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Weak Disposability in DEA-Based Re-Allocation Resources Model Aiming to Reduce Energy Consumption and CO_2 Pollution

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Abstract

We developed a DEA-based resource re-allocation model based on environmental DEA technology for organizations with a central decision-making environment. The proposed model considered a weak disposability axiom for undesirable outputs and combined Data Envelopment Analysis (DEA) with Multiple-Objective Programing (MOP). The objective was to find the appropriate re-allocation model in order to save energy and reduce environmental pollution, so that the next steps could be taken toward improvement. Given that reducing the inputs and outputs of inefficient units is sometimes not achievable and does not seem logical, for the reduction in the values to be logical and achievable, we divided the Decision-Making Units (DMUs) into different levels of efficient frontier using the context-dependent DEA technique. For this purpose, the model was designed to move the DMUs from the current frontier to the efficient frontier of the previous layer, which has better efficiency conditions, or keep them on their own frontier. In addition, the opinion of the central decision maker regarding the amount of reduction in the inputs and outputs was expressed using Goal Programing (GP) in a way that does not make the model infeasible. By implementing the model in 8 regions of the world, suggestions were made regarding the amounts of energy saving and CO_2 pollution reduction based on the conditions determined by the central decision maker aiming improve the efficiency of inefficient units in the next step.

Keywords: Resource allocation, Data envelopment analysis, Pollution reduction, Weak disposability.

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1 | Introduction

Many studies have been carried out on the measurement of environmental performance. In this area, Data Envelopment Analysis (DEA) is one of the most popular methods used for environmental performance measurement. DEA originated with a paper by Charnes et al. [1], and it is used to evaluate the efficiency of a group of homogeneous Decision-Making Units (DMUs) with multiple inputs and outputs [2]. Due to the limitation of energy, the issue of energy saving has become extremely important and has attracted a lot of interest in recent years among industrial and management topics. Resource allocation toward the better use of resources is currently under active research in the DEA literature. Resource allocation is a common problem in organizations that have a central unit with the power to control the DMUs [3], [4]. The problem of resource allocation is a classic example of the applications of management sciences, which has a lot of value for practical purposes [5]–[7]. In this area, Korhonen and Syrjänen [5] developed an approach based on DEA and Multiple Objective Linear Programing (MOLP) and applied it to a resource allocation problem. Lozano and Villa [8] presented two centralized resource allocation models, where one type considered a reduction of the total consumption of every input by all units, and the other related to separate reductions for each input. For other extensions of the model presented by [8], refer to [9]–[11]. Other proposed models for centralized resource allocation can be found in [6]. Amirteimoori and Kordrostami [12] extended a method that implemented the demand and supply changes in a centralized decision-making environment under the predictable assumption. Hosseinzadeh Lotfi et al. [13] proposed an allocation model based on common dual weights. Sun et al. [14] employed an allocation model to control the emission levels of DMUs, where the model did not allow trading emissions among DMUs and only seeker to cut it down. Wu et al. [15] combined context-dependent DEA and MOLP. Wu et al. [16] presented a DEA model to allocate emission permits for each DMU that would ensure production stability before and after allocation. Many findings of DEA have been used to measure environmental performance [17]–[19]. Given that saving energy decreases the desirable and undesirable outputs, energy and environmental policies make recommendations for energy saving and air pollution reduction so that the desirable outputs are reduced as little as possible. In that direction, Li et al. [20] combined energy consumption reduction through resource allocation with DEA models and proposed a multi-objective model for resource allocation. There has been a significant increase in energy consumption and pollution emission in China during recent decades, and China has become the country with the largest amount of energy consumption in the world. It is already evident to the Chinese government that the only way to reduce the severely dangerous environmental pollution across the country is to improve energy efficiency. The Chinese government has implemented a plan to reduce energy consumption by 20% and 16% in a period of five years, which would hopefully result in the reduction of CO_2 emission levels by 2030 [21]. Environmental pollution has become an increasingly challenging policy problem in China. In 2012, there were seven million deaths by air pollution, and 40% of them were Chinese [22]. Therefore, paying attention to environmental issues and reducing the air pollution of industry productions has become the most important challenge for researchers. If inefficiency exists in the production process, the undesirable pollutants should be reduced to improve the inefficiency, and they should be treated differently [23]. There are various articles on how to deal with undesirable outputs. Some studies ignore the undesirable outputs [24], [25], while some studies treat undesirable outputs as normal inputs in the production possibility set. For example, Reinhard et al. [26] estimated the environmental efficiency of Dutch dairy farms based on the nitrogen and phosphate surplus and energy consumption of the dairy farms. Amirteimoori et al. [3] extended the standard CCR model to deal with relative efficiency by increasing the undesirable inputs and decreasing the undesirable outputs. Another approach is to transform the undesirable outputs into a monotone decreasing form [23], [27], [28]. There is also a direct method based on [29], which replaces the weak disposability of desirable and undesirable outputs with the assumption that outputs have strong disposability [17], [30], [31]. Shao et al. [32]' paper aims to evaluate the eco-efficiency of China's industrial sectors between by using the Directional Distance Function (DDF) of network DEA, which contains a two-stage structure that divides industrial processes into three linked sub processes, i.e., the production, wastewater and waste gas treatment processes. Madadi et al. [33] first modified the model presented by [20], then presented another model that defined based on the idea that the required changes should be applied to the totality of the desirable outputs. They stated that there was no need to reduce individual desirable outputs, as the reduction of undesirable outputs may not be within the acceptable standard range. Seiford and Zhou [34] presented context-dependent DEA approach where a set of DMUs are evaluated against a particular evaluation context. By dividing the DMUs into several efficient frontier, they allowed the units to move the efficient frontier of the previous layer. In this paper, we aim to present a model that, in addition to reducing energy consumption and environmental pollution, would achieve a better environmental performance in units after reallocation, or keep their performance constant. For this purpose, and so that the reduction in the values is logical and achievable, we use the context-dependent DEA technique. Since this technique defines a different frontier, we want the units to be improved so that their projections are not necessarily located on the original frontier, but rather moved toward the closest frontier, which has better conditions than the frontier on which the units are located.

