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A New Dynamical Behaviour Modeling for a Four-Level Supply Chain: Control and Synchronization of Hyperchaotic

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Abstract

This paper, presents a mathematical model of a four-level supply chain under hyperchaos circumstances. The analysis of this model shows that the hyper-chaotic supply chain has an unstable equilibrium point. Using Lyapunov's theory of stability, the problem of designing a hyperchaotic supply chain control is investigated. The design of the nonlinear controller is performed first to synchronize two identical hyper-chaotic systems with different initial conditions and then to eliminate the chaotic behavior in the supply chain and move to one of unstable equilibrium points, as well as different desired values at different times. A different supply chain is predicted to demonstrate the performance of the controller. In the next part of numerical simulation, with the control of the distributor as the center of gravity of the model, the stability of the entire chaotic supply chain can be achieved. The most important point in designing a control strategy is the ability to implement it in the real world. Numerical simulation results in all stages show that the applied nonlinear control policy can provide supply chain stability in a short period of time, also, the behavior of control signals has low amplitude and oscillations. In other words, it represents a low cost to control the hyperchaotic supply chain network.

Keywords: Supply chain, Hyperchaotic, Nonlinear, Stability, Lyapunove.

1 | Introduction

Chaotic dynamical systems have complex, nonlinear behaviors. The most important characteristics of chaotic dynamic systems are: high sensitivity to initial conditions, amazing attractions and the presence of at least one positive view of Lyapunov. In modeling natural phenomena, the more we move towards real models, the more complexities and uncertainties we encounter. Hence, the closer we consider the said effects in modeling these phenomena, the closer we get to the real situation. In other words, all-natural phenomena have linear, nonlinear, chaotic and even hyperchaotic models. Depending on what our purpose of modeling is, these models are selected. The famous meteorologist, Edward Lorenz, first proposed the turbulent model of meteorology in [1]. Since then, the notion of chaos has been used in engineering sciences. The chaos in engineering sciences has been conductrd in different applications: aerospace [2] and [3], guidance and navigation [4], electric motors [5], microelectromechanics [6], secure telecommunications [7] and [8], and many others applications [9]

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and [10]. The issue of chaos control was first raised in the synchronization of two chaotic systems in 1990 [11]. As mentioned, chaos behaves inappropriately, therefore, one of the efforts of researchers in this field is to control and eliminate chaotic behavior in dynamic systems. The various control goals for a chaotic system are: 1) Elimination of chaos and stability at one of the equilibrium points. 2) Synchronization of two chaotic systems, 3) control of two branches. For these control purposes, several methods such as nonlinear control [12], fuzzy control [13], adaptive control [14], adaptive slip model control [15], and also sliding fuzzy control [16] were mentioned. The purpose of a supply chain is to deliver a product at the right time and place to customers. Companies need a supply chain to increase competitiveness. The supply, production, distribution, and retail sectors must be fully stable, so that the supply chain become ultimately stable. The hyperchaos system was first introduced by Russler [17]. After that this has been introduced hyperchaos Chua's circuit [18], hyperchaotic Lorenz system [19], hyperchaotic Chen system [20]. Hyper-chaotic systems, characterized as a chaotic attractor with more than one positive Lyapunov exponents, can generate much more complicated dynamics [21].

Over the past decade, supply chain researchers have turned their attention to modeling, planning, analysis, and design with a nonlinear perspective [22] and [23]. Studies in supply chain models show that some of them can have complex or nonlinear behaviors [24] and [25]. For example, how customer behavior, distributor efforts to control inventory levels, as well as factory production or raw material supply, can disrupt the supply chain. The whipping effect in the supply chain model, in which information flow is also disrupted, causes the supply chain behavior to be nonlinear, especially if orders increase at each level. In [26], synchronization of chaotic supply chain is considered by considering the whipping effect by radial base neural network method. The supply chain model of this three-tier method includes manufacturer, distributor and consumer. Uncertainty in all three levels is also considered for this model. In [27], the same model is used with the adaptive sliding model control method to control and synchronize the chaotic supply chain. In this method, an attempt has been made to adjust the control parameters in such a way as to eliminate the chatting phenomenon, which has caused the time to reach zero error to be slightly increased. The control signal also has chattering. The use of neural network method has been suggested in [28]. The linear feedback control method and the neural networks are compared. The important point after designing the controller is the cost of its implementation. In this method, the control signal is not depicted. In other words, the final cost of this method will be unknown. The robust control for the five-level supply chain network is introduced in [29]. As mentioned earlier, the control signal, which represents the cost of design, is not depicted. In [30] the active control method is used to control the hyperchaotic supply chain. In the results of this method, it can be seen that the time to reach zero error is long. In other words, supply chain management will be costly.

This article, consider hyperchaos control in the supply chain network using a nonlinear controller. Control policy (or control signal) is a very important issue in supply chain stability. In the other hand, the cost of control and stability of the supply chain network is related to the design of the control policy. Sometimes the supply chain is stable, but the costs are very high. The behavior of hyperchaotic dynamics is highly oscillated, so it can increase supply chain control costs. Therefore, if the control policy (or control signal) has low amplitude and oscillations, the cost of supply chain control will be low. Numerical simulation refers to three important parts. First, synchronization of two hyperchaotic supply chains with different initial conditions, then, stability and elimination of hyperchaotic behavior in the supply chain network and in the third part, optimization of the controller implementation cost are discussed. The simulation results show that the intended target function is completely achievable by the nonlinear control. Also, the amplitude and oscillations of the hyperchaos supply chain control are illustrated.

This paper is organized as follows: Section 2 refers to modeling and mathematical analysis of the hyper-chaotic supply chain network will be examined. In Section 3, the proposed method for supply chain control and stability is described. In Section 4, the results of numerical simulation will be illustrated and analyzed. Finally, the concluding remarks in the lase section.

