1$	$QSC_{ijt} / t=2$	$QSC_{ijt} / t=3$
1	1	0.78	0.8	0.62
	2	0.86	0.65	0.92
	3	0.68	0.58	0.88
2	1	0.95	0.82	0.51
	2	0.82	0.94	0.53
	3	0.72	0.62	0.55
3	1	0.93	0.76	0.66
	2	0.64	0.89	0.96
	3	0.52	0.6	0.63

Table 16. Answer proceeding from solving model as against objective functions.

	Z₁	Z₂	Z₃	Z₄
p=1	-30602741.4508065	9924770.23363924	7330.800000000000	240.452199760426
p=2	-37240491.6050168	15806267.7158352	15721.500000000000	257.995403149302
p=3	-32559383.6529256	12188671.2093891	15675.300000000000	254.248064660243
p=4	-29473058.8500060	11323108.6526589	7382.400000000000	243.818014726437
Mean	-32468918.89	12310704.45	11527.5	249.1284206
Variance	8808471415452.9	4725470550249.4	17397006. 4	52.1

Table 17. The obtained value for some of decision variables.

Variable	Value	Explanation
B211	1792	Deficit amount (back order) of second product at first period for the first customer.
X21	2601	Amount of producing second family products at regular work hour production at first period.
Y21	6865	Amount of producing second family products at overtime work hour production at first period.
ZD21	222	Amount of supplying second family products by the depot center at first period.
ZM21	8753	Amount of supplying second family products by the RM center at first period
F211	351	Amount of second family shipped product for the first customer at first period.
SC211	100	Amount of second family products that are procured by first supplier at first period.
OT1	42	Overtime work hours needed at first period.
IP21	23	Inventory level of second family product at the end of first period at the producer's site
WL1	10	Number of work laborers needed at first period.
HL1	4	Number of hired laborers at first period.
FL1	2	Number of fired laborers at first period.
ZC211	1055	Number of second family product shipped for first customer at first period from the RM center.
IM21	100	Inventory level of second family product at the end of first period in the RM center
ID21	101	Inventory level of second family product at the end of first period in the depot center.
XD21	9793	Amount of producing second family product at regular work hours at first period in the depot center
YD21	7186	Amount of producing second family product at overtime work hours at first period in the depot center.
XM21	7186	Amount of producing second family product at regular work hours at first period in the RM center.
YM21	9619	Amount of producing second family product at overtime work hours at first period in the RM center.

Pareto-optimal solutions are determined in the *Table 16*. As shown in this table, the solutions are non-dominate. For example, for P=2 and P=4, the best value of cost (first objective) is obtained for P=2 whereas the best value of suppliers' satisfaction (third objective) is obtained for P=4. In addition, the best value of customers' satisfaction (second objective) and the quality of the manufactured products (fourth objective) are for P=2. So, although P=2 optimizes the three objective functions, but the third function objectives is optimized for P=4. In addition, the mean and variance of objective functions are determined in *Table 16*. According to this table the first objective function has the max variance. So, this function, among 4 functions, has the max of dispersion. It should be noted that we can find more solutions for more values of P.

Table 17 shows the value of some of decision variables. Given that there are many variables or indices, we cannot propose all of them and this table is just an example and a part of outputs of model.

6 | Conclusion

This research study presents a multi-objective mathematical model for APP in a closed-loop supply chain under uncertain conditions. Worthy of note in the design of the model – formulated as a non-

linear planning scheme- is the particular attention it pays to creating a depot center, and a center for RM while simultaneously taking into account satisfying customers and suppliers as well as giving particular attention to the quality of the manufactured products and various costs and expenditures. The demand and the parameters related to the demand include uncertainty and the objectives of the model consist of minimizing costs, maximizing the product quality provided by suppliers and the product produced by the producer at regular and overtime work hours together with minimizing the sum total weight of maximum shortage among customers accompanied by minimizing the overall total weight of minimum rate of supplying goods from suppliers with a view to establishing a win-win relation.

The proposed model is first solved by a numerical example, then it is solved by the actual data taken from a closed-loop supply chain related to a specified military industry; and finally, all of variables are valued with the help of the closed-loop supply chain of the proposed model. As is evident from *Table 16*, the Pareto answers related to the problem are established. Also, in order to validate the finding, the above model is investigated with the help of the data taken from a numerical example at greater dimensions. The acquired results contribute greatly to the supply chain in attaining higher profits, better decision-making criteria and an increased level of rendering services to customers. The model can be applied in APP for various industries. Future research works might also add other parameters to the model and uncertain conditions for parameters of uncertain nature can be applied in view of the prevailing conditions of each industry. Furthermore, in proportion to the model becoming more complicated, meta-heuristic algorithms can be invoked to solve the model. It should be noted that proposed objectives and limitations are not limited to these cases and may there are some of new objectives and limitations. In addition, there are many methods for validation the model like LP- metric, epsilon-constraint and etc. So, choice of suitable method for validation was a limitation in this research. Furthermore, there are many methods to deal with uncertain conditions like fuzzy programming, chance programming, and sensitivity analyses, robust and etc. So, choice of suitable method was a limitation, too. Moreover, if there are many indices, like period or product, the model will become complicated and we can not solve it with and have to solve it with meta-heuristic algorithms.

