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A Novel Numerical Approach for Distributed Order Time Fractional COVID-19 Virus Model

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Abstract

In this paper, we proposed a numerical approach to solve a distributed order time fractional COVID 19 virus model. The fractional derivatives are shown in the Caputo-Prabhakar contains generalized Mittag-Leffler Kernel. The coronavirus 19 disease model has 8 Inger diets leading to system of 8 nonlinear ordinary differential equations in this sense, we used the midpoint quadrature method and finite different scheme for solving this problem, our approximation method reduce the distributed order time fractional COVID 19 virus equations to a system of algebraic equations. Finally, to confirm the efficiency and accuracy of this method, we presented some numerical experiments for several values of distributed order. Also, all parameters introduced in the given model are positive parameters.

Keywords: COVID-19 virus, Distributed-order, Finite difference method, Caputo-prabhakar derivative.

1 | Introduction

Newly, Fractional Differential Equations (FDEs) have been widely applied to describe abundant seemingly diverse and different fields because of their vast potential to explain many phenomena in economics [1], biology [2], physics [3], engineering [4] and science [5], etc. Among types of FDEs, the distributed-order ones, for the first time in 2002 studied by Chechkin et al. [6], have been accurately applied to explain the relaxation and anomalous diffusion phenomena where the diffusion exponent can shift in the course of time. Extending numerical and analytical methods for the solutions of FDEs is very significant duty. In fact, it is not feasible to find the exact analytic solutions of FDEs. Thus, several numerical methods have been showed to solve the FDEs. Sousa and Li [7] applied the weighted finite difference method to obtain the fractional diffusion equation; Babolian et al. [8] found the numerical solution of nonlinear FDEs by using Adomian decomposition method; Yang et al. [9] used variational iteration method for the solution of multi-order FDEs; Hosseinnia et al. [10] obtain the numerical solution of FDEs by using Homotopy perturbation method; Bhrawy et al. [11] used Chebyshev and Legendre polynomials method to solve the multi-term FDEs; Rahimkhani et al. [12] used fractional-order Bernoulli functions method for solving the fractional

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Fredholm-Volterra integro-differential equations; Rahimkhani et al. [13] used wavelet method for obtaining solution of the fractional pantograph differential equations; Rahimkhani et al. [14] applied fractional-order wavelet method; Mahboob et al. [15] found a numerical solution of the Benjamin-Bona-Mahony-Burgers equation based on finite difference method of sixth order.

FDEs include distributed-order fractional derivatives can serve as a normal extension of the multi-term fractional different equation and single order fractional different equation. To find solution of the distributed-order time FDEs, Mashayekhi and Razzaghi [16] applide a numerical method based on the Bernoulli polynomials and block-pulse functions; Chen et al. [17] studied a numerical method based on the finite difference/spectral to solve the time-fractional reaction-diffusion equation of distributed-order; Aminikhah et al. [18] found a combined Laplace transform and new homotopy perturbation method for solving a special class of the distributed order fractional Riccati equation; Aminikhah et al. [19] proposed two numerical methods for solving distributed-order fractional Bagley-Torvik equation; Mashoof et al. [20] used a numerical method based on operational matrix of fractional order integration from fixed point for distributed order FDEs; Mashoof et al. [21] found the stability of two classes of distributed-order Hilfer-Prabhakar differential equations; Mashoof et al. [22] applied the stability of distributed order form of Hilfer-Prabhakar differential equations; Li and Wu [23] showed the numerical method for finding solutions of the time-fractional diffusion equation of distributed-order with variable coefficients; Liu et al. [24] studied a numerical method based on the finite volume method to obtain such equation; Fan and Liu [25] used a numerical method based on the finite element method for solving a two-dimensional fractional diffusion equation of distributed-order on an irregular convex domain; Jia and Wang [26] extended a fast finite difference method for the fractional partial differential equation of distributed-order on convex domains; Morgado et al. [27] used a numerical method based on the Chebyshev collocation method for solving distributed order FDEs; Zaky [28] studied a Legendre collocation method for solution of fractional optimal control equations distributed-order; Hu et al. [29] used an implicit numerical method; Ye et al. [30] showed an implicit difference method for solving the distributed-order time fractional equation; Ford et al. [31] investigated an implicit difference method for solving the distributed-order time fractional equation; Gao and Sun [32] showed some alternating direction implicit difference methods for solving the distributed-order time fractional equation; Jin et al. [33] studied a rigorous numerical analysis for the distributed-order time fractional diffusion equation.

