Abstract

Today, logistics costs often make up a major part of large organizations’ expenses. These costs can be reduced with optimal design and its implementation in the supply chain. As a result, in present study, a two-objective mathematical location-routing model is presented, where an objective is to minimize the costs and the next is to maximize the reliability in order to deliver the goods timely to customer according to the probable time and time window. The proposed problem has two levels of distribution. The first level, which is called transportation level, points to the distribution of products from a factory to an open distribution center, and the latter is known as routing level, which is related to a part of the problem in which we deliver products from the warehouse to customers. The proposed mathematical model is solved by Epsilon-constraint and NSGA-II approaches in small and medium, and large scales problem, respectively. The present study has provided the following contributions: concurrent locating and routing in the supply chain in accordance with the customer’s time window, probable travel time in the supply chain and customer’s reliability in the supply chain. The assessment metric results indicate the proper performance of our proposed model.

Keywords: Location-routing problem, reliability, time window, Meta-heuristic algorithm.

1 | Introduction

Over the last years, efficient, reliable, and flexible decisions on locating the warehouses and routing the distribution paths have been of high importance to managers [1], [2]. Many research report that if routing is neglected while warehouses are located, the cost of distribution systems may increase [3], [4]. The Location-Routing Problem (LRP) has overcome this problem by concurrent consideration of location and routing [5], [6]. LRP is used in many fields such as food distribution, newspaper delivery, wastes collection, drug distribution,
military applications, postal parcels delivery, natural disaster relief goods distribution, and distribution of various goods among customers [7], [8], [9].

In the classical models of LRP problems, a number of nodes are deployed as potential locations for warehouses and some nodes are considered as customers in specific locations [10], [11]. In these problems, limited capacity is generally considered for warehouses and vehicles which is called Capacitated Location Routing Problem (CLRP) [12], [13], [14]. One answer to this question is regarding the open warehouses, each customer should be allocated to an active warehouse, and routes should be created for each warehouse and the corresponding customers [15], [16]. The following limitations should be taken into account: 1) The total demand of customers allocated to a warehouse should not exceed the capacity of the warehouse, 2) each route starts from a warehouse and ends at the same warehouse, 3) each vehicle makes a maximum of one trip, 4) each customer is served by only one vehicle (no separate delivery), 5) The total demand of customers who are met by one vehicle should be proportional to the capacity of the vehicle [17], [18]. This two-step LRP includes three sub-problems: Facility Location Problem (FLP), products transportation from factories to warehouse (a transportation proportional to the truck capacity), and vehicle route design in order to meet customers of any intracity distribution center, referred to as the Transportation Location-Routing Problem (TLRP) [19], [20], [21], [22].

Due to traffic jam and road problems, time never can be certainly said as fixed, because in each route from the warehouse to the customer or between customers, the travel time of the vehicle is a probable variable with a uniform distribution [23]. Customers’ time window is embedded for timely product delivery. Maintaining efficiency for logistics distribution not only reduces the procurement costs, but is also very important in the quality of services [4]. Given the time window, customer demand must be met in time. If a vehicle arrives at the customer’s place earlier than the deadline, it incurs the waiting cost, and if it is delayed, the customer’s satisfaction decreases or the contract may even be canceled in strict circumstances. These additional costs of early and delay arrivals are called penalty costs. For this reason, by combining the time window and TLRP, the problem gets closer to the real world. Adding reliability to the model is essential to ensure the level of service to each customer node. By reliability modeling, the method presented in [24], was approved and developed. Reliability of service-delivery is considered as an indicator of quality for transportation, which has been highly regarded in recent years. The variety of real-time travel per vehicle (as a variety of service) affects the reliability of service and travel time of the passenger. Hence, we have developed a transportation location-routing model along with a probable travel time and a time window with two objectives including costs minimization and the maximization of customer’s reliability on timely delivery, attracting the customers’ complete satisfaction. The contributions of papers are as follows:

- Paying attention to location and routing in the supply chain and the customer’s time window
- Paying attention to the probable travel time in the supply chain
- Paying attention to customer’s reliability in the

This paper is organized as follows: the introduction is presented in the first section; in the second section, the literature and the background of the problem will be reviewed; followed by these two sections, three proposed mathematical models are presented in section 3; the problem solving methods and the results of the calculations are mentioned in the sections four and five; and finally the conclusions will be presented in the section six.