References

- [1] Chen, C. L., & Lee, W. C. (2004). Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices. *Computers & chemical engineering*, 28(6-7), 1131-1144.
- [2] Pokharel, S., & Mutha, A. (2009). Perspectives in reverse logistics: a review. *Resources, conservation and recycling*, 53(4), 175-182.
- [3] Ghahremani-Nahr, J., Nozari, H., & Najafi, S. E. (2020). Design a green closed loop supply chain network by considering discount under uncertainty. *Journal of applied research on industrial engineering*, 7(3), 238-266.
- [4] Teimoori, E., Shafiiian Bejestani, J., & Kalb Khani, K. (2002). Simultaneous selecting and developing of suppliers in fuzzy state. *The first national conference of logistics and supply chains*, Tehran, Iran. (In Persian). <https://civilica.com/doc/8764/>
- [5] Hansen, Z. N. L., Larsen, S. B., Nielsen, A. P., Groth, A., Gregersen, N. G., & Ghosh, A. (2018). Combining or separating forward and reverse logistics. *The international journal of logistics management*, 29(1), 216-236. <https://doi.org/10.1108/IJLM-12-2016-0299>
- [6] Zhang, R., Zhang, L., Xiao, Y., & Kaku, I. (2012). The activity-based aggregate production planning with capacity expansion in manufacturing systems. *Computers & industrial engineering*, 62(2), 491-503.
- [7] Paduloh, P., Djatna, T., Sukardi, S., & Muslich, M. (2020). Uncertainty models in reverse supply chain: a review. *International journal supply chain management*, 9(2), 139-149.
- [8] Jang, J., & Do Chung, B. (2020). Aggregate production planning considering implementation error: a robust optimization approach using bi-level particle swarm optimization. *Computers & industrial engineering*, 142, 106367. <https://doi.org/10.1016/j.cie.2020.106367>
- [9] Hafezalkotob, A., Chaharbaghi, S., & Lakeh, T. M. (2019). Cooperative aggregate production planning: a game theory approach. *Journal of industrial engineering international*, 15, 19-37. <https://doi.org/10.1007/s40092-019-0303-0>

- [10] Masud, A. S., & Hwang, C. L. (1980). An aggregate production planning model and application of three multiple objective decision methods. *International journal of production research*, 18(6), 741-752. <https://doi.org/10.1080/00207548008919703>
- [11] Cheraghalikhani, A., Khoshalhan, F., & Mokhtari, H. (2019). Aggregate production planning: a literature review and future research directions. *International journal of industrial engineering computations*, 10(2), 309-330.
- [12] Hatefi, S. M., Jolai, F., Torabi, S. A., & Tavakkoli-Moghaddam, R. (2015). A credibility-constrained programming for reliable forward–reverse logistics network design under uncertainty and facility disruptions. *International journal of computer integrated manufacturing*, 28(6), 664-678. <https://doi.org/10.1080/0951192X.2014.900863>
- [13] Ghorbani, M., Arabzad, S. M., & Tavakkoli-Moghaddam, R. (2014). A multi-objective fuzzy goal programming model for reverse supply chain design. *International journal of operational research*, 19(2), 141-153.
- [14] Rivaz, S., Nasser, S. H., & Ziaseraji, M. (2020). A fuzzy goal programming approach to multiobjective transportation problems. *Fuzzy information and engineering*, 12(2), 139-149. <https://doi.org/10.1080/16168658.2020.1794498>
- [15] Khalifa, H. A. (2018). On solving fully fuzzy multi-criteria De Novo programming via fuzzy goal programming approach. *Journal of applied research on industrial engineering*, 5(3), 239-252. DOI: 10.22105/jarie.2018.148642.1054
- [16] Baykasoglu, A. D. İ. L. (2001). MOAPPS 1.0: aggregate production planning using the multiple-objective tabu search. *International journal of production research*, 39(16), 3685-3702. <https://doi.org/10.1080/00207540110061607>
- [17] Leung, S. C., Wu, Y., & Lai, K. K. (2003). Multi-site aggregate production planning with multiple objectives: a goal programming approach. *Production planning & control*, 14(5), 425-436. <https://doi.org/10.1080/0953728031000154264>
- [18] Gholamian, N., Mahdavi, I., & Tavakkoli-Moghaddam, R. (2016). Multi-objective multi-product multi-site aggregate production planning in a supply chain under uncertainty: fuzzy multi-objective optimisation. *International journal of computer integrated manufacturing*, 29(2), 149-165. <https://doi.org/10.1080/0951192X.2014.1002811>
- [19] Mirzapour Al-E-Hashem, S. M. J., Malekly, H., & Aryanezhad, M. B. (2011). A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. *International journal of production economics*, 134(1), 28-42.
- [20] Rahimi, E., Paydar, M. M., Mahdavi, I., Jouzdani, J., & Arabsheybani, A. (2018). A robust optimization model for multi-objective multi-period supply chain planning under uncertainty considering quantity discounts. *Journal of industrial and production engineering*, 35(4), 214-228. <https://doi.org/10.1080/21681015.2018.1441195>
- [21] Zanjani, B., Amiri, M., Hanafizadeh, P., & Salahi, M. (2021). Robust multi-objective hybrid flow shop scheduling. *Journal of applied research on industrial engineering*, 8(1), 40-55.
- [22] Mirzapour Al-e-hashem, S. M. J. M., Aryanezhad, M. B., & Sadjadi, S. J. (2012). An efficient algorithm to solve a multi-objective robust aggregate production planning in an uncertain environment. *The international journal of advanced manufacturing technology*, 58, 765-782. <https://doi.org/10.1007/s00170-011-3396-1>
- [23] Mirzapour Al-e-hashem, S. M. J., Baboli, A., & Sazvar, Z. (2013). A stochastic aggregate production planning model in a green supply chain: considering flexible lead times, nonlinear purchase and shortage cost functions. *European journal of operational research*, 230(1), 26-41.
- [24] Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research*, 43(2), 264-281.
- [25] Pan, F., & Nagi, R. (2010). Robust supply chain design under uncertain demand in agile manufacturing. *Computers & operations research*, 37(4), 668-683.