The COVID-19 are a large collection of viruses which have a specified corona or ’crown’ of sugary-proteins and because of their form, they were called COVID-19 in 1960. Due to the World Health Organization (WHO), COVID-19 is spreaded via people who have been infected with the corona virus. The virus may quickly transmit via small drops from the mouth compilation or nose of anybody infected via this virus to cough or sneeze. The small drops then land on surfaces or objects which are touched and the healthy person regulates their nose, mouth or eyes. For the first time in the Wuhan city the COVID-19 was appeared that this virus has not been previously known in humans. Bats or snakes have been skepticed as a potential source for the prevalence, though other experts currently consider this unlikely. Cough, fever, breathing difficulties and shortness of breath are the initial signs of this infection. In the next stages, the infection may reason pneumonia, kidney failure, even death and severe acute respiratory syndrome.

In this paper, we introduce a class of distributed order time fractional Coronavirus-19 disease involving one Caputo-Prabhakar derative in time t. For solving this modol, we use the midpoint quadrature method and finite difference method for discretizing the distributed order time fractional derivative and Caputo-Prabhakar derative.

The remainder of the paper is organized as follows. Section 2 we give some notations, basic definitions and lemma of fractional calculus. In Section 3, we show the finite difference method for solving the distributed order time fractional Coronavirus-19 disease. In Section 4, we illustrate some numerical examples using the presented method.

2 | Preliminaries and Some Notations

In this section, we introduce the basic definitions of Prabhakar fractional integral and derivative, Caputo-Prabhakar derative and some basic lemmas which are applied for later.

Definition 1 ([39]). Let $m-1<\Re(\mu)\leq m$ and $u\in L^1[0,b]$, $0<t<b\leq\infty$. Then the left-sided and the right-sided Prabhakar fractional integrals are given as

$$\begin{aligned}(\mathbf{E}^{\gamma}_{\rho,\mu,\omega,0^+}u)(t) &= \int_0^t (t-\tau)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(t-\tau)^{\rho})u(\tau)d\tau,\\ (\mathbf{E}^{\gamma}_{\rho,\mu,\omega,b^-}u)(t) &= \int_t^b (\tau-t)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(\tau-t)^{\rho})u(\tau)d\tau.\end{aligned} \tag{1}$$

Definition 2 ([39]). Let $u\in L^1[0,b]$, then for $m-1<\Re(\mu)\leq m$, the left-sided and the right-sided Prabhakar fractional derivatives are given by

$$\begin{aligned}(D^{\gamma}_{\rho,\mu,\omega,0^+}u)(t) &= \frac{d^m}{dt^m}\mathbf{E}^{-\gamma}_{\rho,m-\mu,\omega,a^+}u(t),\\ (D^{\gamma}_{\rho,\mu,\omega,b^-}u)(t) &= (-1)^m\frac{d^m}{dt^m}\mathbf{E}^{-\gamma}_{\rho,m-\mu,\omega,b^-}u(t).\end{aligned} \tag{2}$$

Also, for the given absolutely continuous function u, the Caputo-Prabhakar fractional derivatives is given by

$$^{CP}D^{\mu}_t u(t) = \mathbf{E}^{-\gamma}_{\rho,m-\mu,\omega,0^+}\frac{d^m}{dt^m}u(t), \tag{3}$$

Where for $m=1$, *Eq. (8)* is obtained.