2 | Dynamic Supply Chain Modeling

In this section, we consider the design of a supply chain model of raw material, manufactures, distributiors and retailers. *Fig. 1* illustrates the information and product flow of a supply chain network. As mentioned earlier, this network is a four-level supply chain. Model parameters are:

i. Time period.

a. Delivery rate of the distributor to the retailer.

b. Estimation of customer request.

c. The degree of distortion of customer request information for products.

d. Distributor inventory correction factor.

e. Safety factor of products produced in the factory.

f. The supply of raw materials in the factory.

g. Safety factor of raw material supplier.

h. The amount of factory demand for raw materials.

x. The amount of the retailer request in the current period.

y. The amount that the distributor can distribute in the current period.

z. Number of products produced in the factory for the current period.

w. The number of raw materials to produce the product at the request of the factory in the current period.

The first step in supply chain modeling is to find the connection between these four elements. In other words, the relationship between the retailer and the supplier and how the uncertainties between the retailers and the distributora are seen in the factory mathematically formulated.

Assumption 1. Information is transmitted along the supply chain with a delay of one-time unit. Thus, the behavior of the model in stage i is affected by the information in stage i-1. The customer requests at a rate and the distributor can meet the demand response with a coefficient.

$$x_i = a y_{i-1} - b x_{i-1} + y_{i-1} z_{i-1}, \tag{1}$$

which, a is the coefficient of delivery of the product from the distributor to the retailer in the previous stage and b is the satisfaction of the retailer in the previous stage. Uncertainty between distributor and manufacturer is $y_{i-1} z_{i-1}$.

Receive retailer requests with a coefficient of information deviation (at the distributor station, retail requests are processed at a rate and will be prepared with a coefficient). And on the other hand, they are always looking to control their inventory level. For this scenario we have *Eq. (2)*. Therefore:

$$y_i = c x_{i-1} + d y_{i-1} - z_{i-1} x_{i-1}. \tag{2}$$

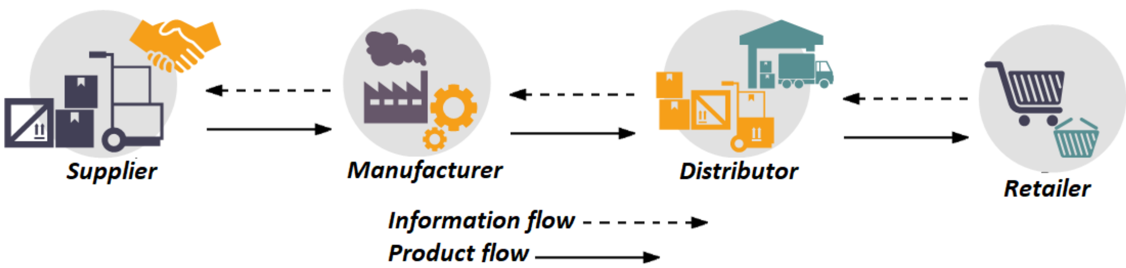

Fig. 1. Schematic representation of a four-level supply chain.

In *Eq. (2)*, c is the rate of distortion of the information of the products required by the retailer that reaches the distributor, d is the coefficient of control of the distributor's inventory level as well as the uncertainty between the retailer and the manufacturer $z_{i-1}x_{i-1}$. The output of each stage of production depends on two major factors. Production line safety factor and the coefficient of raw material supplier. Thus, in each step, these two coefficients are obtained from the previous step.

$$z_i = -ez_{i-1} - fw_{i-1} + y_{i-1}x_{i-1}, \tag{3}$$

which e is the safety factor of the product in the factory, and f is the demand factor for the raw materials supplier. There $y_{i-1}x_{i-1}$ is also uncertainty between the retailer and the distributor. The task of this level is to supply the raw materials of the factory to produce the product. The amount of raw material supply at the supplier level depends on the request of the factory in the previous stage and also the safety factor of raw material supply in each stage depends on the previous stage. So

$$w_i = -gw_{i-1} - hz_{i-1} + y_{i-1}x_{i-1}. \tag{4}$$

Which g is the safety stock of raw material supply for suppliers and h is the amount of factory raw material demand at each stage, which depends on the previous stage. There $y_{i-1}x_{i-1}$ is also uncertainty between the retailer and the distributor. According to *Eq. (1)* to *(4)*, *Eq. (5)* is obtained.

$$\begin{aligned} x_i &= ay_{i-1} - bx_{i-1} + y_{i-1}z_{i-1}. \\ y_i &= cx_{i-1} + dy_{i-1} - z_{i-1}x_{i-1}. \\ z_i &= -ez_{i-1} - fw_{i-1} + y_{i-1}x_{i-1}. \\ w_i &= -gw_{i-1} - hz_{i-1} + y_{i-1}x_{i-1}. \end{aligned} \tag{5}$$

Where a, b, c, d, e, f, g, h are system parameters, always positive and x, y, z, w are system variables. If $\alpha = a = b = 50$ and $\beta = c = d = 24$ and $e = 13, g = 8, f = 33, h = 30$ and if i (time period in the supply chain) is small enough, then *Eq. (5)* is rewritten as a dynamic *Eq. (6)*.

$$\begin{aligned} \dot{x} &= \alpha(y - x) + yz. \\ \dot{y} &= \beta(x_{i-1} + y_{i-1}) - zx. \\ \dot{z} &= -ez - fw + yx. \\ \dot{w} &= -gw - hz + yx. \end{aligned} \tag{6}$$

Also, its initial condition $[x(0), y(0), z(0), w(0)]^T = [3, 1, 2, 2]^T$. *Eq. (6)* is known as the hyperchaotic equations Qi. These equations were introduced by Qi et al. [31]. With the proposed model, and its implementation for the four-level supply chain, a new model of hyperchaotic supply chain is introduced. *Fig. 2* shows the behavior of the hylerchaotic variables of the four-level supply chain. If i is small enough, then *Eq. (5)* is rewritten as dynamic *Eq. (6)*.

Chaotic and hyperchaotic systems have unstable equilibrium points. An equilibrium points for any linear and nonlinear equations is zero. Therefore:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -b & a & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & -e & -f \\ 0 & 0 & -h & -g \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}_{\substack{x=0 \\ y=0 \\ z=0 \\ w=0}} . \tag{7}$$

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By calculating the eigenvalues of the linearized matrix around the equilibrium points: $\lambda_1 = -63.68, \lambda_2 = 37.68, \lambda_3 = -42.06, \lambda_4 = -21.06$. The hyperchaotic dynamic equations of the supply chain at the equilibrium point are unstable.