Therefore, the units will be allowed to move to the efficient frontier of the previous layer. In this case, the environmental performance of the DMUs will be logically improved. The rest of this paper is organized as follows. Section 2 presents DEA- based environmental technology, which assumes the weak disposability of outputs to evaluate the energy and environmental performance. In Section 3, we present a model that, in addition to reducing energy consumption and reducing environmental pollution, would improve the efficiency of units reachable and logical after reallocation or keep it constant. We implemented the new model for eight regions of the word. Finally, the conclusions are presented in Section 4.

2 | Methodology

DEA is a nonparametric method for evaluating the relative efficiency of a set of comparable DMUs with multiple inputs and outputs. Assume that there are n DMUs, DMU_j (j=1,…, n), each consuming m different inputs to produce s different desirable outputs and p different undesirable outputs, such as environmental pollution in the production process. Moreover, $X_j=(x_{1j},...,x_{mj})^T$, $Y_j^g=(y_{1j}^g,y_{2j}^g...,y_{sj}^g)^T$, and $Y_j^b=(y_{1j}^b,y_{2j}^b...,y_{pj}^b)^T$, $X_j>0$, $Y_j^g>0$, $Y_j^b>0$ are the input, desirable output, and undesirable output vectors, respectively. The production possibility set T is defined as

$$T=\left\{\left(x,y^g,y^b\right)\middle| x \text{ can produce } (y^g,y^b)\right\}.$$

According to the assumption made by [35], desirable outputs and undesirable outputs are weakly disposable, and they referred to this kind of production technology as environmental DEA technology. Environmental DEA technology, which assumes the weak disposability of outputs, has been widely used to measure industrial productivity when undesirable outputs are present [36], [37]. The expression of environmental DEA technology exhibiting Constant Returns to Scale (CRS) as proposed by [29] is as follows:

$$T=\left\{\left(x,y^g,y^b\right)\middle| \begin{array}{l} x\geq\sum_{j=1}^{n}\lambda_j x_j \\ y^g\leq\sum_{j=1}^{n}\lambda_j y_j^g \\ y^b=\sum_{j=1}^{n}\lambda_j y_j^b \end{array} ,\lambda_j\geq 0, j=1,...,n\right\}. \quad (1)$$

Many models have been proposed under the CRS assumption to measure environmental DEA technology [17], [35], [38], [39]. Among these models, Tyteca [39]'s *Model (3)* is particularly noteworthy. In their model, only the adjustment of undesirable outputs is allowed. For this reason, it provides a pure environmental performance measure for DMU_o.

$$\begin{aligned} \min \quad & \lambda_o, \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j x_j \le x_o, \\ & \sum_{j=1}^{n} w_j y_j^g \ge y_o^g, \\ & \sum_{j=1}^{n} w_j y_j^b = \lambda_o y_o^b, \\ & w_j \ge 0, \quad j = 1, \ldots, n. \end{aligned} \tag{2}$$

