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A New Interval for Ranking Alternatives in Multi Attribute Decision Making Problems

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Abstract

In this paper, a hybrid method based on a linear programming model for solving Multi-Attribute Decision-Making (MADM) problems by combining two new methods, the COmplex PRoportional ASsessment (COPRAS) and the Multi-Objective Optimization Ratio Analysis (MOORA) and also using the concept of discrimination intensity functions are presented. Further interaction with the Decision Maker (DM) to determine the weights of the attributes and calculate the weights by solving a linear programming problem without determining the predetermined weight are two of the advantages of the new method. In the proposed method, for each alternative, attributes are weighted with optimism for that alternative, and then alternatives are ranked through efficiency intervals. The proposed method is implemented on a real-world problem derived from the subject literature and compared with other MADM methods. The difference in the final results is evident due to the consideration of more details in determining the rankings.

Keywords: Multi-attribute decision-making, Complex proportional assessment method, Multi-objective optimization ratio analysis method, Interval efficiency.

1 | Introduction

Decision-making involves the correct expression of goals or objectives, determining diverse and possible solutions, evaluating their feasibilities and consequences, as well as the results of their implementation. The quality of management is fundamentally a task related to the quality of decision-making, as the quality of plans and programming, effectiveness, the efficiency of strategies, and the quality of results attained from their application all depend on the quality of the decisions made by the manager. In most cases, decision-making is desirable and is to the satisfaction of the Decision-Maker (DM) when decision-making is based on surveying multiple criteria. These criteria could be quantitative or qualitative. In Multi-Criteria Decision-Making

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(MCDM) models, which researchers in recent decades have contemplated, several criteria are used for evaluation instead of employing one optimal measure. The MCDM models are divided into two categories: Multiple-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM). In general, multi-objective models are used for designing, and multi-attribute models are utilized for selecting an enhanced alternative. Soltanifar [1] investigated some of the popular MODM methods and, in each case, made suggestions for better use of DM opinions. The main difference between the MODM and MADM models is that the first is defined in the continuous decision space and the second in the discrete one. Due to the broader applications of MADM in real-world problems than MODM, MADM has been further developed by researchers over the past 60 years. From this point of view, MADM comprises methods and models divided into compensatory and non-compensatory. In the non-compensatory models such as the mastery method (method of domination), max-min, max-max, the Satisfactory inclusion method, specific satisfactory method, and the lexicographic method and the like, exchange between criteria is not allowed. In compensatory methods, exchange between criteria is permitted. This signifies that the score of another criterion could compensate for the weakness of a criterion. Models such as the ELimination Et Choice Translating REality (ELECTRE), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the Analytical Hierarchy Process (AHP), and the Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) and such examples are models of this classification. Many of these models have been developed by researchers with objectives like interval, fuzzy and probabilistic versions. Several of these methods can be found in [2]–[12]. Alinezhad & Khalili [13] have recently expressed the MADM methods in a manuscript containing 27 such methods. Some hybrid methods for solving MADM problems can also be seen in [14]–[22].

What increases the popularity of different MADM methods is the degree of DM satisfaction with the results. Satisfactory results provided by different MADM methods depend on several factors, such as determining the weights of attributes and interaction with DM. In many existing methods, the weights of the attributes, after interacting with the DM, are determined by predetermined weights. This may reduce the acceptance of ranking results by alternatives. In some cases, methods such as Shannon entropy, which does not interact much with DM, are used. This reduces satisfaction with the ranking results. The motivation for presenting the proposed method in this research is to present a method that, while considering more details in presenting the ranking results, determines the weights of the attributes through interaction with DM and considering the optimistic policy for each alternative. This interaction occurs both in determining attributes' priority and determining the discrimination intensity functions in solving a linear programming model with DM. Thus, the final results are satisfactory to DM and acceptable to the alternatives. In fact, in this research, the information related to the attributes is obtained only in the form of a list of priorities from several experts, which greatly increases the accuracy of the available information. Then, using a linear programming model and weight restrictions, the weights of the attributes are determined. Considering the various details in the proposed method leads to providing an efficiency interval for each alternative, which leads to the ranking of alternatives using interval rating rules.

Hence, this paper is organized as hereunder. Section 2 will include the research background and research literature. Section 3 encloses the steps of the proposed method. In Section 4, the proposed method will be implemented for a case study and be compared with the results of several other decision-making methods. Ultimately, the necessary conclusion will be rendered in Section 5.

2 | Research Background and Subject Literature

2.1 | The COPRAS Method

COmplex PRoportional ASsessment (COPRAS) is one of the compensatory methods introduced by [23] and was rapidly utilized by researchers in numerous decision-making problems [24]–[27]. This method is employed for solving multi-criteria healthcare waste treatment problems [28], [29], for the COVID-19 regional safety assessment [30], sustainable supplier selection [31], pharmacological therapy selection for type

2 diabetes [32], the evaluation and selection of hotels [33] selecting the green supplier chain [34] and such aspects.