Lemma 1 ([3]). Suppose that $\rho,\gamma,\mu,\omega\in\mathbb{C}$ that $\Re(\rho),\Re(\mu)>0$. Then the following relation is resulted.

$$\int_0^t \tau^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega\tau^{\rho})d\tau = t^{\mu}E^{\gamma}_{\rho,\mu+1}(\omega t^{\rho}). \tag{4}$$

2.1 | Description of the Distributed Order Time Fractional Coronavirus-19 Disease Model

Here, we study the distributed order time fractional Coronavirus-19 disease (COVID-19) model as

$$\begin{aligned}
^{CP}\mathbb{D}^{\zeta(\mu)}_t S(t) &= -S(t)\big(\alpha_1 I(t)+\alpha_2 D(t)+\alpha_3 A(t)+\alpha_4 R(t)\big),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t I(t) &= S(t)(\alpha_1 I(t)+\alpha_2 D(t)+\alpha_3 A(t)+\alpha_4 R(t))-(\epsilon_1+\zeta_1+\lambda_1)I(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t D(t) &= \epsilon_1 I(t)-(\eta_1+\rho_1)D(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t A(t) &= \zeta_1 I(t)-(\theta_1+\mu_1+\kappa_1)A(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t R(t) &= \eta_1 D(t)+\theta_1 A(t)-(\nu_1+\xi_1)R(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t T(t) &= \mu_1 A(t)+\nu_1 R(t)-(\sigma_1+\tau_1)T(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t H(t) &= \lambda_1 I(t)+\rho_1 D(t)+\kappa_1 A(t)+\xi_1 R(t)+\sigma_1 T(t),\\
^{CP}\mathbb{D}^{\zeta(\mu)}_t E(t) &= \tau_1 T(t).
\end{aligned} \tag{5}$$

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With the initial conditions

$$\mathrm{S}(0) = \mathrm{S}_0, \mathrm{I}(0) = \mathrm{I}_0, \mathrm{D}(0) = \mathrm{D}_0, \mathrm{A}(0) = \mathrm{A}_0, \mathrm{R}(0) = \mathrm{R}_0, \mathrm{T}(0) = \mathrm{T}_0, \mathrm{H}(0) = \mathrm{H}_0, \mathrm{E}(0) = \mathrm{E}_0. \quad (6)$$

Also, all parameters introduced in the given model are positive parameters and their physical interpretation and meaning can be given in [34]. In *Eq. (5)*, $S(t)$ is the class of susceptible, $I(t)$ is the class of infected asymptomatic infected undetected, $D(t)$ is the class of asymptomatic infected, detected, $A(t)$ is ailing symptomatic infected, undetected, $R(t)$ is recognized symptomatic infected, detected, $T(t)$ is the class of acutely symptomatic infected detected, $H(t)$ is the healed class, $E(t)$ is the death class and ${}^{CP}\mathbb{D}_t^{\zeta(\mu)}$ displays the distributed order fractional derivative of given functions in time t, defined by:

$$ {}^{\mathrm{CP}}\mathbb{D}_{\mathrm{t}}^{\zeta(\mu)} = \int_0^1 \zeta(\mu) {}^{\mathrm{CP}}\mathrm{D}_{\mathrm{t}}^{\mu} \mathrm{d}\mu, \mu \in (0,1], \quad (7)$$

where $\zeta(\mu) > 0$ is the weight function and;

$$\int_0^1 \zeta(\mu)\mathrm{d}\mu = \mathrm{C}, \mathrm{C} > 0. \quad (8)$$

Further, the symbol ${}^{CP}D_t^{\mu}$ is the Caputo-Prabhakar derivative of μ order in time t and it is given by [39]

$$\int_0^1 \zeta(\mu)\mathrm{d}\mu = \mathrm{C}, \mathrm{C} > 0. \quad (9)$$

Here $E_{\rho,\mu}^{\gamma}(\omega t^{\rho})$ is a generalization of one-parameter Mittag-Leffler and two-parameter Mittag-Leffler functions and it is defined by [39]:

$$\int_0^1 \zeta(\mu)\mathrm{d}\mu = \mathrm{C}, \mathrm{C} > 0. \quad (10)$$

Due of the plenty application and use of the generalized Mittag-Leffler function in fractional calculus a reason was to select this type of the Caputo-Prabhakar fractional derivative of order μ. Applications of the three-parameter Mittag-Leffler function may be used in mathematical fields as physics and stochastic processes, electromagnetic, various materials, viscosity and several media [35]-[38].