2 | Literature Review

The first studies on LRP date back to the 1970s and early 1980s. Of the first authors analyzing LRP were Watson-Gandy and Dohrn [25], and to date, many studies have been conducted in this field. Few works were found on multi-stage LRP. In Tirkolaee et al. [26], a four-layer LRP model is designed. The optimal solution for a sample was obtained after a limited processing time of 25 hours for small samples and
several days for large samples. In Bashiri et al. [27], a two-stage LRP model is presented. The authors divided the problem into two modules including location/allocation and routing, getting good results for small samples but having very time-consuming calculations.

Panicker et al. [28], presented a two-level model for routing and locating distribution centers in conditions of uncertainty. Considering replenishment at intermediate depots is one of the innovations of their research. Also, they examined the time window as well as loading and docking time for transporting the goods. Finally, due to the NP-hard nature of the model, the ant colony optimization algorithm was used to solve the problem.

Balci [29], provided a mathematical model for locating distribution centers and making routing decisions at uncertainty conditions in the Turkish city Van as the case study. His proposed model aimed to cover all considered areas so that the coverage rate would be maximized. This model is solved with the Tabu search algorithm.

Herazo-Padilla et al. [30], for the first time increased the complexity of LRP by simultaneously considering the assumptions of multi-period, multi-product, and uncertainty of demand. Their three-tier supply chain consists of one supplier, several warehouses and customers. In this model, customers and warehouses use a periodic review policy to replenish the inventory. Therefore, inventory shortage and maintenance costs are included in the objective function for both customers and warehouses. Furthermore, customer demand is uncertain and follows Poisson distribution.

Ghasemi et al. [31], presented a multi-objective, multi-product and multi-period mathematical model for managing crisis relief decisions in Tehran as their case study. The decisions considered in this study included location, allocation of ambulances and investigation of the flow of relief goods. Consideration of the failure of distribution centers was one of the innovations for this research. The proposed mathematical model is solved using NSGA-II and MMOPSO approaches, indicating the proper performance of their proposed model.

Zeng et al. [32], a variable and probabilistic time is included in Vehicle Routing Problem (VRP) schedules where clients do not have a time window. The model was developed by extending the classical VRP objective function and considering the expected time and standard travel time deviation. In Ganesh et al. [33], the VRP model was developed by considering variable and probabilistic times as well as soft time windows with the aim of minimizing shipping and the fines costs.

By considering Tehran as their case study, Ghasemi and Khalili-Damghani [34], proposed a robust mathematical model for locating distribution centers, controlling the inventory of relief goods, and allocating centers to hospitals in earthquake conditions. Estimating the demand for relief goods with the help of computer simulation and interactive design of basic urban infrastructure are of their innovations. The simulation-optimization model is solved using robust optimization. The findings indicated the proper performance of their proposed model. Gerdrodbari et al. [35], presented a multi-level, multi-period, multi-product closed-loop supply chain (CLSC) for timely production and distribution of perishable products. To solve the model the robust optimization method is utilized. To validate the model in small-size problems, the epsilon-constraint method was presented and Non-dominated Sorting Genetic Algorithm was developed for solving large-size problems. Son et al. [36], developed a mathematical model for managing a robust closed-loop supply chain. Minimizing environmental costs and transportation costs along with maximizing social impacts are among the innovations of their research. To this end, two mathematical models including a robust model and an evaluation model were proposed to minimize the system costs. Due to the NP-hard nature of the model, a genetic approach was to solve the model, indicating the proper performance of their proposed model in minimizing the costs. In Florio et al. [37], the cost of unpredicted fines for planning each tour is presented to reflect the costs incurred in actual operations to adjust the arrival
sooner or later than the scheduled time in LRP. In Araghi et al. [38], LRP has been extended in large-scale which was formulated as a multi-commodity integer network flow problem.

Also, a complete and practical location-routing model is proposed in a distribution network to improve customer satisfaction [39]. A fleet size and mixed location-routing problem with time windows (FSMLRPTW) problem is proposed by taking into account the number of vehicles and the different time window that aims to minimize fixed vehicle and warehouse costs as well as routing costs [40].