Let us assume that we want to evaluate *n* homogenous alternatives $A_1, A_2, \ldots, A_n$ by considering m criteria in the form of $C_1, C_2, \ldots, C_m$. The overall configuration of the decision matrix will be as in *Matrix (1).* r_{ji} is the value assigned to A_j in C_i.

$$D = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nm} \end{bmatrix}. \tag{1}$$

The steps of the COPRAS method are as follows:

Step 1. The elements of the decision *Matrix (1)* will be normalized using *Eq. (2).*

$$\hat{r}_{ji} = \frac{r_{ji}}{\sum_{j=1}^{n} r_{ji}}, \ i = 1, 2, \ldots, m. \tag{2}$$

Step 2. Let us presume that the weights attained from the decision-making are $(w_1, w_2, \cdots, w_m)$, $\left(\sum_{i=1}^{m} w_i = 1\right)$. In this case, the elements of the normalized decision *Matrix (2)* are weighted using *Eq. (3).*

$$\overline{r}_{ji} = w_i \times \hat{r}_{ji}, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \tag{3}$$

Step 3. Calculate the profit criteria's total rate and each alternative's costs. If we presume that I^+ is a set of positive criteria signifying (profit) and I^- as a set of negative criteria for (costs), then these values will be computed by *Eqs. (4)* and *(5).*

$$S_j^+ = \sum_{i \in I^+} \overline{r}_{ji}, \ j = 1, 2, \ldots, n. \tag{4}$$

$$S_j^- = \sum_{i \in I^-} \overline{r}_{ji}, \ j = 1, 2, \ldots, n. \tag{5}$$

Step 4. The relative significance value of each alternative is calculated from *Eq. (6).*

$$Q_j = S_j^+ + \frac{\sum_{p=1}^{n} S_p^-}{S_j^- \sum_{p=1}^{n} \frac{1}{S_p^-}}, \ j = 1, 2, \ldots, n. \tag{6}$$

Step 5. Perform the ranking of alternatives based on their relative significance value; the higher the value stipulated from an alternative, the better the ranking of that alternative proves to be.

2.2 | MOORA Method

The Multi-objective Optimization Ratio Analysis (MOORA) method is one of the compensatory methods presented by Brauers and Zavadkas [35] and was used for numerous decision-making problems [36]–[39]. This method has been used for issues such as healthcare management [40], [41], optimization in various industries [42]–[44], engineering designs [45], selecting a laptop [46], and determining the recipients of the smart app [47]. There are two MOORA methods: the 'Ratio System' and the 'reference point approach.' We shall define and investigate both these methods in this paper.

Let us presume we wish to evaluate n homogenous alternatives $A_1, A_2, \ldots, A_n$ by considering m criteria in the form of $C_1, C_2, \ldots, C_m$. The general outline of the decision matrix will be in the form of *Matrix (1)*. The steps of the MOORA method are given below:

Step 1. Normalize the elements of the decision *Matrix (1)* by using *Eq. (7).*

$$\hat{r}_{ji} = \frac{r_{ji}}{\sqrt{\sum_{j=1}^{n}\left(r_{ji}\right)^2}}, \ i = 1, 2, \ldots, m. \tag{7}$$

Step 2. Assuming that the weights obtained from the decision-making criteria are $(w_1, w_2, \cdots, w_m)$, $\left(\sum_{i=1}^{m} w_i = 1\right)$. In this case, it balances the elements of the normalized decision *Matrix (7)* by utilizing *Eq. (8).*

$$\overline{r}_{ji} = w_i \times \hat{r}_{ji}, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \tag{8}$$

Step 3. Compute the total rate of profit and cost criteria for each alternative. If we presume that I^+ is a set of positive criteria signifying (profit) and I^- as a set of negative criteria for (costs), then these values will be calculated by *Eqs. (9)* and *(10).*

$$S_j^+ = \sum_{i \in I^+} \overline{r}_{ji}, \ j = 1, 2, \ldots, n. \tag{9}$$

$$S_j^- = \sum_{i \in I^-} \overline{r}_{ji}, \ j = 1, 2, \ldots, n. \tag{10}$$

Step 4. Compute the overall performance score of each alternative from *Eq. (11).*

$$y_j = S_j^+ - S_j^-, \ j = 1, 2, \ldots, n. \tag{11}$$

Step 5. Rank the alternatives according to their overall performance score. The higher the value specified from an alternative, the better the ranking of that alternative.