3 | The Numerical Method for Solving Distributed Order Time Fractional Coronavirus-19

In this section, we apply a finite difference method to approximate the solution of *Eq. (5)*. For this aim, first, we apply a quadrature method to approximate the distributed order fractional derivative term on the left-hand side of *Eq. (5)*. We consider a segmentation of $[0,1]$, the interval $[0,1]$ where the order of the derivative in time t lies, into N sub-intervals $[\varpi_{j-1}, \varpi_j]$ that $j = 1,2, \dots, N$ with $h = \frac{1}{N}$. Showing the midpoints of any one of sub-intervals $[\varpi_{j-1}, \varpi_j]$ as

$$\mu_j = \frac{\varpi_{j-1} + \varpi_j}{2}, \quad (11)$$

where $j = 1,2, \dots, N$. Therefore, we apply the midpoint method for the approximation of the distributed order fractional derivative term in *Eq. (5)*, then we have

$$
\begin{aligned}
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}S(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}S(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}S(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}S(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}I(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}I(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}I(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}I(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}D(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}D(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}D(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}D(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}D(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}D(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}A(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}A(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}R(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}R(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}R(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}R(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}T(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}T(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}T(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}T(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}H(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}H(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}H(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}H(t))'',\\
&{}^{CP}\mathbb{D}_t^{\zeta(\mu)}E(t) = \int_0^1 \zeta(\mu){}^{CP}D_t^{\mu}E(t)d\mu = h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}E(t) - \frac{h^2}{24}(\zeta(\mu)^{CP}D_t^{\mu}E(t))''.
\end{aligned}
\tag{12}
$$

Putting *Eq. (12)* into *Eq. (5)* by neglecting $O(h^2)$, we obtain

$$
\begin{aligned}
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}S(t) = -S(t)(\alpha_1 I(t) + \alpha_2 D(t) + \alpha_3 A(t) + \alpha_4 R(t)),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}I(t) = S(t)(\alpha_1 I(t) + \alpha_2 D(t) + \alpha_3 A(t) + \alpha_4 R(t)) - (\epsilon_1 + \zeta_1 + \lambda_1)I(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}D(t) = \epsilon_1 I(t) - (\eta_1 + \rho_1)D(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}A(t) = \zeta_1 I(t) - (\theta_1 + \mu_1 + \kappa_1)A(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}R(t) = \eta_1 D(t) + \theta_1 A(t) - (\nu_1 + \xi_1)R(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}T(t) = \mu_1 A(t) + \nu_1 R(t) - (\sigma_1 + \tau_1)T(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}H(t) = \lambda_1 I(t) + \rho_1 D(t) + \kappa_1 A(t) + \xi_1 R(t) + \sigma_1 T(t),\\
&h\sum_{j=1}^{N}\zeta(\mu_j)^{CP}D_t^{\mu_j}E(t) = \tau_1 T(t).
\end{aligned}
\tag{13}
$$