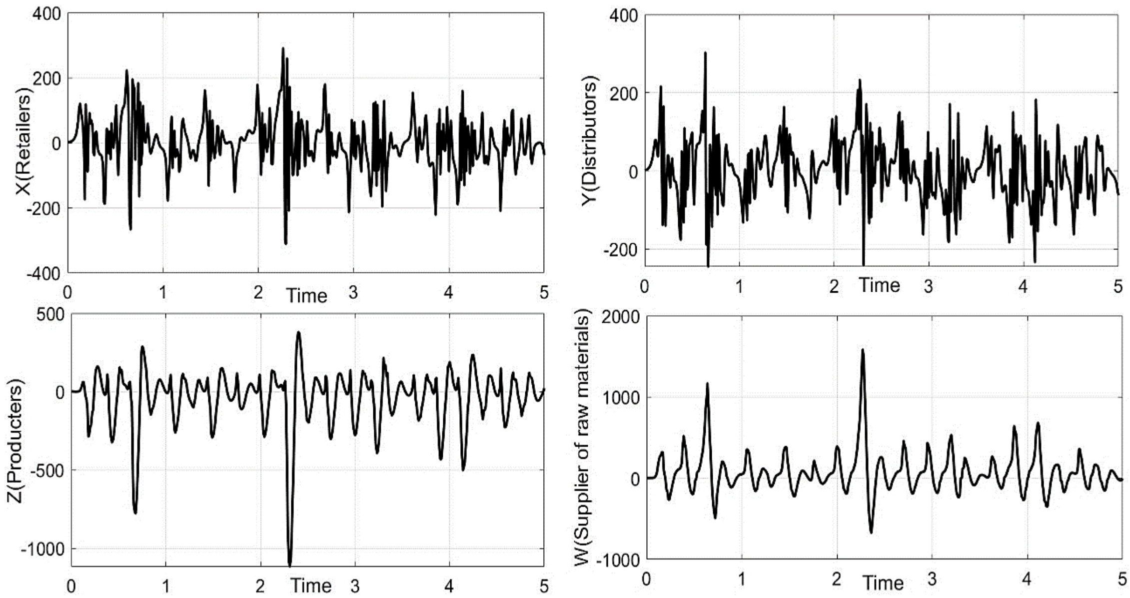

Fig. 2. Hyperchaotic supply chain behavior.

3 | Synchronization of Qi Hyperchaotic Supply Chain

Synchronization of two hyperchaotic system, means controlling the output of the following system so that it follows the output of the master system. Given that the hyperchaotic supply chain model was proven in the previous section, it will now be designed to control the supply chain stability. *Fig. 3* shows the synchronization scheme of two hyperchaotic supply chains.

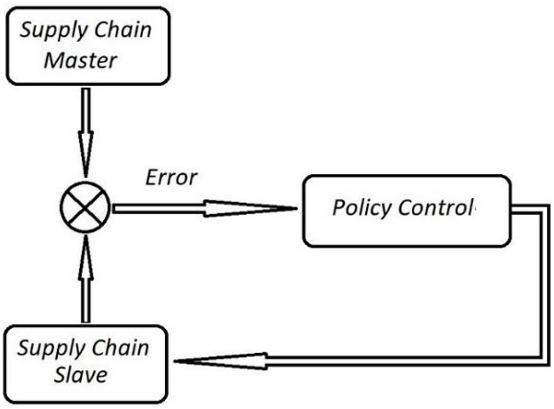

Fig. 3. The concept of chaos synchronization.

The master supply chain is

$$
\begin{aligned}
\dot{x}_1 &= \alpha(y_1 - x_1) + y_1 z_1. \\
\dot{y}_1 &= \beta(x_1 + y_1) - z_1 x_1. \\
\dot{z}_1 &= -e_1 z_1 - f_1 w_1 + y_1 x_1. \\
\dot{w}_1 &= -g_1 w_1 - h_1 z_1 + y_1 x_1.
\end{aligned}
\tag{7}
$$

Also, the slave supply chain is

$$\begin{aligned} \dot{x}_2 &= \alpha(y_2 - x_2) + y_2 z_2 + u_x. \\ \dot{y}_2 &= \beta(x_1 + y_1) - z_1 x_1 u_y. \\ \dot{z}_2 &= -e_2 z_2 - f_2 w_2 + y_2 x_2 + u_z. \\ \dot{w}_2 &= -g_2 w_2 - h_2 z_2 + y_2 x_2 u_{w.} \end{aligned} \tag{8}$$

That x, y, z, w are variables supply chain, α, β, e, f, g are parameters and u_x, u_y, u_z, u_w are control policies for management of hyper chaotic supply chain system.

In order, supply chain *Eq. (8)* to follow supply chain *Eq. (7)*, they must u_x, u_y, u_z, u_w be designed. As mentioned earlier, one of the most important characteristics of chaotic and hyperchaotic systems is sensitivity to initial conditions. *Eq. (7)* and *(8)* are identical systems, but if the initial conditions of *Systems (7)* and *(8)* are different, their behavior will be different. See *Fig. (4)*.

The first step is to calculate the error between the master and slave systems.

$$\begin{aligned} e_1(t) &= x_2(t) - x_1(t). \\ e_2(t) &= y_2(t) - y_1(t). \\ e_3(t) &= z_2(t) - z_1(t). \\ e_4(t) &= w_2(t) - w_1(t). \end{aligned} \tag{9}$$

The objective function for the model is defined as:

$$\lim_{t\to\infty} \| e_i(t) \| = 0, \quad i = 1,2,3,4, \text{for all} \quad e(0) \in \Re^n. \tag{10}$$