The abovementioned method is known as the 'ratio system'. The algorithm for the 'reference point approach' is as hereunder:

Step 1. Normalize the elements of the decision *Matrix (1)* by using *Eq. (7).*

Step 2. Presume that the weights obtained from the decision-making criteria are $(w_1, w_2, \cdots, w_m)$, $\left(\sum_{i=1}^{m} w_i = 1\right)$. In this case, it balances the elements of the normalized decision *Matrix* (7) by using *Eq. (8).*

Step 3. The ideal positive alternative (optimum alternative feasible for A^+) is the alternative that carries the optimum value in all the criteria. It is apparent that this alternative could be a virtual alternative and does not have a physical presence. If we suppose that I^+ is a set of positive criteria suggesting (profit) and I^- as a set of negative criteria for (costs), then A^+ will be calculated by *Eq. (12).* This alternative will be contemplated as a reference point.

$$A^+ = \left\{\max_j r_{ji}; i \in I^+\right\} \cup \left\{\min_j r_{ji}; i \in I^-\right\} = \left\{A_1^+, A_2^+, \ldots, A_m^+\right\}. \tag{12}$$

Step 4. Find the distance of each alternative from the reference point in the (Tchebycheff min–max metric) infinity norm concept from *Eq. (13).*

$$y_j = \max_{1 \le i \le m} \left| w_i A_i^+ - \overline{r}_{ji} \right|, \ j = 1, 2, \ldots, n. \tag{13}$$

Step 5. Rank the alternatives based on the distance of each alternative to the reference point. The lower the value for an alternative, the better the ranking of that alternative.

Amidst the MADM methods, many consider the distance to this point as a basis for evaluating alternatives by contemplating this point as a reference point. The most reputed method is the TOPSIS method, introduced by Hwang & Yoon [3]. This model defined the ideal positive alternatives (best possible alternative) and the negative ideal (worst possible alternative), and in assuming an unvarying increase or reduction in the desirability of each criterion, proximity to the positive ideal alternative (best possible alternative) and by evading the negative ideal alternative (worst possible alternative), which forms the basis for decision-making and alternatives are ranked by calculating the relative proximity, to the positive ideal solution. The concept of the ideal positive and negative alternatives has been utilized in the VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method, presented by [48], and has also been employed. Yet, another method is the Evaluation based on the Distance from Average Solution (EDAS) method. This was offered by Keshavarz Ghorabaee et al. [49]. In the mentioned method, the positive and negative distance with this alternative will provide the grounding for evaluating the alternative by considering an average solution alternative. In the numerical taxonomy method, a concept known as the development pattern is considered a reference point [50]. In several of these methods, the two-point distance concept is utilized. Numerous functions measure the distance between objects, with few specialities. Distance functions have several applications relative to data mining techniques, particularly in clustering. Possibly, the most popular overall concept of distance can be known as the Minkowski distance, which Hermann Minkowski offered. In this concept, the distance between two assumed points $(a_1, a_2, \cdots, a_n)$ and $(b_1, b_2, \cdots, b_n)$ are in the form of $\sqrt[p]{\sum_{i=1}^{n}(|a_i - b_i|)^p}$ which in lieu of $p = 1$ for taxicab distance and $p = 2$ as the Euclidean distance, as well as $p = \infty$ which is reputedly known as the Tchebycheff distance. In the MOORA and TOPSIS methods, the Tchebycheff and Euclidean concepts are used, respectively. However, in the EDAS method, the concept of a distinctive distance to the average alternative is defined and cannot be described in the form of the Minkowski distance concept. It is obvious that varied ways of decision-making can be achieved based on varied reference points, including diverse concepts of distance.

In Section 3, by utilizing the COPRAS method and the ratio system, we shall render an efficiency interval for each alternative when the DM overt weight cannot be secured, and the priority of the criteria is only determined. In Section 4, the mentioned method and the various reference points will be launched on a real-world problem, and we shall compare them.

3 | Presenting an Efficiency Interval for the Ranking of Alternatives

Determining the weight of criteria is an imperative step in MADM methods, directly impacting the final results. Though methods such as the Shannon entropy approach [51] are available to determine such weights, collaborating with the DM in determining the weights of criteria is certainly considered essential. By prioritizing the criteria from numerous experts, in an approach called KEmeny Median Indicator Ranks Accordance (KEMIRA), Krylovas et al. [52] selected the final priority and, based on which, determined the weights of the criteria were determined. This method quickly drew the attention of other researchers and was utilized in its alleviated versions in numerous decision-making problems[53]–[59]. In this section, we intend to employ the COPRAS and the ratio system for efficiency interval for each alternative, where the explicit weight for criteria has not been determined, and also, by taking advantage of the KEMIRA method, render this concept. Next, using the conceptual interval data, measures shall be taken to rank the alternatives. Let us assume that we want to evaluate n homogenous alternatives $A_1, A_2, \ldots, A_n$, which are under consideration for m criteria as $C_1, C_2, \ldots, C_m$. The general form of the decision matrix will be configured as in *Matrix (1)*. We will presume the elements of this matrix to be positive. The algorithm of the proposed method is as given hereunder:

Step 1. Normalize the decision *Matrix (1)* using *Eq. (7)*.