Next we discretize the time derivatives. For simplicity, $S(t_j)$, $I(t_j)$, $D(t_j)$, $A(t_j)$, $R(t_j)$, $T(t_j)$, $H(t_j)$, $A(t_j)$ and $E(t_j)$ are define as $S_j, I_j, D_j, A_j, R_j, T_j, H_j$ and E_j, respectively. Then the value of Caputo-Prabhakar derivative ${}^{CP}D_t^{\mu_j}$ for given functions as $S(t)$ at the nodes $t_l = l\Delta t, l = 0,1, \dots, N, \Delta t = \frac{1}{N}$ can be calculated as

$$
\begin{aligned}
{}^{CP}D_t^{\mu_j} S(t)|_{t=t_l} &= \int_0^{t_l} (t-\tau)^{-\mu_j} E_{\rho,1-\mu_j}^{-\gamma}(\omega(t-\tau)^{\rho}) S\prime(\tau) d\tau, \\
{}^{CP}D_t^{\mu_j} S(t)|_{t=t_l} &\simeq \sum_{m=0}^{l} \frac{S_{m+1} - S_m}{\Delta t} \int_{t_l}^{t_{l+1}} (t-\tau)^{-\mu_j} E_{\rho,1-\mu_j}^{-\gamma}(\omega(t-\tau)^{\rho}) d\tau.
\end{aligned}
\tag{14}
$$

Applying *Eq. (4)*, we obtain

$$
{}^{CP}D_t^{\mu_j} S(t)|_{t=t_l} \cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} S_m, \tag{15}
$$

where the coefficients $a_{m,l}^{(\mu_j)}$ are obtained by

$$
a_{m,l}^{(\mu_j)} = (\Delta t)^{-\mu_j}
\begin{cases}
0 & \text{for } l = 0, \\
(l-1)^{1-\mu_j} E_{\rho,2-\mu_j}^{-\gamma}(\omega(l-1)^{\rho}(\Delta t)^{\rho}), & \\
-l^{1-\mu_j} E_{\rho,2-\mu_j}^{-\gamma}(\omega l^{\rho}(\Delta t)^{\rho}) & \text{for } l > 0 \ \wedge m = 0, \\
(l-m+1)^{1-\mu_j} E_{\rho,2-\mu_j}^{-\gamma}(\omega(l-m+1)^{\rho}(\Delta t)^{\rho}), & \\
-2(l-m)^{1-\mu_j} E_{\rho,2-\mu_j}^{-\gamma}(\omega(l-m)^{\rho}(\Delta t)^{\rho}), & \\
+(l-m-1)^{1-\mu_j} E_{\rho,2-\mu_j}^{-\gamma}(\omega(l-m-1)^{\rho}(\Delta t)^{\rho}) & \text{for } l > 0 \ \wedge m = 1,2,\cdots,l-1, \\
E_{\rho,2-\mu_j}^{-\gamma}(\omega(\Delta t)^{\rho}), & l > 0 \ \wedge m = l.
\end{cases}
\tag{16}
$$

Similarly, for functions I, D, A, R, T, H and E, we have

$$
\begin{aligned}
{}^{CP}D_t^{\mu_j} I(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} I_m, \\
{}^{CP}D_t^{\mu_j} D(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} D_m, \\
{}^{CP}D_t^{\mu_j} A(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} A_m, \\
{}^{CP}D_t^{\mu_j} R(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} R_m, \\
{}^{CP}D_t^{\mu_j} T(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} T_m, \\
{}^{CP}D_t^{\mu_j} H(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} H_m, \\
{}^{CP}D_t^{\mu_j} E(t)|_{t=t_l} &\cong \sum_{m=0}^{l} a_{m,l}^{(\mu_j)} E_m.
\end{aligned}
\tag{17}
$$

Substituting *Eqs. (15)* and *(17)* into *Eq. (13)*, we get the following finite difference method to approximate of the solution of *Eq. (5)* as