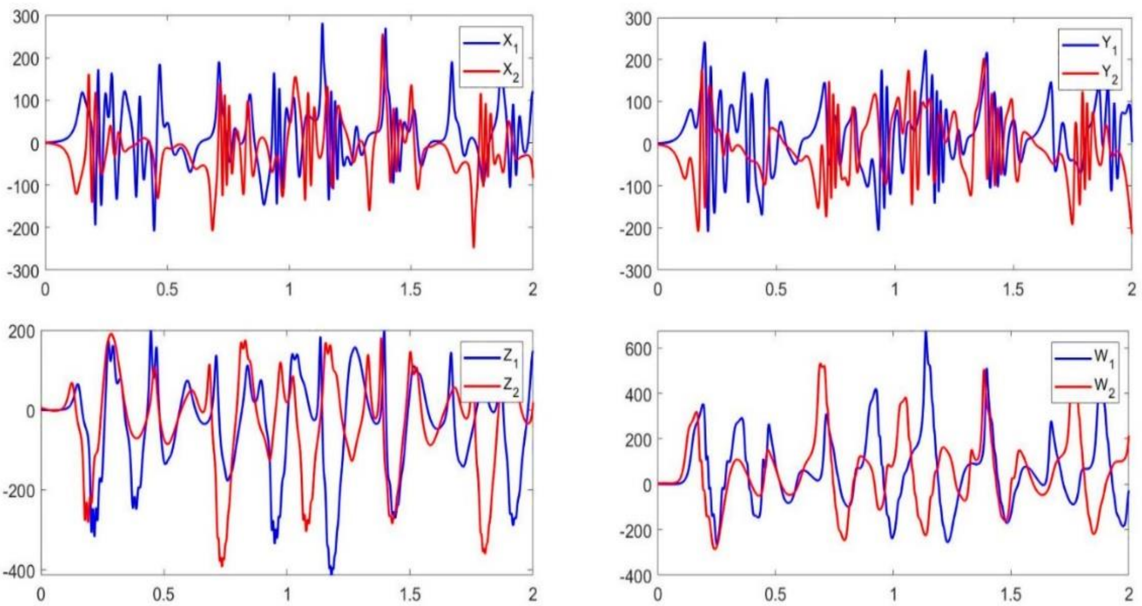

Fig. 4. Hyperchaotic supply chain behavior.

If derived from *Eq. (9)*, and also by substituting *Eq. (7)* and *(8)* in *Eq. (9)*.

$$\begin{aligned} \dot{e}_1(t) &= \alpha(y_2 - x_2) + y_2 z_2 + u_x - (\alpha(y_1 - x_1) + y_1 z_1). \\ \dot{e}_2(t) &= \beta(x_2 + y_2) - z_2 x_2 + u_y - (\beta(x_1 + y_1) - z_1 x_1). \\ \dot{e}_3(t) &= -ez_2 - fw_2 + y_2 x_2 + u_z - (-ez_1 - fw_1 + y_1 x_1). \\ \dot{e}_4(t) &= -gw_2 - hz_2 + y_2 x_2 + u_w - (-gw_1 - hz_1 + y_1 x_1). \end{aligned} \tag{11}$$

Theorem 1. The error of the hyperchaotic supply chain system asymptotically move to zero, if the control policies are designed as follows:

$$
\begin{aligned}
u_x &= -(\alpha(y_2 - x_2) + y_2 z_2) + \alpha(y_1 - x_1) + y_1 z_1 + \lambda_1 e_1.\\
u_y &= -(\beta(x_2 + y_2) - z_2 x_2) + \beta(x_1 + y_1) - z_1 x_1 + \lambda_2 e_2.\\
u_z &= -(-ez_2 - fw_2 + y_2 x) - ez_1 - fw_1 + y_1 x_1 + \lambda_3 e_3.\\
u_w &= -(-gw_2 - hz_2 + y_2 x_2) - gw_1 - hz_1 + y_1 x_1 + \lambda_4 e_4.
\end{aligned}
\tag{12}
$$

There $\lambda_i (i = 1, 2, 3, 4)$ are gains controller.

Proof. According to Lyapunov, if a positive function is definite and its derivative moves to zero and eventually negative over time, the function is shrinking over time.

Consider Lyapunov's candidate function as follows:

$$
V(e_1, e_2, e_3, e_4) = \frac{1}{2}\sum_{i=1}^{4} e_i^2. \tag{13}
$$

By deriving *Eq. (13)*, and substituting *Eq. (11)*:

$$
\begin{aligned}
\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4,\\
\Rightarrow &= e_1(\alpha(y_2 - x_2) + y_2 z_2 + u_x - (\alpha(y_1 - x_1) + y_1 z_1))\\
&\quad + e_2(\beta(x_2 + y_2) - z_2 x_2 + u_y - (\beta(x_1 + y_1) - z_1 x_1))\\
&\quad + e_3(-ez_2 - fw_2 + y_2 x_2 + u_z - (-ez_1 - fw_1 + y_1 x_1))\\
&\quad + e_4(-gw_2 - hz_2 + y_2 x_2 + u_w - (-gw_1 - hz_1 + y_1 x_1)).
\end{aligned}
\tag{14}
$$

Also, if *Eq. (12)* is placed in *Eq. (14)*:

$$
\dot{V} = \lambda_1 e_1^2 + \lambda_1 e_3^2 + \lambda_1 e_3^2 + \lambda_1 e_4^2 \Rightarrow \quad \dot{V} < 0 \quad \text{if} \quad \lambda_1, \lambda_2, \lambda_3, \lambda_4 < 0. \tag{15}
$$

The proof was complete.

3 | Stability of Qi Hyperchaotic Supply Chain

Sometimes the only goal is the stability of the supply chain model. Therefor, the hyperchaotic misbehavior must now be removed from the supply chain model and the supply chain must move toward one of its stable equilibrium points. Thus:

$$
\begin{aligned}
\dot{x}_1 &= \alpha(y_1 - x_1) + y_1 z_1 + u_x.\\
\dot{y}_1 &= \beta(x_1 + y_1) - z_1 x_1 + u_y.\\
\dot{z}_1 &= -ez_1 - fw_1 + y_1 x_1 + u_{z.}\\
\dot{w}_1 &= -gw_1 - hz_1 + y_1 x_1 + u_w.
\end{aligned}
\tag{16}
$$

See *Fig. 5* for a better understanding.

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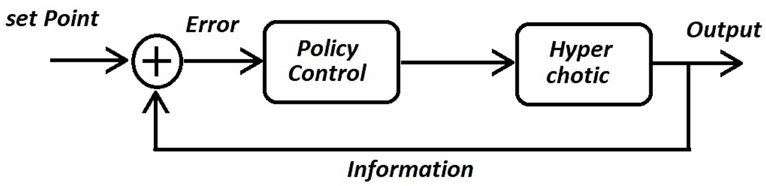

Fig. 5. Supply chain control policy for eliminate hyperchaotic behavior.

