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Equitable Resource Allocation Combining Coalitional Game and Data Envelopment Analysis

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Abstract

In this paper, while taking into account the cooperative relationships between units, the problem of revenue allocation is considered as a coalitional game. In order for the allocation to be equitable, by relying on the concept of DEA efficiency, a new characteristic function is presented, and then using the concept of the Shapley value, which is a well-recognized concept in coalitional game theory, a unique solution is obtained for the revenue allocation problem. And finally, to evaluate the equitability of the performed revenue allocation, the Gini coefficient is utilized. A comparison of the Gini coefficient obtained for our method with those of some existing methods showed that our method is more equitable than the previous ones. This demonstrates how impactful the wise and accurate selection of the characteristic function is in the equitability of the results.

Keywords: Resource allocation, Data envelopment analysis, Revenue allocation, Cooperative game, Shapley value.

1 | Introduction

Strategic management refers to a set of managerial decisions and actions that determine the long-term performance of an organization [1]. Each organization has at least four types of resources that must be allocated to achieve the organizational goals: financial resources, physical resources, human resources, and technological resources [2]. The matter to be considered here is the limitation of resources. Therefore, studying the matter of resource allocation, which refers to the art and science of allocating existing limited resources for various uses [2], would require a careful scientific and logical approach that would provide an appropriate justification for using various mathematical-management models and approaches. Resource allocation is one of the main activities of strategic management, and it is an extremely difficult and sensitive task that can decide the failure or success of a project. Technically, the issue of resource allocation mainly exists in organizations in which a set of units are operating under the management of a central decision maker, and the central decision maker has the power to control the decision parameters, such as resources [3], [4].

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Game theory, which was introduced in 1921 with the publication of a number of papers by the French mathematician Émile Borel on the predictability of casino games, has gained extensive application nowadays in various areas, such as economics, management, social sciences, biology, and so forth. Generally, a game can be defined as follows: "whenever the payoff of an entity is not solely dependent on its own behavior and is affected by the behaviors of one or multiple other entities, meaning that the decisions of others can have positive or negative effects on the entity's payoff, a game has started between two or more entities" [5]. Based on this definition, the problem of resource allocation creates a strategic situation, which can be considered as a game and for which solutions can be provided.

Generally, games can be divided into the two categories of cooperative and non-cooperative. The nature of the cooperative approach in dealing with problems has gained attention in numerous researches [41]. In game theory, this approach is more compatible with both Data Envelopment Analysis (DEA) and resource allocation problems. Thereby, it seems appropriate to use techniques related to cooperative games in order to arrive at a logical solution to the resource allocation problem [6]. In cooperative games, the players decide to behave in a way that would be optimal for the community. The main issue in these games is the way the payoff resulting from this cooperation is divided between the members. Cooperative games fall into two categories themselves: Transferable Utility (TU) games and bargaining games. In TU games, utility is transferred from one player to another with no reduction in the utility of the whole community [7]. Based on these explanations, the problem of allocation can be discussed from the perspective of TU games.

The DEA technique, which was first introduced in 1978 by Charnes et al. [8] is a nonparametric method for evaluating the efficiency of homogenous and comparable Decision-Making Units (DMUs) [9]. Nowadays, the use of DEA is becoming rapidly widespread, as it has found applications in various areas. In recent years, one of the most important applications of DEA has been in solving the problem of resource allocation to a set of DMUs [10], [11].

The history of DEA-based approached in game theory goes back to the 1980 paper by Banker [12]. In that paper, Banker [12] proved that the efficiency score produced by DEA is the equivalent of a two-person zero-sum game in which the players consist of an external evaluator and the DMUs. In this paper, it is assumed that all players have access to the inputs and outputs of the DMUs [5]. Banker et al. [13] reformulated the approach presented by Banker [12] by constraining the previous approach to consider non-zero slack variables. Cook and Kress [9] studied fixed cost allocation problems within a DEA framework for the first time. Their proposed approach was based upon two axioms:

I. Efficiency invariance: meaning that the efficiency of each DMU shall remain unchanged after allocation.
II. Pareto-minimality: a cost allocation is said to be Pareto-minimal if no cost is transferrable from one DMU to another, unless the first axiom is violated [9].