$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}S_m(t_l) = -S(t_l)\big(\alpha_1 I(t_l) + \alpha_2 D(t_l) + \alpha_3 A(t_l) + \alpha_4 R(t_l)\big),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}I_m(t_l) = S(t_l)(\alpha_1 I(t_l) + \alpha_2 D(t_l) + \alpha_3 A(t_l) + \alpha_4 R(t_l)) - (\epsilon_1 + \zeta_1 + \lambda_1)I(t_l),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}D_m(t_l) = \epsilon_1 I(t_l) - (\eta_1 + \rho_1)D(t_l),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}A_m(t_l) = \zeta_1 I(t_l) - (\theta_1 + \mu_1 + \kappa_1)A(t_l), \quad (18)$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}R_m(t_l) = \eta_1 D(t_l) + \theta_1 A(t_l) - (\nu_1 + \xi_1)R(t_l),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}T_m(t_l) = \mu_1 A(t_l) + \nu_1 R(t_l) - (\sigma_1 + \tau_1)T(t_l),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}H_m(t_l) = \lambda_1 I(t_l) + \rho_1 D(t_l) + \kappa_1 A(t_l) + \xi_1 R(t_l) + \sigma_1 T(t_l),$$
$$h\sum_{j=1}^{N}\zeta(\mu_j)\sum_{m=0}^{1}a_{m,l}^{(\mu_j)}E_m(t_l) = \tau_1 T(t_l).$$

Then, the finite difference numerical method for solving *Eq. (5)* provided by *Eq. (18)*.

4 | Numerical Experiments

In this section, we present numerical experiments to study the performance and efficiency of the finite difference method which is introduced in Section 3 for the distributed order time fractional Coronavirus-19 disease (5). We use here all the computations done in Matlab (R2020b) software for the problems implemented in numerical experiments. For this end, we let that the general population is $N = 100$. Here, we show numerical experiments for various values of distributed order μ. The numerical experiments are reported in *Figs. (1)-(8)*. We observed that with this given method all types are increasing exponentially. The numerical results obtained in *Figs. (1)-(8)* illustrates the comparison between the numerical experiments demonstrated for the distributed order time fractional COVID-19 virus model with Caputo-Prabhakar derivative for different values of μ and the distributed order time fractional COVID-19 virus model with Caputo-Prabhakar derivative for for one value of $\mu = 1$, arbitrarily selected. The graphical numerical simulations present that the model depends especially to the fractional order μ and the chosen system parameters. The new described model with generalized Mittag Leffler function kernel authorizations give anomalous spread like that infection biological systems. This new given model authorizations a better explanation of the history of the biological process.

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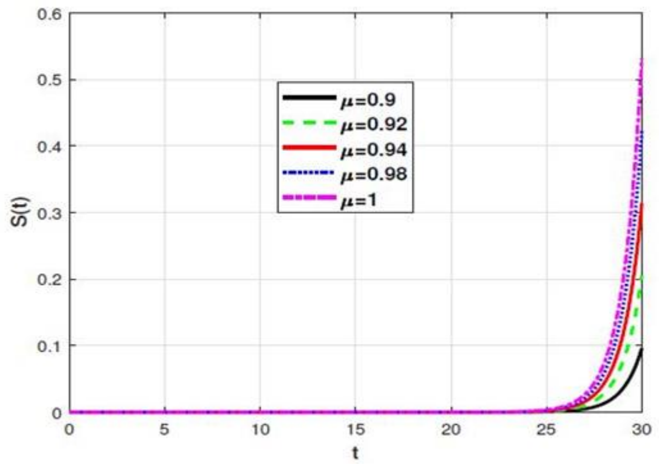

Fig. 1. Numerical experiments of susceptible class for various value of fractional order μ.

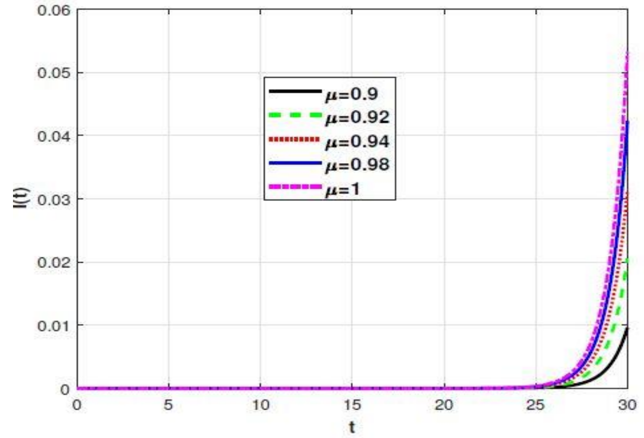

Fig. 2. Numerical experiments of infected asymptomatic infected undetected class for various value of fractional order μ.