Here, control policies must be designed so that the behavior of hyperchaotic supply chain variables moves toward the desired value. For this purpose, we again go to the definition of the error *Function (10)*.

$$\begin{aligned}
&e_1(t)=x(t)-x^*(t).\\
&e_2(t)=y(t)-y^*(t).\\
&e_3(t)=z(t)-z^*(t).\\
&e_4(t)=w(t)-w^*(t).
\end{aligned} \tag{17}$$

There $x^*(t), y^*(t), z^*(t), w^*(t)=0$ are desire or sets points. Derived from the recent equation:

$$\begin{aligned}
&\dot{e}_1(t)=\dot{x}(t)-\dot{x}^*(t).\\
&\dot{e}_2(t)=\dot{y}(t)-\dot{y}^*(t).\\
&\dot{e}_3(t)=\dot{z}(t)-\dot{z}^*(t).\\
&\dot{e}_4(t)=\dot{w}(t)-\dot{w}^*(t).
\end{aligned} \tag{18}$$

Theorem 2. The objective function expressed in *Eq. (10)* moves asymptotically to zero if the control policy in the supply chain model is designed as follows:

There $\lambda_i (i=1,2,3,4)$ are gain controller.

Proof 2. Consider Lyapunov's candidate function as follows:

$$\begin{aligned}
&u_x=-\alpha(y_1-x_1)+y_1 z_1+\lambda_1 e_1.\\
&u_y=-(\beta(x_1+y_1)-z_1 x_1)+\lambda_2 e_2.\\
&u_z=-(-ez_1-fw_1+y_1 x_1)+\lambda_3 e_3.\\
&u_w=-(-gw_1-hz_1+y_1 x_1)+\lambda_4 e_4.
\end{aligned} \tag{19}$$

$$V(e_1,e_2,e_3,e_4)=\frac{1}{2}\sum_{i=1}^{4} e_i^2. \tag{20}$$

By deriving *Eq. (20)*, and substituting *Eq. (18)*:

$$\begin{aligned}
&\dot{V}=e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4=\\
&\Rightarrow e_1(\dot{x}(t)-\dot{x}^*(t))+e_2(\dot{y}(t)-\dot{y}^*(t))+e_3(\dot{z}(t)-\dot{z}^*(t))+e_4(\dot{w}(t)-\dot{w}^*(t)).
\end{aligned} \tag{21}$$

Also, if *Eq. (16)* and *(19)* is placed in *Eq. (21)*:

$$\begin{aligned}
\dot{V}=\;&e_1((\alpha(y_1-x_1)+y_1 z_1+u_x)-\dot{x}^*(t))+\\
&e_2((\beta(x_1+y_1)-z_1 x_1+u_y)+\lambda_2 e_2)-\dot{y}^*(t))+\\
&e_3((-ez_1-fw_1+y_1 x_1+u_z)-\dot{z}^*(t))+\\
&e_4((-gw_1-hz_1+y_1 x_1+u_w)-\dot{w}^*(t)).
\end{aligned} \tag{22}$$

Simplification and with replaced *Eq. (19)* in to *(22)*, assuming the desired values are a constant level, then its derivative will always be zero.

$$\begin{aligned} &\dot{V} = \lambda_1 e_1^2 + \lambda_1 e_3^2 + \lambda_1 e_3^2 + \lambda_1 e_4^2 \\ &\qquad \Rightarrow= \dot{V} < 0 \quad \text{if} \quad \lambda_1, \lambda_2, \lambda_3, \lambda_4 < 0. \end{aligned} \tag{23}$$

5 | Numerical Simulation

In this section, the numerical simulation results prove that the two Qi hyperchaotic supply chains synchronized with different initial conditions and the proposed nonlinear controller. For numerical simulation in the Qi hyperchaotic supply chain model, the fourth-order RangKutta method with step 0.01 was used.

The parameters of the hyperchaotic Qi supply chain model are $\alpha = 50, \beta = 24, c = 13, d = 8, e = 33, f = 30$, Also, the initial conditions of the master and slave hyperchaotic supply chain are equal $[x_1(0), y_1(0), z_1(0), w_1(0)]^T = [-1, 1, 0, 1]^T$ and $[x_2(0), y_2(0), z_2(0), w_2(0)]^T = [1, -2, 5, 3]^T$. The parameters are controller $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -10$.

Fig. 6 shows the synchronization of the two Qi hyperchaotic supply chains. The control policy from t=1 has been added to the follower chaotic supply chain model. On average, in less than t <0.5, the synchronization error converged to zero, and as well as the slave system tracked the master system.

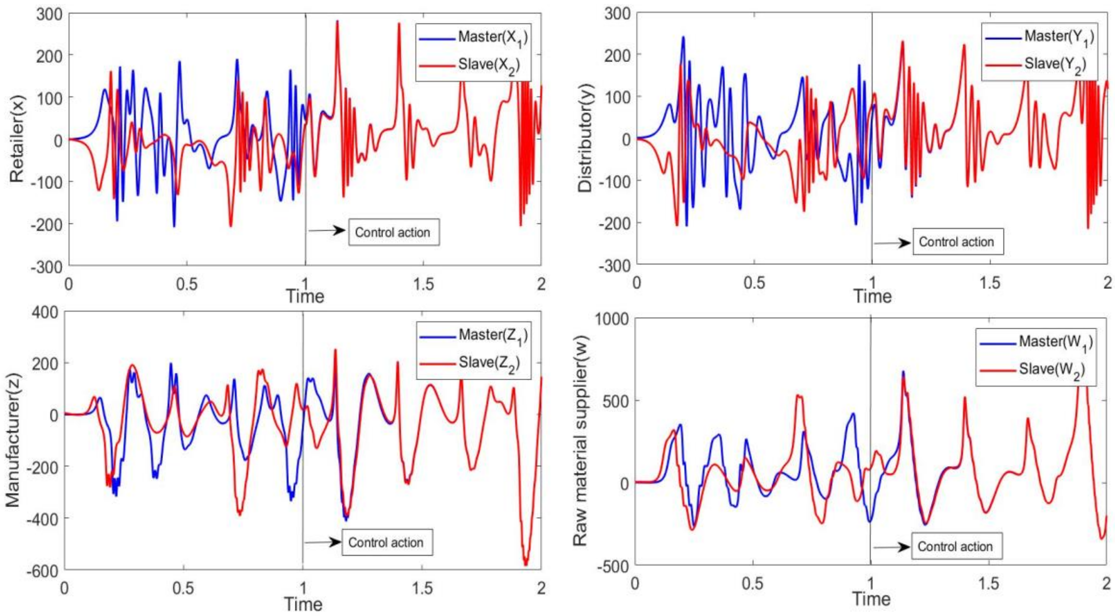

Fig. 6. Synchronization of two hyper chaotic Qi systems with different initial conditions under the proposed control policy.