Beasley [14] presented a cost allocation method by considering the axiom of efficiency maximality, based on which the efficiency of all DMUs increases following allocation. By presenting an example, Jahanshahloo et al. [15] proved that in the study by Cook and Kress [9], the Pareto-minimality axiom has been violated, and then proposed a method for fixed cost allocation by presenting a formula and without any need to solve the model. Amirteimoori and Kordrostami [16] presented a method based on common weights and efficiency invariance.

Cook and Zhu [17] extended the approach proposed by Cook and Kress [9] from input orientation to output orientation, and presented a feasible cost allocation. In this study as well, it is assumed that the cost is an additional input for each DMU. Li et al. [18] proposed combining the allocated cost with other costs so that they make up a single input value in the efficiency calculation. In addition, based on DEA and coalitional game concepts such as Nash bargaining, core, and nucleolus, they presented corresponding fixed cost allocation methods. Amirteimoori and Tabar [10] presented a DEA-based approach for resource allocation with fixed costs. They believed that resource allocation and benchmarking should be carried out in a way that every DMU has an efficiency of one. Lin [16] proved that the method proposed by Cook and Zhu [17] does not have a feasible solution in specific situations, and improved the method. Lin [19] also proved through a numerical example that the method presented by Jahanshahloo et al. [20] is not acceptable in many applications. Furthermore, he [19] proposed allocating a ratio of fixed costs to common revenue to each DMU, and adopted the axiom of minimum deviation to ensure that the efficiency invariance condition is met [19]. Lozano [21] presented a cooperative game based on DEA. That study was based on the idea that different organizations would benefit from sharing the input and output data of their units that are under investigation. Now, since not all organizations benefit from the outcome of this information exchange to the same degree, a cooperative game should be utilized. It was proven that the cooperative DEA game presented was balanced and subadditive [21]. Mostafaee [22] presented an alternative allocation method in which the efficiency and returns to scale remained unchanged in all DMUs following allocation. After testing the equitability of the proportional sharing method [14], Si et al. [23] extended the method from the one-dimensional case to a multi-input and multi-output case, and then studied the relationship between the presented method and other DEA-based allocation methods. Li et al. [24] proved there are some cost allocations that can make all DMUs or sets of DMUs efficient. In a one-dimensional case, such an allocation would be unique and equivalent to the proportional sharing method. Next, they defined the concept of satisfaction degree, and presented an algorithm for a unique fixed cost allocation by proposing a maxmin model [24]. Du et al. [25] used the concept of cross-efficiency in DEA to discuss the matters of resource allocation and cost allocation. They proved that their proposed method is always feasible, and that after allocating the fixed cost as an input, all DMUs would become efficient. Khodabashshi and Aryavash [26] stated that the allocation of a common fixed cost or revenue must be carried out based on three principles: direct proportionality of the allocation with inputs and outputs that are directly proportional to the common fixed cost or revenue; inverse proportionality of the allocation with inputs and outputs that are inversely proportional to the common fixed cost or revenue; and finally, eliminating the effect of inputs and outputs that have no effect on the common fixed cost or revenue in the allocation. Yang and Zhang [27] introduced a characteristic function based on DEA efficiency, and then presented a modified Shapley value for solving resource allocation problems using cooperative games. In that study, they also presented a new Gini coefficient to demonstrate the equitability of the resource allocation project based on DEA efficiency. Lin and Chen [25] showed that the Pareto-minimality axiom presented by Cook and Kress [9] is not suitable. Therefore, based on the concepts of super efficiency and practical feasibility, they presented a novel cost allocation method [19]. Jahanshahloo et al. [20] showed that in the method proposed by Amirteimoori and Kordrostami [16], the principle of efficiency invariance does not necessarily hold, and thereby presented an equitable approach to cost allocation based on the principles of efficiency invariance and common weights. Lin and Chen [28] studied fixed cost allocation from the perspective of efficiency analysis. For this purpose, they first introduced an enhanced additive general model using the DEA technique, and presented an algorithm for unique fixed cost allocation. Li et al. [29] presented a new mechanism for adopting the principles of common weights and efficiency invariance in the allocation of multiple resources and the determination of multiple objectives. This method was based on a minimization of the deviation between the possible design based on common weights and the possible design based on the efficiency invariance principle. Pendharkar [30] presented a hybrid framework of the genetic algorithm and DEA for solving fixed cost allocation problems. Their proposed framework allowed managers to enter various objective subfunctions for both efficient and inefficient DMUs, and solve the fixed resource allocation problem in a way that would maximize the overall resource allocation entropy for efficient DMUs while minimizing the correlation between resource allocation and efficiency score [30]. By presenting two centralized DEA methods in a central decision-making environment, Ding et al. [31] introduced a novel approach to fixed cost allocation and resource allocation that considered technology heterogeneity. Using DEA and game theory, Zhang et al. [32] presented a DEA-game programming and solved it using nucleolus and the Shapley value.