In Ebrahim Qazvini et al. [41], reliability is defined as the lower limit of the probability that if an accident occurs in one of the demand nodes, it will be handled immediately by a vehicle located in one of the warehouses. Accordingly, the desired level of reliability can be achieved by limiting the probability of failure. Furthermore, a framework is presented along with cost-effective design tools to develop the level of reliability of service from a passenger’s perspective. Service reliability is known as one of the indicators based on the schedule, which is perceived by the user and is one of the most important indicators of public transport quality. Table 1 summarized the literature as follows:

<table>
<thead>
<tr>
<th>Author</th>
<th>Number of objective functions (SO)/(MO)</th>
<th>Type of objective functions (Cost (C)/Reliability (R))</th>
<th>Echelon (Multi (M), Single (S))</th>
<th>Time window</th>
<th>Decision types (Allocation (A) / Routing (R))</th>
<th>Meta-heuristic (M), Exact algorithm (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qazvini et al. [41]</td>
<td>SO</td>
<td>C</td>
<td>M</td>
<td></td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Almouhanna et al. [40]</td>
<td>SO</td>
<td>C</td>
<td>S</td>
<td>*</td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Araghi et al. [38]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td>*</td>
<td>*</td>
<td>M</td>
</tr>
<tr>
<td>Florio et al. [37]</td>
<td>SO</td>
<td>C</td>
<td>S</td>
<td>*</td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Son et al. [36]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td>*</td>
<td>M</td>
<td>E</td>
</tr>
<tr>
<td>Ghasemi and Khalili-Damghani. [34]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td></td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Tirkolaee et al. [26]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td></td>
<td>*</td>
<td>M</td>
</tr>
<tr>
<td>Bashiri et al. [27]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td>*</td>
<td>*</td>
<td>M</td>
</tr>
<tr>
<td>Ganesh et al. [33]</td>
<td>SO</td>
<td>C</td>
<td>S</td>
<td>*</td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Zeng et al. [32]</td>
<td>SO</td>
<td>C</td>
<td>S</td>
<td>*</td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>Ghasemi et al. [31]</td>
<td>MO</td>
<td>C</td>
<td>M</td>
<td>*</td>
<td>*</td>
<td>M</td>
</tr>
<tr>
<td>Balcik [29]</td>
<td>SO</td>
<td>C</td>
<td>S</td>
<td>*</td>
<td>*</td>
<td>E</td>
</tr>
<tr>
<td>This Study</td>
<td>MO</td>
<td>CR</td>
<td>M</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

Therefore, according to the literature review, the research gap is described as follows:

- Not concurrent paying attention to location and routing in the supply chain and the customer’s time window
- Not paying attention to the probable travel time in the supply chain
Not paying attention to customer’s reliability in the supply chain.

3| The problem definition

The customers have a definite demand. Their product demand is supplied from a range of factories. Each customer $j$ has a specific demand $b_j$ and each factory $f$ can supply a maximum of $a_f$ units of the product. On the other hand, the factory cannot deliver the products directly to the customer; for this reason, the warehouse acts as an intermediate point so that the products are sent from the factory to these centers and these centers deliver the products to the customer. This is a classic situation for the urban logistics problem, where trucks carry the final products from the factory but they cannot deliver the products to customers located in the city. Therefore, they carry the goods to the warehouses, which are usually located in the suburbs, and unload the products.

Warehouse candidate centers are places where warehouse facilities are located or places where warehouses can be built. Each warehouse $i$ has a reopening cost of $g_i$ and a capacity of $b_i$. In general, not all candidate warehouses need to be reopened and used, and only a few will be employed. Figure 1 shows the framework of proposed supply chain.

The problem has two levels of distribution, which is a part of linked FLP. The first level, which is called transportation level, points to the distribution of products from a factory to an open distribution center, and the latter is routing level, which is the part of the problem that we deliver products from the warehouse to customers. In the second objective, we look at the problem differently. For competition, service providers need to deliver the right level of customer service, which often depends on the choice of centers, the distance of customers from the warehouse, the reliability of shipping the requested products to support the customer in a given time, and the relationship between shipping routes.

3.1| Assumptions

Specifications of level 1 of the objective 1:

- There is a number of $f$ factories for production.
- It includes only one type of product.
- Each factory $f$ can supply a maximum of $a_f$ products.
• There is a set of \( m \) candidate sites where the warehouse can be located. Each candidate site has a capacity of \( b_i \) and a fixed cost of \( g_i \) to stay open.

• The final product must be shipped from the factory to open warehouse. Transportation at this level is done by trucks. Each truck has a capacity \( CR \) and a cost \( d_{fi} \) to transport from the factory to the warehouse.