Zhou and Ang [17] pointed out that in the case of Variable Returns to Scale (VRS) assumption, we cannot just add one constraint to the traditional DEA forms to achieve environmental DEA technology. They presented the following mixed model for measuring environmental performance:

$$\begin{aligned} \min \quad & \frac{\lambda_o}{\theta_o}, \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j x_j \le x_o, \\ & \sum_{j=1}^{n} w_j y_j^g \ge \theta_o y_o^g, \\ & \sum_{j=1}^{n} w_j y_j^b = \lambda_o y_o^b, \\ & \sum_{j=1}^{n} w_j = 1, \\ & w_j \ge 0, \quad j = 1, \ldots, n. \end{aligned} \tag{3}$$

Since *Model (3)* is NLP, they converted the above model to the following equivalent LP model:

$$\begin{aligned} \min \quad & \lambda_o', \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j x_j \le \beta_o x_o, \\ & \sum_{j=1}^{n} w_j y_j^g \ge y_o^g, \\ & \sum_{j=1}^{n} w_j y_j^b = \lambda_o' y_o^b, \\ & \sum_{j=1}^{n} w_j = \beta_o, \\ & w_j \ge 0, \quad j = 1, \ldots, n. \end{aligned} \tag{4}$$

The strength of this model is that it simultaneously considers the improvement of desirable and undesirable outputs, but since $\theta_o^* \ge 1$ is not necessarily obtained in $\min \frac{\lambda_o}{\theta_o}$, there may be no improvement in y_o^g. Because of this, *Model (3)* does not necessarily produce an improved projection for y_o^g. Hence, the authors added the constraints $\theta_o \ge 1, \quad \lambda_o \le 1$ to *Model (3)*, and subsequently added $\beta_o \le 1, \quad \lambda_o' \le \beta_o$ to *Model (4)*. By inserting $\theta_o^* = \frac{1}{\beta_o^*}, \lambda_o^* = \theta_o^* \lambda_o'^*$, various cases can be expected for the optimal solutions. The optimal solution may be located on the frontier in relation to both desirable and undesirable outputs, or it may have inefficiency in relation to at least one of them. For example, if $\theta_o^* > 1$ and $\lambda_o^* < 1$, DMU_0 has had inefficiency in relation to both desirable and undesirable outputs, and when $\theta_o^* = 1$ and $\lambda_o^* < 1$, the inefficiency is only in relation to the undesirable outputs of DMU_0.

We also used the concept of context-dependent DEA presented by [34]. They divide a set of DMUs into different levels of efficient frontiers. If the efficient frontier is removed, then the remaining DMUs will form a new second-level efficient frontier. If this new second-level efficient frontier is removed, a third-level efficient frontier is formed, and so on, until no DMU is left. Then, the DMUs located in the second-level efficient frontier are evaluated relative to the DMUs located on the first-level efficient frontier and the performance of DMUs on the third-level efficient frontier are evaluated with respect to the first-or second-level efficient frontier.

3 | Proposed Model: Resource Allocation Model with Energy Saving while Environmental Performance Is Not Diminished

Korhonen and Syrjänen [5] referred to a decision problem in which a DM aims to allocate additional resources or re-allocate the current resources to a set of existing units to achieve the maximal output. Li et al. [20] proposed a resource allocation model as a Multiple Objective Linear Problem (MOLP) which considered reductions in the inputs, desirable outputs, and undesirable output. Technically, they were looking for a re-allocation model that would have the maximum amount of input saving and achieve the minimum reduction of desirable outputs. Suppose that there is a decision-making environment in the organization with the power to control the resources of the DMUs. Now, assume that the decision maker wants to apply energy saving and environmental pollution reduction in the production process, so that the performance of the units in the next period of re-allocation is not worse than before. In other words, the performance will improve or stay the same. In this paper, we aim to present a model that, in addition to reducing energy consumption and reducing environmental pollution, would improve the efficiency of units after reallocation or keep it constant. For this purpose and in order to the reduction values are logical and achievable, we divide the DMUs into different levels of efficient frontier, using the context-dependent DEA technique [34]. VRS has been assumed, because each level of efficient frontier has similar performance in production [40]. The units will be allowed to move to the efficient frontier of the previous layer. Certainly, efficient units still remain on the efficient frontier, but they may be allowed a reduction in some of the inputs or outputs. Inefficient units move as far as possible toward the previous layer frontier. Accordingly, we first evaluate the DMUs and obtain the efficiency scores θ_j^*, λ_j^* for each unit using *Models (3)* and *(4)*. Then, according to the following algorithm [34], we determine the all efficient frontier (DMU_j that $\theta_j^* = 1, \lambda_j^* = 1$ is recognized efficient and it is on the efficient frontier).