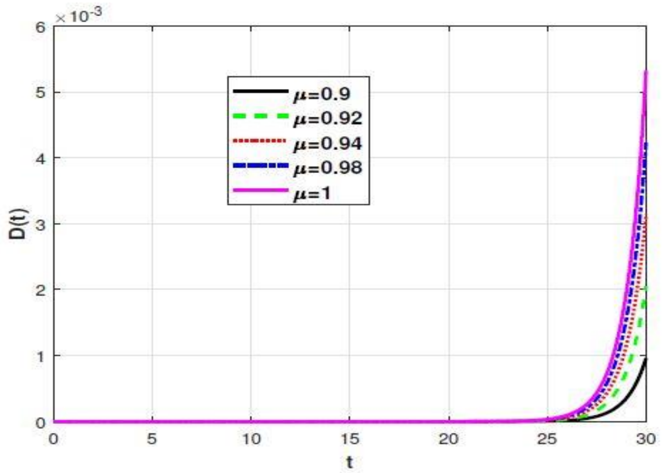

Fig. 3. Numerical experiments of asymptomatic infected class for various value of fractional order μ.

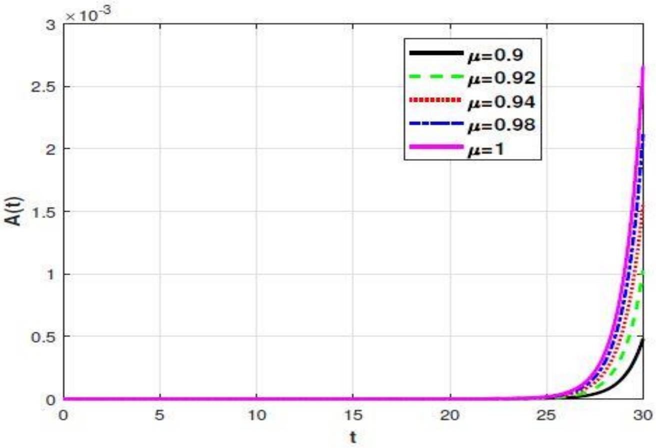

Fig. 4. Numerical experiments of healed class for various value of fractional order μ.

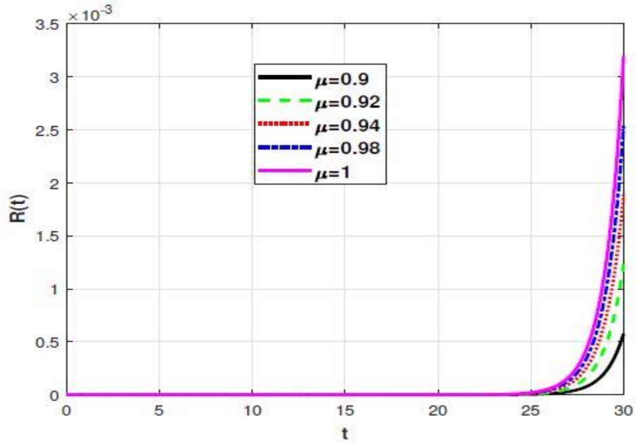

Fig. 5. Numerical experiments of ailing symptomatic infected class for various value of fractional order μ.