Specifications of level 2 of the objective 1:

• There is a set of \( n \) customers that each customer \( j \) has a demand \( b_j \).

• From every warehouse in operation, products are shipped for customers by vehicles. Delivery of products is done through a series of routes. Vehicles must leave the warehouse, meet the customer on the way and return to the warehouse.

• Each vehicle has a limited loading of \( Q \) units of the product.

• There is a cost \( c_{ij} \) which points to the distance between warehouse and customer or between the two customers.

• The vehicles are considered same.

• A set of \( k \) vehicles delivers the products to the customers at the level of the route and returns to the warehouse.

• Travel time from the warehouse to the customer or between customers is probabilistic.

Specifications of objective 2:

• A time window has been considered for timely delivery to customers.

• Possible reliability function is uniform with the distribution function.

• Uncertainty should not be lower than \( \beta \).

A problem has been formulated with two objectives, cost minimization and maximizing customer service assurance at the right time. We defined a mathematical model for this two-objective problem:

**Sets**

\[ p \]  
Number of factories

\[ i \]  
Number of candidate sites for routing the warehouses

\[ N \]  
Number of customers

\[ f = \{1,2,...,p\} \]  
Set of indicators for factories

\[ D = \{1,2,...,m\} \]  
Set of indicators for warehouses

\[ J = \{m+1,m+2,...,m+n\} \]  
Set of indicators for customers

**Parameters**

\[ a_f \]  
The product capacity in the factory \( f \)

\[ d_{fi} \]  
The cost of shipping a truck from the factory \( f \) to the warehouse \( i \)

\[ CR \]  
Capacity of truck, factory \( f \), warehouse \( i \)

\[ g_i \]  
The fixed opening and operating cost of the warehouse \( i \)

\[ b_i \]  
Capacity of the warehouse \( i \)
Variables:

- $h_i$: Demand of the customer $j$
- $c_{ij}$: Customer meeting cost/warehouse $i$ exactly after customer/distribution center $j$
- $Q$: Capacity of the vehicle in the route level
- $d_{fi}$: The cost of travel from the factory $f$ to the warehouse $i$
- $s_j$: The time of arriving to the customer $j$
- $e_j$: Cost of fines per unit of late arrival to the customer $j$
- $cl_j$: Cost of fines per unit of early arrival to the customer $j$
- $C_{\text{wait}_j}$: Cost of fines per unit of late arrival to the customer $j$
- $\beta$: Minimum reliability
- $\text{down}_j$: Lower limit of time window for the customer $j$
- $\text{up}_j$: Upper limit of time window for the customer $j$
- $V_{fi}$: The amount of shipped products from factory $f$ to the warehouse $i$
- $W_{fi}$: Number of transformed trucks from the factory $f$ to the warehouse $i$
- $e_j$: Time of leaving customer $j$
- $ar_j$: Time of arriving to the customer $j$
- $\text{wait}_j$: Waiting time of customer $j$ (free in sign)
- $\text{waitt}_j$: Waiting time of customer $j$
- $\text{late}_j$: Delaying time of customer $j$ (free in sign)
- $\text{latee}_j$: Delaying time of customer $j$
- $u_i$: Variable used to prevent vehicle intrusion and entrance to the sub-tour. In fact, it expresses the loading of vehicles after meeting the warehouse $i$.
- $T_{ij}$: Travel time from the customer site/from warehouse $i$ to the customer/from warehouse $j$ in the route level
- $R_j$: Reliability of the timely arrival of vehicle form warehouse/to the customer $j$
\[ y_i = \begin{cases} 1 & \text{In case of open warehouse } i \\ 0 & \text{Otherwise} \end{cases} \]

\[ z_{ij} = \begin{cases} 1 & \text{In case of assigning customer } j \text{ to the warehouse } i \\ 0 & \text{Otherwise} \end{cases} \]

\[ o_{jk} = \begin{cases} 1 & \text{In case of assigning vehicle } k \text{ to the customer } j \\ 0 & \text{Otherwise} \end{cases} \]

\[ x_{ijk} = \begin{cases} 1 & \text{In case of meeting customer or warehouse } j \text{ after customer or warehouse } i \text{ by the vehicle } k \\ 0 & \text{Otherwise} \end{cases} \]