Step 1. Set k=1, Assuming that the set of all DMUs is L (1). Evaluate the DMUs using the *Models (3) and (4)* to obtain the first-level efficient DMUs F (1).

Step 2. L (k+1) = L (k) – F (k), if L (k+1) =∅, then algorithm stops, else go to *Step 3*.

Step 3. Evaluate the DMUs in L (k+1), to obtain a new set of efficient DMUs and new-layer efficient frontier.

Step 4. Let k=k+1, go to *Step 2*.

After defining the efficient layers, then designed the model in a way that the performance score after re-allocation is not worse than before. In other words, we designed the model so that

$$\begin{pmatrix} (x_j - \Delta x_j) \\ \theta_j^{new}(y_j^g - \Delta y_j^g) \\ \lambda_j^{new}(y_j^b - \Delta y_j^b) \end{pmatrix} \in T(k).$$

T (k) is the changed production possibility set based on layers k and k-1.

$$T(k)=\left\{(x,y)\in L(k)\Big|\sum_{j\in L(k)\cup L(k-1)}\lambda_j x_{ij}\le x \quad, \sum_{j\in L(k)\cup L(k-1)}\lambda_j y_{rj}\le y \quad, \sum_{j\in L(k)\cup L(k-1)}\lambda_j=1 \quad, \lambda_j\ge 0\right\}.$$

We contract that L (0) =L (1). θ_j^{*new} , λ_j^{*new} are the efficiency scores related to desirable and undesirable outputs obtained for the transformed unit with the input and output values $\begin{pmatrix} x_j-\Delta x_j \\ y_j^g-\Delta y_j^g \\ y_j^b-\Delta y_j^b \end{pmatrix}$. We want the two values θ_j^{*new} , λ_j^{*new} to be improved as much as possible in the next stage. That is, in general, we want to have $1\le\theta_j^{new}\le\alpha_{j2}$, $\alpha_{j1}\le\lambda_j^{new}\le 1$.

Where α_{j1},α_{j2} are parameters related to the performance score of the undesirable and desirable outputs obtained in the previous step for each DMU_j , j=1,…, n, respectively.

For this purpose, we formulate the following allocation model as a multi-objective model:

$$\begin{aligned}
\min \quad & Z_2=\Delta Y^g,\\
\max \quad & Z_3=\Delta X,\\
\max \quad & Z_4=\Delta Y^b,\\
\text{s.t.} &
\end{aligned} \tag{5}$$

$$x_{ij}-\Delta x_{ij}\ge\sum_{l\in F(k)\cup F(k-1)} w_{jl}x_{il}, \qquad j\in F(k), i=1,..,m, \tag{5.1}$$

$$\theta_j^{new}(y_{rj}^g-\Delta y_{rj}^g)\le\sum_{l\in F(k)\cup F(k-1)} w_{jl}y_{rl}^g, \qquad j\in F(k), r=1,...,s, \tag{5.2}$$

$$\lambda_j^{new}(y_{pj}^b-\Delta y_{pj}^b)=\sum_{l\in F(k)\cup F(k-1)} w_{jl}y_{pl}^b, \qquad j\in F(k), p=1,...,P, \tag{5.3}$$

$$1\le\theta_j^{new}\le\alpha_{j2}, \qquad j\in F(k), \tag{5.4}$$

$$\alpha_{j1}\le\lambda_j^{new}\le 1, \qquad j\in F(k), \tag{5.5}$$

$$0\le\Delta x_{ij}\le x_{ij}, \qquad j\in F(k), i=1,...,m, \tag{5.6}$$

$$0\le\Delta y_{rj}^g\le y_{rj}^g, \qquad j\in F(k), r=1,...,s, \tag{5.7}$$

$$0\le\Delta y_{pj}^b\le y_{pj}^b, \qquad j\in F(k), p=1,...,P, \tag{5.8}$$

$$\sum_{l\in F(k)\cup F(k-1)} w_{jl}=1, \qquad j\in F(k), \tag{5.9}$$

$$w_{jl}\ge 0, \qquad j\in F(k), l\in F(k)\cup F(k-1).$$

$$\Delta X=\sum_{j=1}^{n}\sum_{i=1}^{m}\Delta x_{ij} \quad , \quad \Delta Y^g=\sum_{j=1}^{n}\sum_{r=1}^{s}\Delta y_{rj}^g \quad , \quad \Delta Y^b=\sum_{j=1}^{n}\sum_{p=1}^{P}\Delta y_{pj}^b. \tag{6}$$