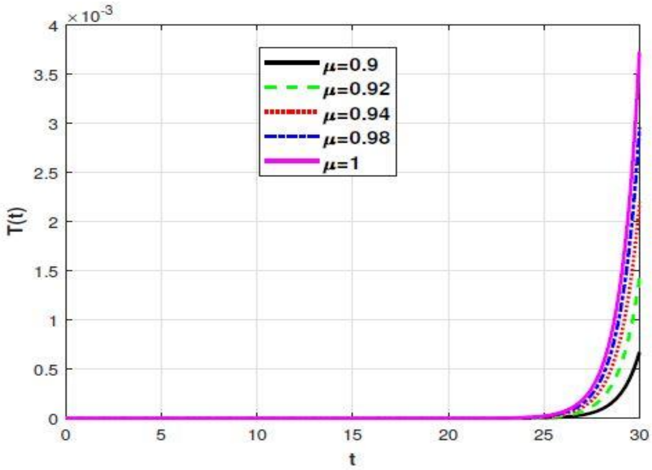

Fig. 6. Numerical experiments of recognized symptomatic infected class for various value of fractional order μ.

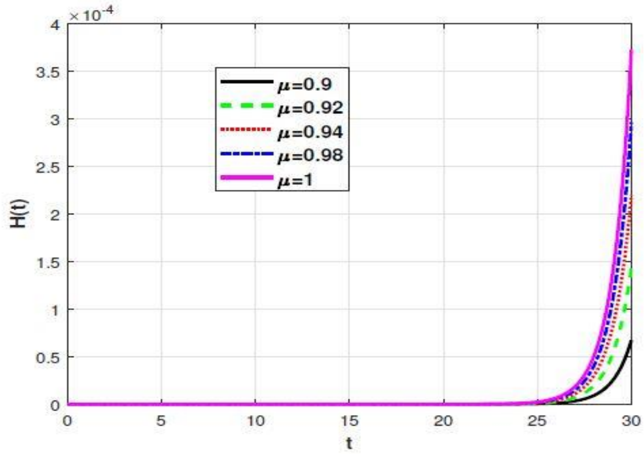

Fig. 7. Numerical experiments of acutely symptomatic infected detected class for various value of fractional order μ.

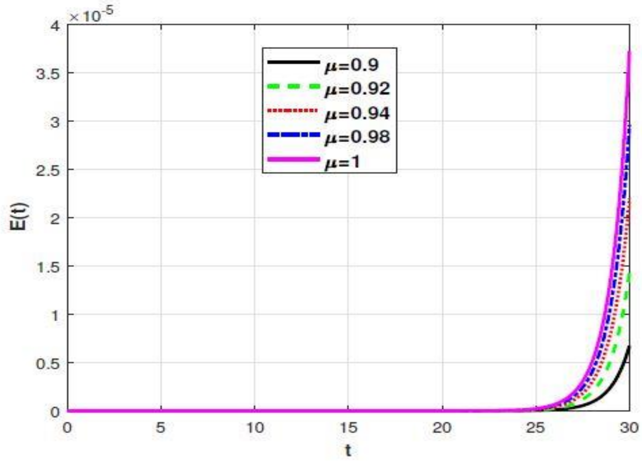

Fig. 8. Numerical experiments of death class for various value of fractional order μ.

5 | Conclusion

Because of recent expansion in the fractional calculus, scientists and authors are many interested in the comparative study of the integer order models with the fractional order models. In this paper, we shown a numerical approach based on the midpoint quadrature method and finite difference method to obtain the solution of distributed order time fractional Coronavirus-19 disease. The fractional order derivative is used in the Caputo-Prabhakar sense. We get numerical solutions in convergent series. The midpoint quadrature and finite difference techniques reduces the introduced systems to a equation of non-linear algebraic equations. The obtained equation is then solved by Newton-Raphson method. In all of the items, we obtained a remarkably well agreement. Finally, the dynamic behavior of the distributed order time fractional COVID-19 virus model was showed by allocating various values to the order of the fractional derivative as well as for various values of the other parameters involved. With the numerical experiments, we show a picture for COVID-19, which presents the rapid transmission of the virus to various sets of people. The high applicability of the discussed results in Section 4 demonstrate the high accuracy and practical applicability of the considered method.

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Conflicts of Interest

There is no conflict of interest in connection with this paper.

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