\[
\begin{align*}
\text{Min} f_1 &= \sum_{ij \in D} \sum_{v \in V} d_{ij} w_{fi} + \sum_{ij \in D} g_i + \sum_{ij \in D} \sum_{v \in V} f_i x_{ijk} + \sum_{ij \in D} \sum_{v \in V} \sum_{k \in K} c_{ijk} x_{ijk} \\
&\quad + \sum_{j \in J} c_{j, \text{late}} + \sum_{j \in J} C_{\text{wait}, j, \text{wait}} \\
\text{Max} f_2 &= \prod_{i \in D} \prod_{j \in J} \prod_{k \in K} R_{ij}^{x_{ijk}} \\
\sum_{i \in D} v_{fi} &\leq a_f \quad \forall f \in F \\
w_{fi} &\geq \frac{v_{fi}}{CR} \quad \forall f \in F, \forall i \in D \\
\sum_{j \in J} v_{fi} &\leq b_i y_i \quad \forall i \in D \\
\sum_{j \in J} v_{fi} &\leq \sum_{j \in J} h_j z_{ij} \quad \forall i \in D \\
\sum_{i \in D} z_{ij} &\leq 1 \quad \forall j \in J \\
\sum_{i \in D} \sum_{v \in V} x_{ijk} &\leq 1 \quad \forall k \in k \\
\sum_{i \in D} \sum_{v \in V} x_{ijk} &\leq \sum_{i \in D} \sum_{v \in V} x_{ijk} = 1 + z_{ij} \quad \forall i \in D, \forall j \in J, \forall k \in k \\
\sum_{i \in D} \sum_{v \in V} x_{ijk} &\leq \sum_{i \in D} \sum_{v \in V} x_{ijk} = \sum_{i \in D} \sum_{v \in V} x_{ijk} = 1 + z_{ij} \quad \forall j \in v, i \neq j, \forall k \in k \\
\sum_{i \in D} \sum_{v \in V} x_{ijk} &\leq u_i - u_j + Q \sum_{v \in V} x_{jk} \leq Q - h_j \quad \forall i, j \in J, i \neq j \\
h_j &\leq u_j \leq Q \quad \forall j \in J \\
\sum_{v \in V} x_{ijk} &\leq o_{jk} \quad \forall k \in k, \forall j \in J, i \neq j \\
\sum_{v \in V} x_{ijk} &\leq o_{jk} \quad \forall k \in k, \forall i \in v, i \neq j \\
10 \leq T_v &\leq 20 \quad \forall i, j \in v, i \neq j 
\end{align*}
\]
Objective function (1) points to the economic objective including the fixed and operating costs of reopening the warehouse and operating a vehicle at candidate site $i$, the varying costs including transporting from the
factory to the warehouse and then to the customer, and the costs of fines for early or late delivery of products to the customer.

Objective function (2) calculates reliability over time at the route level.

Constraint (3) ensures that the product is shipped not more than the factory capacity. Constraint (4) expresses transforming trucks from any factory to any warehouse according to the capacity of the trucks. The quantity of the product shipped from each factory to each warehouse should not exceed the warehouse capacity, according to Constraint (5), if \( y_i = 0 \), then no product will be shipped to the warehouse \( i \). In accordance with Constraint (6), the quantity of products shipped to a warehouse must be equal to the total demand of the customers allocated to that warehouse. Constraint (7) assures us that each customer is allocated to only one warehouse. The Constraints (8) to (10) first ensure that each customer must be met immediately after a warehouse or after other customers. Second, the structure of the routes takes place between the customers assigned to the common warehouse. According to the Constraint (11), the number of vehicles leaving the warehouse should be equal to the number of vehicles returning to the warehouse. Constraints (12) and (13) are relevant to the sub-tours, so that the capacity of vehicles while traveling between two customers should not be exceeded. Constraints (14) and (15) state allocation of the vehicle to each customer. The Constraint (16) expresses that the travel time is a probable variable. Given the Constraints (17) to (19), the time of departure from the warehouse should be the beginning of work and in fact equal to zero, expressing the time of departure of the first and the rest of customer. Constraint (20) and (21) calculate the soft time window. Constraints (22) to (25) calculate the amount of waiting and delay times. Constraints (26) to (28) express the reliability from the time of delivering services to the customers. Constraints (29) to (36) specify the type of decision variables.