The matrices X, Y^g, Y^b as follows:

$$X=\left[x_1,x_2,...,x_n\right]\in R^{m\times n}, \quad Y^g=\left[y_1^g,y_2^g,...,y_n^g\right]\in R^{s\times n} \quad , \quad Y^b=\left[y_1^b,y_2^b,...,y_n^b\right]\in R^{p\times n},$$

$$X>0, \quad Y^g>0, \quad Y^b>0.$$

Δx_j represents the saving amount of inputs, and $\Delta y_j^g, \Delta y_j^b$ represent the reduction amounts of desirable and undesirable outputs in DMU_j, respectively. w_{jl}, $l \in F(k) \bigcup F(k-1)$ is a factor contribution of DMU_j. *Model (5)* maximizes the sum of input changes, minimizes the sum of desirable output changes, and maximizes the sum of undesirable output changes. Now, assume the manager imposes the following conditions on the problem:

$$\Delta x_j \le A_j, \Delta x_j \ge B_j, \Delta y_j^g \le C_j, \sum_{j=1}^{n} \Delta y_j^b \ge D \;; D \in R_{\ge 0}^{P}, A_j, B_j \in R_{\ge 0}^{m}, C_j \in R_{\ge 0}^{s}.$$

If these conditions are added to the *Problem (5)*, it may become infeasible. Therefore the model is modified by Goal Programing (GP) in a way that it becomes feasible. Incidentally, this model is formulated under VRS assumption. The resource allocation model is presented using GP as follows:

$$\begin{aligned}
\min \quad & Z_1 = \sum_{j \in F(k)} n_{1j} + \sum_{j \in F(k)} n_{2j} + \sum_{j \in F(k)} v_j + q, \\
\min \quad & Z_2 = \Delta Y^g, \\
\max \quad & Z_3 = \Delta X, \\
\max \quad & Z_4 = \Delta Y^b, \\
\text{s.t.} &
\end{aligned} \tag{7}$$

$$x_{ij} - \Delta x_{ij} \ge \sum_{l \in F(k) \bigcup F(k-1)} w_{jl} x_{il}, \qquad j \in F(k), i = 1,\dots,m, \tag{7.1}$$

$$\theta_j^{new}(y_{rj}^g - \Delta y_{rj}^g) \le \sum_{l \in F(k) \bigcup F(k-1)} w_{jl} y_{rl}^g, \qquad j \in F(k), r = 1,\dots,s, \tag{7.2}$$

$$\lambda_j^{new}(y_{pj}^b - \Delta y_{pj}^b) = \sum_{l \in F(k) \bigcup F(k-1)} w_{jl} y_{pl}^b, \qquad j \in F(k), p = 1,\dots,P, \tag{7.3}$$

$$1 \le \theta_j^{new} \le \alpha_{j2}, \qquad j \in F(k), \tag{7.4}$$

$$\alpha_{j1} \le \lambda_j^{new} \le 1, \qquad j \in F(k), \tag{7.5}$$

$$\Delta x_j \le A_j + n_{1j}, \qquad j \in F(k), \tag{7.6}$$

$$\Delta x_j \ge B_j - n_{2j}, \qquad j \in F(k), \tag{7.7}$$

$$\Delta y_j^g \le C_j + v_j, \qquad j \in F(k), \tag{7.8}$$

$$\sum_{j \in F(k)} \Delta y_j^b \ge D - q, \tag{7.9}$$

$$0 \le \Delta x_j \le x_j \qquad j \in F(k), \tag{7.10}$$

$$0 \le \Delta y_j^g \le y_j^g, \qquad j \in F(k), \tag{7.11}$$

$$0 \le \Delta y_j^b \le y_j^b, \qquad j \in F(k), \tag{7.12}$$

$$\sum_{l \in F(k) \bigcup F(k-1)} w_{jl} = 1, \qquad j \in F(k), \tag{7.13}$$

$$w_{jl} \ge 0, \qquad j \in F(k), l \in F(k) \bigcup F(k-1).$$

Where $\Delta Y^g, \Delta X$ is defined in *Eq. (6)*, $n_{1j}, n_{2j} \in R_{\ge 0}^{m}$, $v_j \in R_{\ge 0}^{s}$, $q \in R_{\ge 0}^{P}$, $\Delta x_j = \left[\Delta x_{1j}, \Delta x_{2j}, \dots, \Delta x_{mj}\right]$, $\Delta y_j^g = \left[\Delta y_{1j}^g, \Delta y_{2j}^g, \dots, \Delta y_{s\,j}^g\right]$, $\Delta y_j^b = \left[\Delta y_{1j}^b, \Delta y_{2j}^b, \dots, \Delta y_{Pj}^b\right]$.