Li et al. [33] considered both competitive and cooperative relationships between DMUs and presented a combined method of cross efficiency and cooperative games. This approach seems attractive for large organizations. Li et al. [34] presented a novel common weights model for solving fixed resource allocation problems in a decentralized environment based on goal programming and DEA with a non-egoistic approach. Furthermore, by considering the fact that game relations exist in the process of allocation, Li et al. [34] presented a cooperative game approach for solving cost allocation problems, and used the nucleolus concept to obtain the solution of the proposed game. After defining the concept of utility, Chu and Jiang [35] presented an approach for fixed cost allocation to each DMU based on the framework and advantages of common weight estimation and DEA. Their proposed approach ensured allocation uniqueness [35]. Ding et al. [36] discussed the matter of fixed cost allocation under the control of a central authority for a general two-stage production network structure with external inputs and outputs. Zhu et al. [37] presented three different procedures for equitable fixed cost allocation when the DMUs consist of two stages.

Furthermore, using the concept of relative efficiency in DEA and considering common weights, Li et al. [34] presented an optimal and unique design for fixed cost allocation in a case where the DMUs have a two-stage network structure, while taking into account the size of the operational units. Qingxian et al. [38] presented a fixed cost allocation approach for two-stage systems under the condition of efficiency invariance, once in a case where the two stages have a cooperative relationship, and another time in a case where the stages have a non-cooperative relationship. Then, they extended the approach to general two-stage systems.

As mentioned in the beginning of this section, one type of resources available to organizations is financial resources. Financial resources are not necessarily related to costs, but rewards or merit pay in income-generating organizations are also considered as financial resources. In the current study, by combining DEA with cooperative games, we intend to present an equitable method for revenue allocation. Our motive for the study is that in our opinion, in an organization, the efficiency of a given unit being lower or higher would have an impact on the revenue of the other units, and therefore, each unit must necessarily be considered in relation to the other units. Thereby, revenue allocation can be discussed as a game theory problem. However, the important issue is that equitability must be maintained in revenue allocation, i.e. in a way that the more efficient units do not lose their motivation, while the weaker units are encouraged to continue their cooperation and increase their efficiency. To this end, based on DEA efficiency, we first proposed a new characteristic function, and then using the Gini coefficient, which is a well-known concept in economy and management, the equitability of our revenue allocation was evaluated. Comparing the results of our allocation with other methods shows that the Gini coefficient of our allocation was smaller than that of all other methods. This shows how important the correct choice of the characteristic function is in the equitability of the allocation.

The rest of the paper is structured as follows: Section 2 provides the basic definitions necessary for understanding the topics discussed herein. In Section 3, a characteristic function is presented based on the state of overall and partial efficiencies in each unit. Section 4 presents a revenue allocation algorithm using the Shapley value in combination with the introduced characteristic function. In Section 5, numerical results are presented, and the equitability of the allocation carried out based on our proposed characteristic function is evaluated by calculating the Gini coefficient and making relative comparisons

2 | Preliminaries

In order for the reader to have a proper understanding of the contents and analyses presented in this paper, it is necessary to be familiar with the basic concepts of DEA and TU games; therefore, in this section, we first present the basic definitions in DEA, and then discuss those related to cooperative games.

Definition 1. Assume that an organization is consisted of n homogenous and comparable DMUs that consumes the input vector $X=\left(x_1,x_2,...,x_m\right)$ to produce the output vector $Y=\left(y_1,y_2,...,y_t\right)$ .The CCR production possibility set (PPS) is defined as follows [9]:

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$$P=\left\{(X,Y)\middle| X\geq\sum_{j=1}^{n}\lambda_j X_j,\ Y\leq\sum_{j=1}^{n}\lambda_j X_j,\ \lambda\geq 0\right\}. \tag{1}$$

According to production technologies there are several PPSs. In this paper, the CCR production technologies has been considered.