Fig. 6 shows the synchronization error. In other words, the duration of stability and reaching the objective function expressed in *Eq. (10)*.

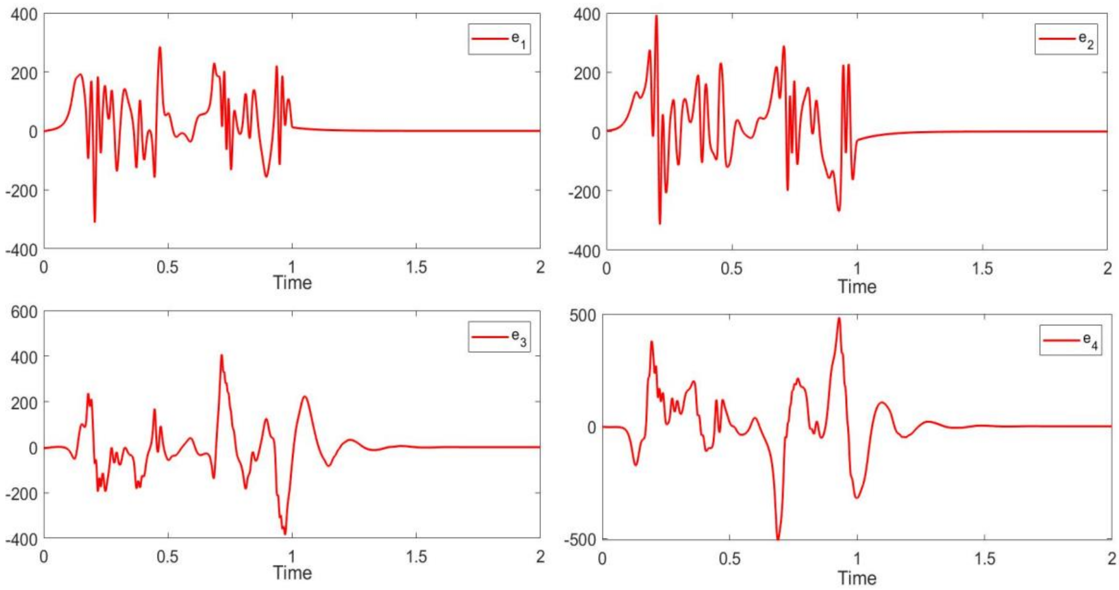

Fig. 7. Error of the Synchronization of two hyper chaotic Qi systems.

The any design method that is completed must be evaluated for the cost of implementing it in the real world. The simple method of analyzing the controller behavior can be examined in two areas. The first domain is the oscillations of the controller signal, and the second domain is the amplitude of the controller signal. *Fig. 8* shows the proposed controller signal. Mathematically, the behavior of the control signal does not have extreme oscillations and large amplitudes. Therefore, it will not cost much to implement the method in the real world.

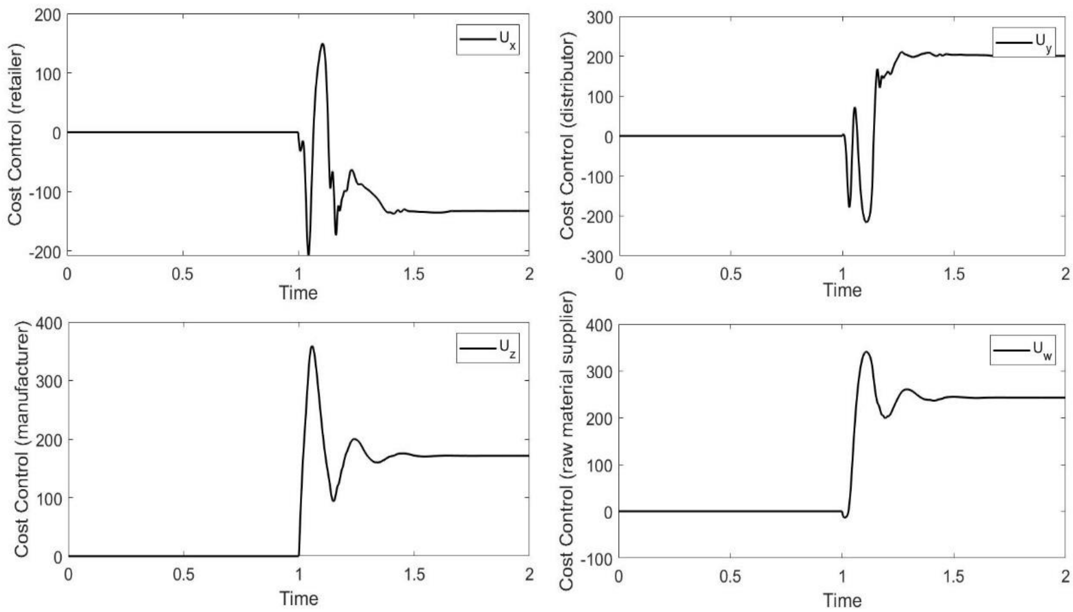

Fig. 8. Control cost for synchronization of hyperchaos supply chain (or controller signaa).

In the second part of numerical simulation, the goal is to eliminate the hyperchaotic behavior and going to the desired value in the hyper chaotic supply chain model. Also, the proposed controller parameters are considered unchanged and like the first part of numerical simulation and the initial conditions are also unchanged. *Fig. 9* shows the stability and elimination of hyper chaotic behavior in the supply chain model. The control policy is applied from t = 1. As can be seen from *Fig. 9*, the system stability of t <0.5 is achievable.