The *Eq. (7.4)* and *Eq. (7.5)* constraints guarantee that the performance will not be worse in the next stage. In *Eqs. (7.6)-(7.8)*, if the management's expectations that $\Delta x_j \le A_j, \Delta x_j \ge B_j, \Delta y_j^g \le C_j$, and $\sum_{j=1}^{n} \Delta y_j^b \ge D$ are unattainable, the deviation variables n_{1j}, n_{2j}, v_j, q will modify them, and they will prevent the problem from being infeasible. As can be seen, the objective function is multi-objective, and hence we can get the optimal solution in two steps according to the rules of lexicography. First, we solve the model with the first objective function as the first priority to make the model feasible. The second step is to optimize the weighted sum of the three next objective functions based on the optimal value of the previous step. Suppose $(\theta_j^{*new}, \lambda_j^{*new}, w_j^*, \Delta x_j^*, \Delta y_j^{g*}, \Delta y_j^{b*}, j = 1,...,n)$ are obtained by solving the model above.

Theorem 1. If $\theta_o^{*new} = 1, \lambda_o^{*new} = 1$, then DMU_o is efficient with the new input and output values $\begin{pmatrix} x_o - \Delta x_o^* \\ y_o^g - \Delta y_o^{g*} \\ y_o^b - \Delta y_o^{b*} \end{pmatrix}$.

Proof: Consider the re-allocated optimal values $x_j - \Delta x_j^*, y_j^g - \Delta y_j^{g*}, y_j^b - \Delta y_j^{b*}$ obtained from *Model (7)*, where $\theta_o^{*new} = 1, \lambda_o^{*new} = 1$.

If $\begin{pmatrix} x_o - \Delta x_o^* \\ y_o^g - \Delta y_o^{g*} \\ y_o^b - \Delta y_o^{b*} \end{pmatrix}$ is inefficient, then there exists a linear combination of DMUs that would dominate $\begin{pmatrix} x_o - \Delta x_o^* \\ y_o^g - \Delta y_o^{g*} \\ y_o^b - \Delta y_o^{b*} \end{pmatrix}$, meaning that there exists $(w_{o1}, w_{o2}, ..., w_{on})$ where $x_{io} - \Delta x_{io}^* \ge \sum_{l=1}^{n} w_{ol} x_{il}$ (i=1,…, m), $y_{ro}^g - \Delta y_{ro}^{g*} \le \sum_{l=1}^{n} w_{ol} y_{rl}^g$ (r=1,…,s), and $y_{po}^b - \Delta y_{po}^{b*} = \sum_{l=1}^{n} w_{ol} y_{pl}^b$, (p=1,…,P). Moreover, at least one of the inequalities strictly holds true in the first and second restrictions. If this strict inequality occurs in the first restriction, it would be in contradiction with the third objective function, and if it occurs in the second restriction, it would be in contradiction with the second objective function.

Theorem 2. If DMU_o was efficient in the past, it will be efficient in the next period as well.

Proof: If $\alpha_{o2} = 1$, $\alpha_{o1} = 1$ based on *Eq. (7.4)* and *Eq. (7.5)*, we have $\theta_o^{*new} = 1$ and $\lambda_o^{*new} = 1$. Therefore, according to theorem 1, DMU_o is an efficient unit.

3.1 | Illustrative Example

Table 1 shows the inputs and outputs of seven DMUs, which consume the same inputs to produce different desirable and undesirable outputs. The results obtained using *Model (7)* are shown in *Fig. 1*. The figure is displayed based on the desirable and undesirable outputs. The DMUs are classified into three layers using the variables obtained from *Model (4)* and placement in *Model (3)*. At the first, DMU_A, DMU_F and DMU_C are efficient and stay on layer 1. If we remove this layer, then if the remaining inefficient DMUs are evaluated, a second efficient layer is formed. And leaving aside layers 1 and 2, an efficient frontier of layer 3 is created. DMU_A, DMU_F and DMUC are projected onto the layer 1 efficient frontier. DMUE and DMUG are projected onto layer1. DMU_B and DMU_D are projected onto layer 2. As can be seen, after running *Model (7)*, y^g and y^b in DMU_A remain unchanged and DMUA is placed on its previous location, but units C and F, although they are on the first efficient frontier layer, change their location on the layer they are on. The desirable output of DMUC is reduced from 6 to 4.8, and the undesirable output is reduced from 5 to 3.8 (C'). Also, the desirable and undesirable outputs of DMU_F are reduced from 4 and 3 to 3.2 and 2.8, respectively (F'). The desirable output of DMU_D does not change, but its undesirable output is decreased from 7 to 4.66 (D'). Similarly, DMU_E is projected on F, DMU_B on G, and DMUG on G'. The units on layer3 have been moved to layer 2 and the units on layer 2 have been moved to layer1, but the units on layer1 can either change or not.