Definition 2. The output-oriented CCR model for measuring the relative efficiency of the k^th^ unit is as follows [39]:

$$\begin{aligned} & d_k^* = \max\ d_k, \\ & \text{s.t.} \\ & \sum_{j=1}^{n} x_j\lambda_j \leq x_k, \\ & \sum_{j=1}^{n} y_j\lambda_j \geq d_k y_k, \\ & \lambda_j \geq 0,\ j=1,\ \dots,\ n. \end{aligned} \tag{2}$$

The constraints of the *Model (2)* ensure that the optimal value satisfies $d_k^* \geq 1$. Moreover, DMUk is DEA efficient if and only if $d_k^* = 1$. In the output-oriented *Model (2)*, it is obvious that the larger the optimal value of DMU is, the lower the efficiency will be.

Definition 3. Assume that *N* set of players are present in a game; now, each subset of *N* such as *S*, who are cooperating with each other based on an agreement to maximize the profits for the group, is called a coalition. Therefore, the sets φ and *N* are also coalitions of the game, and are called the empty coalition and the grand coalition, respectively. The set of all possible coalitions in a game is the power set of N denoted by $P(N)$ [40].

Definition 4. The TU game is denoted by $\langle N,\ \nu\rangle$, in which $\nu : P(N) \rightarrow \mathbb{R}$ is the characteristic function of the TU game, which assigns a real number to each subset (coalition) of *N*. It is always assumed that $\nu(\varphi) = 0$ [40].

Technically, if in this definition, S is a subset of *N*, then $\nu(S)$ shows the overall TU of the coalition *S*.

Definition 5. If in a game, the players prefer higher earnings, that game is a profit game. Meanwhile, in a cost game, a lower level of earnings is preferred [41].

Definition 6. The characteristic function of a coalitional game is super-additive if it has the following property [42]:

$$\nu(S\cup T) \geq \nu(S) + \nu(T)\ ,\ S,T \subseteq N,\ S\cap T = \phi. \tag{3}$$

Technically, having this property requires for the best coalition to be the coalition N, i.e., the grand coalition. This property is useful in organizations that consist of different units, and in which all these units must necessarily cooperate with each other as a group.

Definition 7. The Shapley value, presented in 1953 by Shapley [43], provides a unique answer to the question that in which way the earned profits should be divided between the coalition members. The Shapley value is based on the following three principles:

I. The determining factor for the share of each player from the profits is their role in earning the game profits.

II. If a player plays no role in reaching the profits of the game or in the formation of the coalition, their share of the profits will be equal to zero.

III. The profits expected by each player before and after the profits are earned by the coalition shall not be affected by their bargaining about their strategy.

Thereby, the Shapley value can be calculated using the vector function φ, which adheres to the three principles above and is defined as follows [43]:

$$\varphi_k(\nu)=\sum_{S\subset N}\frac{s!(n-s-1)!}{n!}\{\nu(S\cup\{k\})-\nu(S)\},\quad k\notin S,\quad k=1,\ 2,\ \dots,\ n. \tag{4}$$

The Shapley value is technically the marginal average of the share of each player in earning the coalition's profits.

Definition 8. The Gini coefficient, which is considered as a criterion for the equitability of an allocation program in management sciences, is a number between zero and one. The closer this number is to zero, the more equitable the allocation in question will be. The Gini coefficient can be calculated as follows [44]:

$$G=\frac{\sum_i\sum_{j>i}|q_jS_i-q_iS_j|}{\sum_i S_i}.$$

In the formula above, q_j and S_j are the equity and service units, respectively. In the current study, the service and quality units are the amount of resources allocated and the efficiency of the units, respectively; therefore, q_j and S_j are defined as follows [27]:

$$q_j=\frac{d_j^*}{\sum_{i=1}^n d_i^*},\quad S_j=\frac{\varphi_j(\nu)}{\sum_{i=1}^n\varphi_i(\nu)}R,\quad (j=1,\dots,n).$$

If we enter the equations above into the Gini coefficient formula, the following equation will be resulted:

$$G=\frac{\sum_k\sum_{h>k}|q_h\varphi_k-q_k\varphi_h|}{\sum_k\varphi_k}. \tag{5}$$

3 | The Proposed Characteristic Function

In this section, based on the concept of efficiency in DEA, we introduce a profit characteristic function for TU games. Given that the units of the organization are not in a similar situation in relation to each other in terms of efficiency, to maintain equitability, this matter must be considered when defining the characteristic function. By considering the overall efficiency and partial efficiencies (efficiency of a DMU in the coalition it is a part of), the proposed characteristic function will show the real value of a coalition.