4 | Problem Solving Method

- **Epsilon constraint**

Epsilon constraint is one of the most accurate methods for obtaining Pareto Optimal Solutions, which was first proposed by Aljdan. This method is one of the well-known approaches to dealing with multi-objective problems, which solves these types of problems by transferring all but one of the objective functions to the constraints at each stage. The Pareto boundary can be created by the \( \varepsilon \)-constraint method as the following equation [42]:

\[
\min_{x \in X} f_1(x) \\
\text{s.t. } f_2(x) \leq \varepsilon_2 \\
\vdots \\
\text{s.t. } f_n(x) \leq \varepsilon_n
\]

The steps of this method are as follows:

1. Select one of the objective functions as the main objective function, solve the problem each time according to one of the objective functions and obtain the optimal value of each objective function.
2. Divide the interval between the two optimal values of the sub-objective functions into a predetermined number and obtain a table of values for \( \varepsilon_2, \ldots, \varepsilon_n \).
3. Solve the problem each time by the main objective function with any of the values \( \varepsilon_2, \ldots, \varepsilon_n \).
4. Report the found Pareto solutions.
**- NSGA-II Method**

Non-dominated Sorting Genetic Algorithm II (NSGA-II) is one of the most popular and widely used optimization algorithms in the field of multi-objective optimization. This algorithm is introduced by Deb [43]. This algorithm uses only the values of the objective function to perform the optimization process and does not require any additional information such as the derivative of the function. It is also very fast and efficient due to the simplicity of the search process. Solving the problem by this algorithm requires a number of elements, one of which is the archive of reasonable answers [43]. On the other hand, because the computer memory is limited, the number of archives cannot be allowed to grow as much as it desires. As a result, an archive size control mechanism is needed to control the number of archived responses. If these two features are combined with the search feature, an intelligent optimization algorithm will be obtained. For example, Figure 2 shows the chromosome corresponding to the variable \( o_{jk} \). If vehicle \( k \) is allocated to the customer \( j \), the amount of gene will be 1, otherwise it will be zero.

![Fig. 2. The chromosome corresponding to the variable \( o_{jk} \).](image)

The main operators of this method are mutation and crossover, which are defined in this study as follows: Figure 3 shows the crossover operator. Double-point type of crossover was used in this study. The mechanism of this operator is in such a way that two points are randomly selected and the strings of each chromosome will be displaced.

![Fig. 3. Double-point crossover.](image)

Figure 4 shows the mutation operator. For this purpose, a row is selected optionally and the selected row will be reversed.

![Parent](image)
4.1 | Assessment Metric

- Space Metric (SM)

This metric measures the uniformity of NSGA-II solutions. It is calculated using the following equation:

$$SM = \frac{\sum_{i=1}^{n-1} |d_i - \bar{d}|}{(n-1)\bar{d}}$$

In this equation, $d_i$ is the Euclidean distance between two adjacent Pareto points and $\bar{d}$ is the mean Euclidean distances. Therefore, the closer the SM value to zero, the equal the distance between adjacent Pareto points.

The mean distance from the ideal point (MID)

This metric is a qualitative indicator and calculates the distance between the Pareto edges and the ideal point. Undoubtedly, the lower the value of this indicator, the better the quality of the answer. It is calculated as follows:

$$MID = \frac{\sum_{i=1}^{n} \left(\frac{f_{1i} - f_{1\text{best}}}{f_{1\text{max}} - f_{1\text{min}}}\right)^2 + \left(\frac{f_{2i} - f_{2\text{best}}}{f_{2\text{max}} - f_{2\text{min}}}\right)^2}{n}$$

Where $n$ is the number of Pareto points, $f_{1i}$ and $f_{2i}$ are the values of the first and second targets for the $i^{th}$ Pareto answer, respectively. Also, $f_{1\text{max}}$ and $f_{1\text{min}}$ are the maximum and minimum values of the $j^{th}$ objective function between Pareto points, respectively. According to this definition, a lower MID value indicates better performance of the algorithm.

Table 2 shows the results of solving small and medium problems. The first five samples are Pareto points of small problems and the next five ones are those of medium size problems. The columns 2-4 are the values of the first and second objective functions as well as the solution time by the Epsilon constraint method, and the columns 5-7 are the first and second objective functions, as well as the solution time by NSGA-II method. The last two columns are the amount of error due to NSGA-II. As can be seen, the mean error for both objective functions is lower than one percent.