Table 1. Input and output data of 7 DMUs and the results obtained from Model (7).

DMU	x	y^g	y^g	$\theta_j^* = \frac{1}{\beta_j^*}$	$\lambda_j^* = \theta_j^* \lambda_j^{\prime *}$	$y^g - \Delta y^g$	$y^b - \Delta y^b$
A	7	1	2.5	1	1	1	2.5
B	7	2	6	2	0.5	2	4
C	7	6	5	1	1	4.8	3.8
D	7	3.5	7	1.14	0.42	3.5	4.66
E	7	5	5	1	0.8	4	3
F	7	4	3	1	1	3.2	2.8
G	7	2	4	2	0.75	1.6	2.60

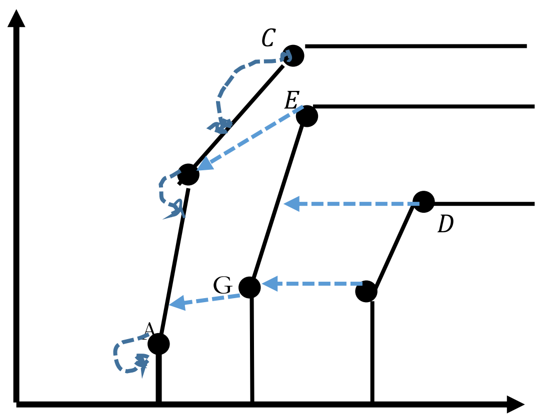

Fig. 1. Illustration of a simple example using Model (7).

3.1.1 | Practical example

In this section, we apply our *Model (7)* to analyze a practical example. *Table 2* shows the input and output data of eight regions of the world in 2002. Energy consumption (Mt) is considered as the input, regional GDP (billion 1995 US$) as the desirable output, and CO_2 emissions (Mt) as the undesirable output. These sources are compiled from [17]. The fourth column presents the optimal values obtained when assessing the environmental performance of each region using *Model (4)*. We assume that the centralize decision maker considers the following parameters:

$$A_j = 0.3x_j \quad B_j = 0.00 \quad ,C_j = 0.2y_j^g, \quad D = 0.4\sum_{j=1}^{n} y_j^b.$$

Table 2. Input and output data of 8 region in the world in 2002 and their efficiency score.

Unit	Energy Consumption (Mtoe)	GDP (Billion in PPP)	CO_2 Emissions (Mt)	$\alpha_{j2} = \theta_j^*$	$\alpha_{j1} = \lambda_j^*$
Middle East	290.90	1025.83	1092.84	2.04	0.65
OECD	3696.50	25374.85	12554.03	1	1
Non-OECD Europe	63.86	358.26	252.84	1	1
Asia excludes China	851.40	5507.94	2257.41	1	1
China	823.02	5359.02	3307.42	1	0.66
Former USSR	610.17	1552.10	2232.17	1.66	0.36
Africa	404.42	1668.75	743.12	1.31	1
Latin American	354.75	2566.74	844.61	1	1

The feasibility of the problem under the conditions that the centralize manager makes is the top priority of the problem. In other words, $Z_1^* > 0$ means that if we do not consider the deviation variables, then the problem would be infeasible. Minimizing the total output changes is the second objective, maximizing the total input changes is the third objective, and maximizing the total undesirable output changes is the last objective. Hosseinzadeh Lotfi et al. [41], Yang et al. [42] and Amirteimoori and Kordrostami [12] investigated the associations between multi-objective problems and DEA. Bal and Orkcu [43] proposed using a lexicographic model to solve GP problems and allocating priority to objective functions of Multi-Criteria Decision Making (MCDM). Bal et al. [44] proposed the weighted Goal Programming and Data Envelopment Analysis (GPDEA) method. The optimal values of this model can be obtained by lexicography's prioritization method. The second step is to obtain the weighted sum of the three next objective functions in order to minimize the desirable output reduction, maximize the input saving, and maximize the undesirable output reduction in the optimal solution, which is obtained from the first step. The results using Gams software are as follows (here, we consider all weights equal to 1 for convenience).

The reduction values of inputs and outputs are indicated in *Tables 3* and *4*.

Table 3. Reduction value of inputs and outputs.