Assume that $N=\{1,\ 2,\ \dots,\ n\}$ is the set of all units in the organization and $S\subseteq N$ is a coalition in the form of $S=\{1,\ 2,\ \dots,\ s\}$; (in general assume that $|S|=s$). The proposed characteristic function is as follows:

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$$\nu(S)=\sum_{i=1}^{s}\frac{d_i^S}{d_i^P}. \tag{6}$$

In this function, d_i^S and d_i^P are the partial and overall efficiencies of player i, respectively, which are obtained by solving the *Models (7)* and *(8)*, respectively:

$$\begin{aligned} d_i^S = \max\ & d_i, \\ \text{s.t.}\ & \\ & \sum_{j=1}^{s} x_j\lambda_j \le x_i, \\ & \sum_{j=1}^{s} y_j\lambda_j \ge d_i y_i, \\ & \lambda_j \ge 0,\ j=1,\ \dots,\ s. \end{aligned} \tag{7}$$

$$\begin{aligned} d_i^P = \max\ & d_i, \\ \text{s.t.}\ & \\ & \sum_{j=1}^{n} x_j\lambda_j \le x_i, \\ & \sum_{j=1}^{n} y_j\lambda_j \ge d_i y_i, \\ & \lambda_j \ge 0,\ j=1,\ \dots,\ n. \end{aligned} \tag{8}$$

In other words, d_i^S is the optimal value of the objective function of the output-oriented CCR model in the PPS formed by all members of the coalition S (in this research, these PPSs are called partial PPSs), and d_i^P is the optimal value of the objective function of the output-oriented envelopment CCR model in the PPS formed by all members of N (in this research, this PPS is called the grand PPS). Note that depending on the production technology, other DEA models, including radial, non-radial, linear, or nonlinear models, can also be used.

Consider the ratio $\frac{d_i^S}{d_i^P}$ in the defined characteristic function. As previously mentioned, the numerator and denominator in this fraction are output-oriented efficiency scores; therefore, $d_i^S \ge 1$ and $d_i^P \ge 1$. On the other hand, $S \subseteq N$, so obviously, $PPS(S) \subseteq PPS(N)$; therefore, $d_i^S \le d_i^P$, and obviously, $\frac{d_i^S}{d_i^P} \le 1$. Hence, $\nu(S) \le s$; it is obvious that the closer this ratio is to one, the more valuable DMU_i is in terms of efficiency. Thereby, the closer $\nu(S)$ is to s, the more valuable the coalition S will be. Obviously, s is an ideal for $\nu(S)$. We demonstrated that higher $\nu(S)$ values are more desirable for us; therefore, the defined characteristic function is a profit function.

Note: It should be noted here that if as in the case of certain researches, such as Yang and Zhang [27], the characteristic function is considered as $\nu(S)=\sum_{i=1}^{s} d_i^P$, since d_i^P indicates the output-oriented efficiency of the members of the coalition S, then $d_i^P \ge 1$. Therefore, the more inefficient a DMU is, the larger the value of d_i^P will be. Thus, a coalition with more inefficient members would obtain a higher characteristic function than a coalition in which the members have higher efficiency scores, and this is incorrect; thereby, the characteristic function above is not a profit characteristic function.

In the following, we will prove that the proposed characteristic function is super-additive.

Lemma 1. The defined characteristic function is super-additive.