<table>
<thead>
<tr>
<th>No</th>
<th>Epsilon-Constraint Function 1</th>
<th>Epsilon-Constraint Function 2</th>
<th>SM</th>
<th>MIDS</th>
<th>Time (s)</th>
<th>NSGA-II Function 1</th>
<th>NSGA-II Function 2</th>
<th>SM</th>
<th>MIDS</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>128.5</td>
<td>4.84</td>
<td>0</td>
<td>6.45</td>
<td>2</td>
<td>128.5</td>
<td>4.84</td>
<td>0</td>
<td>6.45</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>131.7</td>
<td>4.48</td>
<td>0</td>
<td>6.48</td>
<td>26</td>
<td>131.8</td>
<td>4.48</td>
<td>0</td>
<td>6.48</td>
<td>6</td>
</tr>
</tbody>
</table>
5 | Numerical results

In this paper, an example was generated to evaluate the proposed model, which is solved by Epsilon constraint and NSGA-II methods in the platform of LINGO ver.9.0 and MATLAB software packages. Preliminary data on this problem include the cost of travel between nodes, the cost of using the vehicle, the time of delivering services to customer, the cost of penalty for delayed arrival and the cost of penalty for early arrival which have the values 10, 12, 5, 30 and 10, respectively. The capacity of the vehicle is equal to 160 and that of the truck is equal to 220. The amount of factories production of the desired product is 800 and 600 and the rest of the data is given in Table 3.

Table 3. The problem nodes-related parameters.

<table>
<thead>
<tr>
<th>Nodes Qty.</th>
<th>Parameters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customers demand</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>50</td>
<td>80</td>
<td>60</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Warehouse reopening cost</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Warehouse capacity</td>
<td>240</td>
<td>220</td>
<td>180</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Lower limit of the customer's time window</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>25</td>
<td>10</td>
<td>10</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Upper limit of the customer's time window</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>53</td>
<td>38</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 and Figure 5 present the results of Pareto points’ extraction. As it turns out, 5 Pareto points have been extracted. For example, the third one is equal to (134, 0.8).

Table 4. Extracted Pareto Points for both objective functions.

<table>
<thead>
<tr>
<th>Objective Function 1 (Cost)</th>
<th>Objective Function 2 (Reliability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>1</td>
</tr>
<tr>
<td>125</td>
<td>0.9</td>
</tr>
<tr>
<td>134</td>
<td>0.8</td>
</tr>
<tr>
<td>149</td>
<td>0.75</td>
</tr>
<tr>
<td>160</td>
<td>0.65</td>
</tr>
</tbody>
</table>
6 | Conclusion

One of the most important issues in logistics networks is the design and analysis of distribution networks. In recent years, two main issues in the design of distribution networks, namely the location of warehouse and routing of distributors with each other have been considered and have created the location-routing problem or LRP. In this paper, we presented a nonlinear mixed integer programming model for the transportation location-routing problem by considering the probable travel time and the time window. This model aims to achieve two objectives, namely costs minimization and the customer reliability maximization which includes timely delivery to customers and their complete satisfaction.

The proposed mathematical model is developed in small and medium dimensions by epsilon constraint approach and in large dimensions by NSGA-II algorithm. Among the innovations considered in this research, we may refer to paying attention concurrently to location and routing in the supply chain with the customer's time window, paying attention to the probable travel time in the supply chain and considering the customer's reliability in the supply chain. The assessment metric results indicate the proper performance of our proposed model. The results of this research can be useful for the food industry, all medicine distribution companies, food companies, etc. Managers can also consider strategic decisions such as location in the form of sustainable development. Managers can also dynamically consider operational decisions such as routing to minimize supply chain costs. Considering the time window of customers can also increase customer satisfaction, which allows managers to increase supply chain profits. The research results are as follows:

- The cost of travel between nodes, the cost of using the vehicle, the time of delivering services to customer, the cost of penalty for delayed arrival and the cost of penalty for early arrival which have the values 10, 12, 5, 30 and 10, respectively.
- The capacity of the vehicle is equal to 160 and that of the truck is equal to 220.
- The amount of factories production of the desired product is 800 and 600.

In this study, as there was no systematic database for some parts of transportation cost elements, driver's estimations and transportation officers were asked to help.

The following items are proposed for the future studies:

- Considering a competitive game between supply chain members in the proposed model

![Fig. 5. The set of Pareto Points.](image-url)
- Considering other objectives in the model such as maximizing the resilience of supply chain or minimizing the delivery time
- Considering uncertainties in prices and problem solving with a two-tier planning approach
- Consider multiple transportation models, such as lorry, container/trailer or rail and air transportation

References