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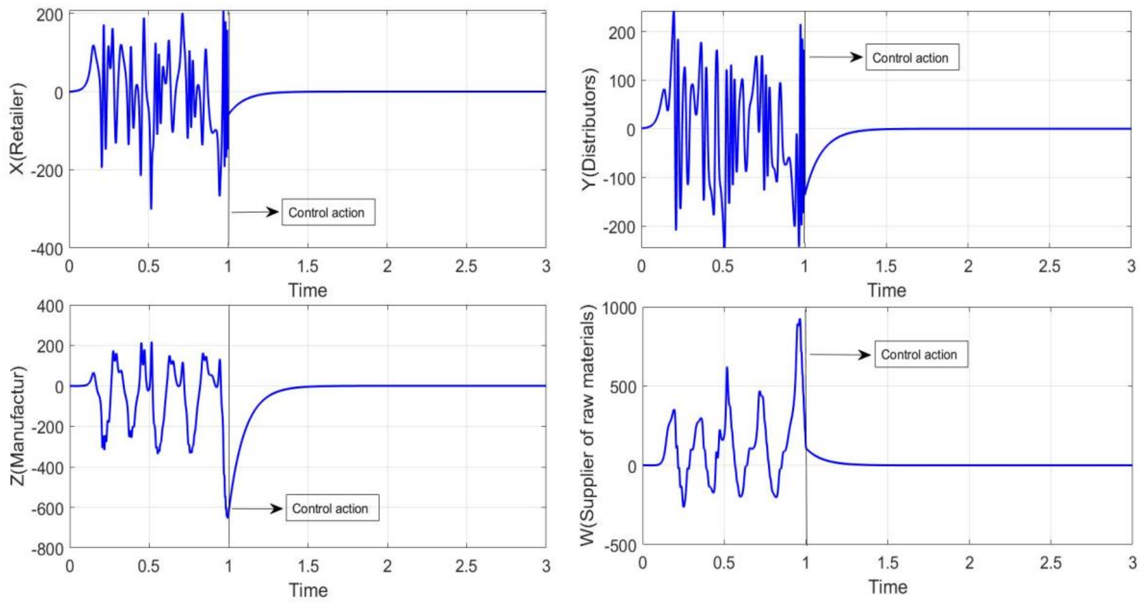

Fig. 9. Elimination of hyper chaotic behavior in the supply chain.

Fig. 10 shows the control signal to eliminate the hyperchaotic behavior in the supply chain. Because the controller has been applied to the supply chain since t = 1, the controller value will be zero at times t<1.

In this model, the distributor is known as the center of gravity of the model. In other words, the distributor in this model can take control of prices or in other words control of supply and demand in the model. The proof of this claim is described in the following simulation. Consider *Eq. (14)* as follows:

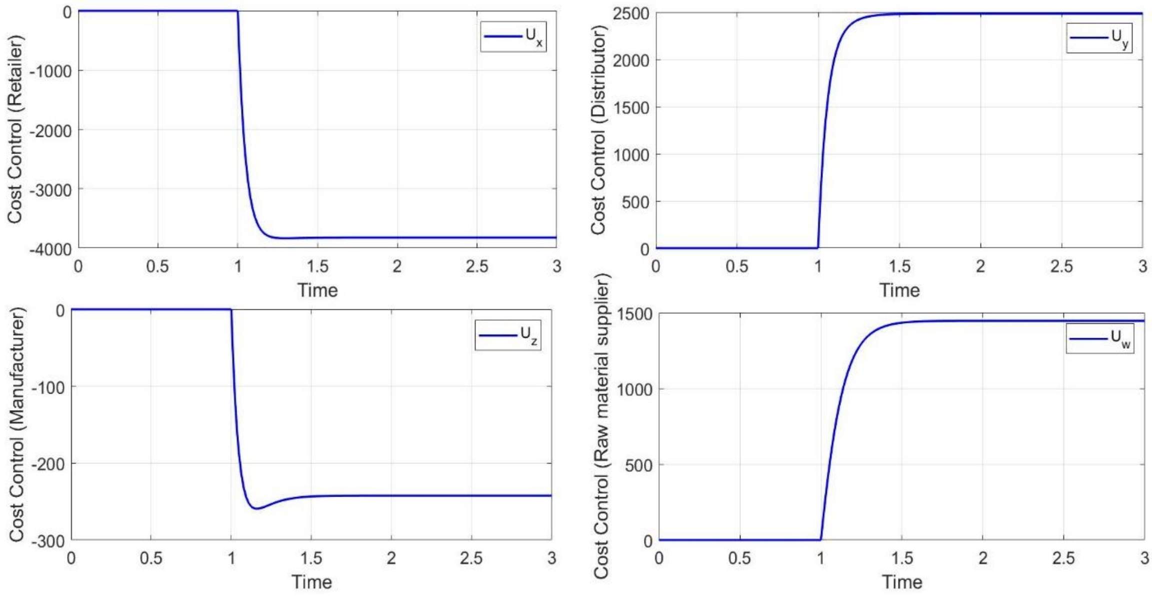

Fig. 10. Cost control for elimination of hyperchaos supply chain.

$$\begin{aligned}
\dot{x}_1 &= \alpha(y_1 - x_1) + y_1 z_1. \\
\dot{y}_1 &= \beta(x_1 + y_1) - z_1 x_1 + u_y. \\
\dot{z}_1 &= -e z_1 - f w_1 + y_1 x_1. \\
\dot{w}_1 &= -g w_1 - h z_1 + y_1 x_1.
\end{aligned} \tag{24}$$

As can be seen, the control policy applies only to the distributor. All its parameters are the same as before. Initial conditions $[x_1(0), y_1(0), z_1(0), w_1(0)]^T = [2, 3, 4, 1]^T$. Due to the complexity and highly dependent supply chain equations in this model, the distributor alone can eliminate the chaotic behavior. See *Fig. 11*.