Proof: We intend to prove that:

$$v(S \cup T) \geq v(S) + v(T) \quad , \ S, T \subseteq N, \ S \cap T = \phi.$$

Without losing generality, assume that $S = \{1, 2, ..., s\}$ and $T = \{s+1, s+2, ..., s+t\}$; therefor:

$$A: \ v(S \cup T) = \sum_{i=1}^{s+t} \frac{d_i^{P^{S\cup T}}}{d_i^P} = \left(\frac{d_1^{P^{S\cup T}}}{d_1^P} + \frac{d_2^{P^{S\cup T}}}{d_2^P} + \ldots + \frac{d_s^{P^{S\cup T}}}{d_s^P} \right) + \left(\frac{d_{s+1}^{P^{S\cup T}}}{d_{s+1}^P} + \frac{d_{s+2}^{P^{S\cup T}}}{d_{s+2}^P} + \ldots + \frac{d_{s+t}^{P^{S\cup T}}}{d_{s+t}^P} \right).$$

$$B: \ v(S) + v(T) = \sum_{i=1}^{s} \frac{d_i^S}{d_i^P} + \sum_{i=s+1}^{s+t} \frac{d_i^T}{d_i^P} = \left(\frac{d_1^S}{d_1^P} + \frac{d_2^S}{d_2^P} + \ldots + \frac{d_s^S}{d_s^P} \right) + \left(\frac{d_{s+1}^T}{d_{s+1}^P} + \frac{d_{s+2}^T}{d_{s+2}^P} + \ldots + \frac{d_{s+t}^T}{d_{s+t}^P} \right).$$

On the other hand, it is obvious that:

$$P^S \subseteq P^{S\cup T} \qquad \& \qquad P^T \subseteq P^{S\cup T}$$

$$P^S \subseteq P^{S\cup T} \Rightarrow \begin{cases} P^S = P^{S\cup T} \Rightarrow d_i^S = d_i^{S\cup T} \ , \ i = 1, 2, ..., s \\ P^S \subset P^{S\cup T} \Rightarrow d_i^S \leq d_i^{S\cup T} \ , \ i = 1, 2, ..., s \end{cases} \Rightarrow d_i^S \leq d_i^{S\cup T}; \ i = 1, 2, ..., s. \tag{9}$$

$$P^T \subseteq P^{S\cup T} \Rightarrow \begin{cases} P^T = P^{S\cup T} \Rightarrow d_i^T = d_i^{S\cup T} \ , \ i = s+1, s+2, ..., s+t \\ P^T \subset P^{S\cup T} \Rightarrow d_i^T \leq d_i^{S\cup T} \ , \ i = s+1, s+2, ..., s+t \end{cases} \Rightarrow d_i^T \leq d_i^{S\cup T} \ , \ i = s+1, s+2, ..., s+t. \tag{10}$$

Now, consider A and B. The denominators of the fractions are the same in both equations; however, based on *Eqs. (9)* and *(10)*, $A \geq B$, and this completes the proof.

4 | Resource Allocation Based on the Proposed Characteristic Function and the Shapley Value

As previously explained, in management, especially strategic management, due to the limitation of resources, the topic of resource allocation is of great importance, so much so that not adopting an appropriate and scientific approach can lead to the failure of a project and impose heavy losses on the organization. The significance of this matter in revenue allocation is also undeniable, as all units participating in a coalition need motivation to continue their cooperation and increase the quality of their services in the future. A logical and acceptable revenue allocation can create such motivation. In this section, using the proposed characteristic function and the Shapley value formula, an algorithm will

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be presented for revenue allocation. Our motive for using the Shapley value lies in the fact that in practice, managers are after simple solutions that are easy to understand, and it is also important to them to have a unique solution. Given that the Shapley value provides a unique solution for cooperative games, it was used in this research to determine the share of each unit from the revenue.

Step 1. Using the *Models (7)* and *(8)*, calculate the overall and partial efficiencies for all DMUs.

Step 2. Using the results from step one and the *Formula (6)*, obtain the value of the characteristic function for all non-empty coalitions N.

Step 3. Using the results from step two and the *Formula* (4), calculate the Shapley value for each DMU.

Step 4. Assuming that R is the resource being allocated, calculate the share of each DMU as follows:

$$r_k = \frac{\varphi_k(v)}{\sum_{i=1}^{n} \varphi_i(v)} \times R \ , \quad k = 1,\ 2,\ \ldots,\ n.$$

Step 5. Evaluate the equitability of the allocation by calculating the Gini coefficient in the *Eq. (5)*.

5 | Numerical Results

In this section, by presenting a numerical example and comparing the Gini coefficient with certain resource allocation methods, we demonstrate that using the proposed characteristic function would lead to a more equitable allocation.