Unit	Δx	Δy^g	Δy^b
Middle East	58.18	0	661.122
OECD	739.30	5091.273	2503.487
Non-OECD Europe	12.772	0	0
Asia excludes China	170.28	854.148	0
China	164.604	993.9462	1598.983
Former USSR	122.034	0	1257.305
Africa	80.884	333.750	228.55
Latin American	70.95	613.986	0
Total amount	1419.004	7887.1032	624.447
Proportional	0.20	0.18	0.27

Table 4. Results of allocation value for inputs and outputs and their evaluation.

Unit	$x - \Delta x$	$y^g - \Delta y^g$	$y^b - \Delta y^b$
Middle East	232.72	1025.83	431.717
OECD	2957.20	20283.576	10050.542
Non-OECD Europe	51.088	358.26	252.84
Asia excludes China	681.12	4653.791	2257.41
China	658.416	4365.073	1708.436
Former USSR	488.136	1552.10	974.864
Africa	323.536	1335	514.560
Latin American	283.80	1952.753	844.61

After evaluating the units, units OECD, Non-OECD Europe, Asia excludes China and Latin American are efficient in the first stage and form the efficiency layer 1. After discarding the units on the layer 1, among the remaining units, Middle East, China and Africa create the efficiency layer 2. Naturally, only unit Former USSR is on the layer 3. Units on the layer 1 are allowed to change on the same layer, while units on the layer 2 can be projected on layer 1 and layer 2, but due to the constraints of performance improvement, they tend to apply the reductions so that they are projected on leyer1 in the next step. Finally, unit Former USSR, which is on the layer 3, project on the layer 2.

Results from *Table 3* show that with a reduction of 739.30 units in the inputs of the OECD, 5091.273 units are reduced in the desirable outputs, and 2503.487 units are reduced in the undesirable outputs. Meanwhile, before re-allocation, the OECD unit had efficiency in relation to both the desirable and undesirable outputs (based on *Table 2*). Although OECD was efficient, it was allowed to change on the efficient layer 1 and remains on the layer1. China is projected by reducing input by 164.604, desirable output by 993.946 and undesirable output by 1598.983 from layer 2 to layer 1, which indicates an improvement its efficiency. This suggestion can be considered by policy makers in order to reduce environmental pollution and save energy. Overall, with 20% reduction of energy consumption in these 8 regions of the world, we will observe 18% reduction in GDP and 27% reduction in air pollution, and this is while all units are located on the previous frontier in the next stage. Depending on the degree to which policy makers' care about reducing environmental pollution and energy consumption, or the degree to which the World Health Organization (WHO) obligates countries to reduce environmental pollutants, this article may be of interest to them. Thus, by executing the proposed *Model (7)* with data collected from 8 regions of the world, we can gain a proper perspective on pollution reduction and the obligation of policy makers by WHO regarding this matter. Although, it should be taken into account that for any of these 8 regions, not only the efficiency has not been reduced but it has also moved toward the previous efficient frontier in cases of inefficiency (according to *Table 4*).

4 | Conclusion

Reducing pollution in developed and developing countries has become an important area of research for researchers. Governments in developed countries have a vision for controlling pollution. In this regard, studies in this field are very necessary and important. However, pollution reduction must follow patterns and models that are logical and applicable for managers. In the current study, a resource allocation model for energy saving and environmental pollution reduction was presented in an environment with a central decision maker. This model is recommended to managers and policy makers for reducing environmental pollution. When dealing with undesirable outputs, some researchers have recommended considering weak disposability for desirable and undesirable outputs. Thereby, the presented model considers weak disposability for the undesirable outputs while trying to reduce energy consumption and environmental pollution. The model was designed in a way that would allow the efficiency of the units to improve after re-allocation, while at the same time, this improvement would be logical and feasible from the manager's point of view. In other words, after re-allocation, the efficiency of a given unit could remain constant or improve. In this regard, we used the concept of context-dependent DEA to divide the units into different frontiers, and designed a model in which each unit is allowed to move from its current frontier to its previous frontier, which has better conditions than the frontier on which the unit is located. In this situation, the units will try to reach the previous efficient frontier and not necessarily to the original efficient frontier. This is the most important point in the proposed model, which creates a logical reduction in the inputs and outputs. This is because projecting an inefficient unit that is too far from the first (original) efficient frontier on the original frontier may be very costly for the central manager and not logical. In this model, the efficient units are still located on the efficient frontier, though they might move on the frontier and be relocated to another point on the same frontier in the next stage. Furthermore, in this model, the preferences of the central management regarding the amount of input and output changes are applied. Given that these manager's preferences may cause the model to become infeasible, the model was modified using GP in a way that it would always be feasible. In this study, by combining DEA with MOP, we attempted to achieve multiple objectives desired by the central management.

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