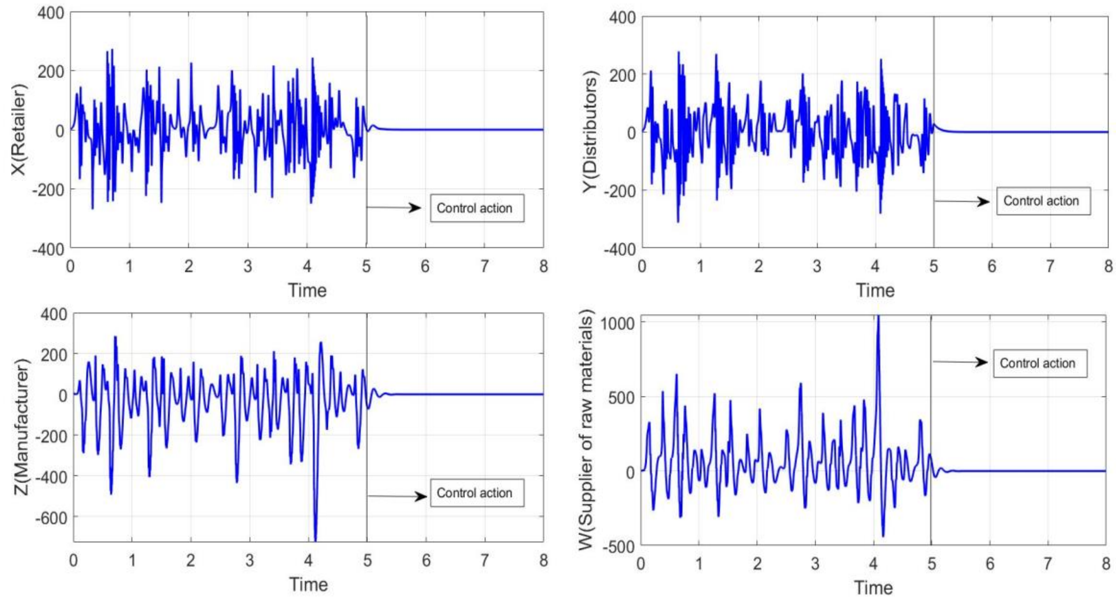

Fig. 11. Elimination of hyper chaotic behavior in the supply chain with control policy in the distributor.

The initial conditions in this section are selected differently. *Fig. 12* shows that the other three controllers are zero at all stages of the simulation and only the controller is applied to the distributor equation. The control signal in the distributor does not have much fluctuations and amplitude that can be implemented in the real world. Also, economically, its design and implementation costs have been greatly reduced.

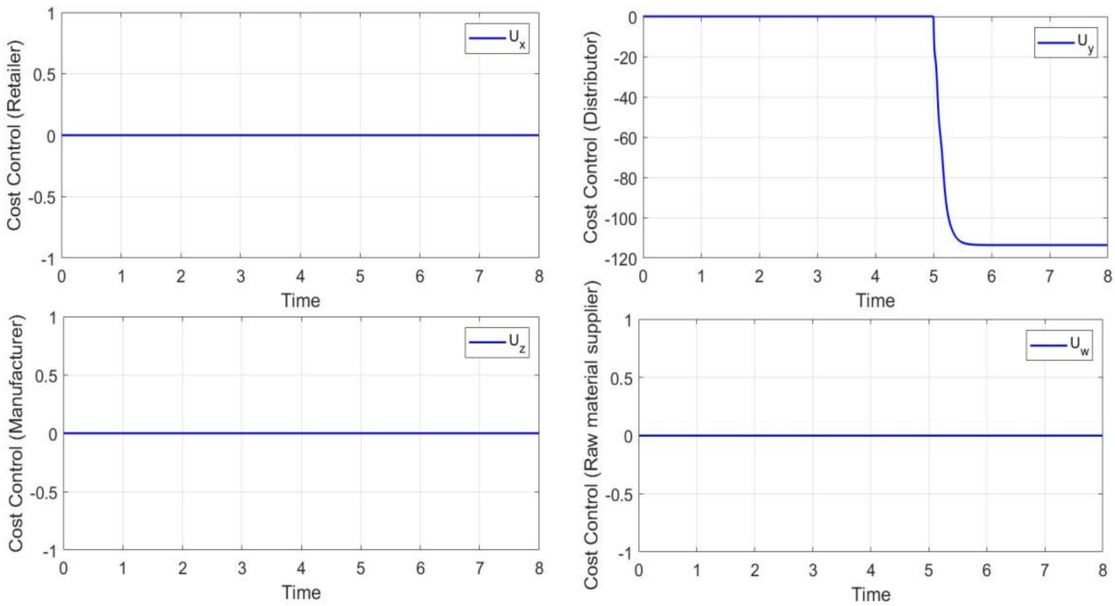

Fig. 12. Policy control in the only distributor for eliminate of the hyperchaos supply chain.

6 | Conclusion

In this paper, a new four-levels supply chain model based on hyperchaotic dynamics is introduced. This model has been shown to have an unstable equilibrium point. In the first part of the simulation, the synchronization of two hyperchaotic supply chains with different initial conditions was investigated. The simulation results showed that the synchronization error (objective function) moves to zero in a short time

(T<0.5) under the nonlinear control method. The controller signal was depicted in synchronization. These signals are the same control policies applied in the real world. Therefore, the lower the amplitude and oscillations of these signals, the more appropriate they will be. Since supply chain stability and control is the most important goal of investors, in the second part of the simulation, the main goal is to eliminate the hyperchaotic behavior in the supply chain model. In this section, the simulation results show that the control signal can be implemented in the real world to eliminate this behavior. The third part of the simulation shows that this supply chain model has a center of gravity. In other hands, by using the control policy (control signal) in the distributor, it is possible to achieve stability and eliminate chaotic behavior. So, the control signal is only added to the distributor equation. This shows that simple solutions can be used in the management of the hyperchaotic supply chain network. Also, economically, its design and implementation costs have been greatly reduced. In future research, it is possible to study the supply chain of more levels and other hyper-chaotic models. It is useful to investigate the uncertainties in the proposed hyper-chaotic supply chain model. The use of the proposed nonlinear method is highly dependent on the information flow (system feedback). Hence, if the flow of information is interrupted, the control policy can not work well. It is suggested to use adaptive control methods to solve this case. Also, for the supply chain model, the use of meta-heuristic methods and fuzzy neural networks can reduce the cost of control policy.

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