Example 1. *Table 1* shows the data for 12 DMUs, each of which have 3 inputs and 2 outputs [12]. The efficiency scores of units, the Shapley value for each unit, and finally, the share of each unit from the determined revenue can be observed in *Table 2*. We assumed a revenue of R=100 for allocation.

To provide a comparison between the equitability of the allocation carried out based on our proposed characteristic function and equitability in some of the existing methods, *Table 3* has been dedicated to the results of the Gini coefficient calculation.

Observing *Table 2*, we find that the allocation is not similar for all the efficient units, and this is one of the advantages to our allocation, as it differentiates between the efficient DMUs as well. In this relation, the DMUs with a higher impact in the coalitions receive higher profits and, naturally, a higher motivation for continuing cooperation with the other DMUs. With a little attention to the inefficient DMUs, we find that the inefficient DMUs are also differentiated based on how close they are to efficiency. In this respect, the more inefficient a DMU has been, the smaller the share it has received from the revenue. On the other hand, the difference between the largest share and the smallest share of the profits is not that great to cause a lack of motivation in the more inefficient units, a fact that is obvious from the calculated Gini coefficients in *Table 3*. It is noteworthy that our allocation has produced smaller Gini coefficients than all the other methods mentioned in *Table 3*, which indicates a more equitable allocation. The advantage of our allocation is using a suitable characteristic function in TU game.

Table 1. Sample DMUs.

DMU$_j$	Input 1	Input 2	Input 3	Output 1	Output 2
DMU 1	350	39	9	67	751
DMU 2	298	26	8	73	611
DMU 3	422	31	7	75	584
DMU 4	281	16	9	70	665
DMU 5	301	16	6	75	445
DMU 6	360	29	17	83	1070
DMU 7	540	18	10	72	457
DMU 8	276	33	5	78	590
DMU 9	323	25	5	75	1074
DMU 10	444	64	6	74	1072
DMU 11	323	25	5	25	350
DMU 12	444	64	6	104	1199

Table 2. Revenue allocation.

DMU$_j$	d_j^*	$\phi_k(v)$	Revenue Allocation
DMU 1	1.3215	0.2218	7.6752
DMU 2	1.0834	0.2236	7.7390
DMU 3	1.3387	0.2205	7.6318
DMU 4	1	0.2425	8.3921
DMU 5	1	0.2529	8.7521
DMU 6	1.0403	0.2262	7.8288
DMU 7	1.1622	0.2227	7.7053
DMU 8	1	0.2481	8.5847
DMU 9	1	0.3647	12.6186
DMU 10	1.2022	0.2226	7.7042
DMU 11	3	0.2083	7.2068
DMU 12	1	0.2358	8.1605

Table 3. The results of the Gini coefficient calculation.

Gini	Ours	Yang and Zhang [27]	Lin [19]	Baesly [14]	Li et al. [24]	Li et al. [18]	Li et al. [45]	Ding et al. [31]	Feng et al. [46]
G	0.2171	0.2362	0.3230	0.3520	0.352	0.2695	0.4227	0.5886	0.2795

Gini	Li et al. [34]	Lin et al. [47]	Li et al. [48]	Cook and Kress [9]	Cook and Zhu [17]	Du et al. [25]	Amirteimoori and Kordrostami [16]	Lin [49]	Hosseinzadeh et al. [50]
G	0.3904	0.2644	0.2325	0.2593	0.562	0.4753	0.2653	0.5371	0.512

6 | Conclusion

In this study, considering the cooperative relations between DMUs (players) the revenue allocation problem was assumed as a cooperative game. To achieve a fair allocation, based on DEA efficiency concept, we introduced a new characteristic profit function for cooperative game. Then using the well-known concept of Shapley value as a solution concept in cooperative game, the share of each DMU of revenue calculated uniquely. To evaluate the equitability of our allocation, we used the Gini coefficient, which is a widely used concept in economics and management sciences. In compare with other methods, the Gini coefficient of our allocation was lower; which shows that our allocation is more equitable. On the other hand, Shapley value for efficient DMUs was not equal; so we have a criterion to rank efficient

DMUs. This is the advantage of our characteristic function and shows the importance of employing a suitable characteristic function in cooperative games. Interested researchers can also present appropriate characteristic functions for cooperative games based on profit efficiency and inefficiency. Moreover, the approach adopted in this study can also be extended to cost games.

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