Step 2. In this step, with the help of experts initially, we will prioritize the criteria of the issue. Let us suppose that *K* experts are utilized in this relative. It is evident that in the new indexing, each criterion will have less priority than a criterion with a lower index. Subsequently, based on this prioritization, the priority matrix of each expert is determined in the form of *Eq. (14)*.

$$R_k = \left[a_{ii'}^k\right]_{m\times m}, a_{ii'}^k = \begin{cases} 0 & \text{if } C_i^k \prec C_{i'}^k \\ 1 & \text{if } C_i^k \succ C_{i'}^k \end{cases}, \ i, i' = 1, 2, \ldots, m, k = 1, 2, \ldots, K. \tag{14}$$

Step 3. Now, we locate the distance between prioritizing each expert with other experts through *Eq. (15).*

$$\rho_k = \sum_{k'=1}^{K}\sum_{i=1}^{m}\sum_{i'=1}^{m}\left|a_{ii'}^{k} - a_{ii'}^{k'}\right|, \ k = 1,2,\ldots K. \tag{15}$$

Step 4. In this step, expert prioritization with a minimal prioritization distance with other experts is determined as reference prioritization. In other words, if *Eq. (16)* is taken under consideration, the k^* expert will be the reference expert; the prioritization of this expert, namely, the median priority components for criteria, will be the basis for determining the weights of criteria.

$$\rho_{k^*} = \min_{1\le k\le K} \rho_k. \tag{16}$$

Step 5. By utilizing the entire matter elaborated upon in the COPRAS and MOORA methods, the efficiency interval for each alternative is formed as in interval *Eq. (17).*

$$y_j = S_j^+ - S_j^- \le E_j \le Q_j = S_j^+ + \frac{\sum_{p=1}^{n} S_p^-}{S_j^- \sum_{p=1}^{n} \frac{1}{S_p^-}} \ j = 1,2,\ldots,n. \tag{17}$$

In actual fact, the interval rendered in interval *Eq. (17)* is in the form of interval *Eq.(18),* in which the criterion weights are not determined, and the prioritization of criteria has only been determined in the fourth step following the opinions and viewpoints of reference experts is available.

$$\left[\sum_{i\in I^+} w_i \hat{r}_{ji} - \sum_{i\in I^-} w_i \hat{r}_{ji}, \sum_{i\in I^+} w_i \hat{r}_{ji} + \frac{\sum_{p=1}^{n}\sum_{i\in I^-} w_i \hat{r}_{pi}}{\sum_{i\in I^-} w_i \hat{r}_{ji} \sum_{p=1}^{n}\sum_{i\in I^-} \frac{1}{w_i \hat{r}_{pi}}}\right]. \tag{18}$$

In continuation, we intend to determine the weights of the criteria with an optimistic view and by resolving and determining a linear planning problem for each alternative. For this reason, we initially present *Lemma 1* and *Lemma 2*, including *Theorem 1,* to develop the terminating interval *Eq. (17).*

Lemma 1. If $(a_1, a_2, \cdots, a_n)$ and $(b_1, b_2, \cdots, b_n)$ are two series of integers belonging to genuine and positive numerical, then, $\frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i} \le \sum_{i=1}^{n} \frac{a_i}{b_i}$.

Proof: We utilize inductive reasoning to prove. The inductive basis for $k = 1$ is established in accordance with *Eq. (19).*

$$P(1): k = 1 \Rightarrow \frac{a_1}{b_1} \le \frac{a_1}{b_1}. \tag{19}$$

As an induction assumption, presume that *Eq. (20)* is maintained and established.

$$P(k): \frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} b_i} \le \sum_{i=1}^{k} \frac{a_i}{b_i}. \tag{20}$$

We must prove that *Eq. (21)* holds as an inductive imperative.

$$P(k+1): \frac{\sum_{i=1}^{k+1} a_i}{\sum_{i=1}^{k+1} b_i} \le \sum_{i=1}^{k+1} \frac{a_i}{b_i}. \tag{21}$$

With due attention to the fact that the values of b_{k+1} and $\sum_{i=1}^{k} b_i$ are positive, *Eq. (22)* holds

$$\frac{\sum_{i=1}^{k+1} a_i}{\sum_{i=1}^{k+1} b_i} = \frac{\sum_{i=1}^{k} a_i + a_{k+1}}{\sum_{i=1}^{k} b_i + b_{k+1}} = \frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} b_i + b_{k+1}} + \frac{a_{k+1}}{\sum_{i=1}^{k} b_i + b_{k+1}} \le \frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} b_i} + \frac{a_{k+1}}{b_{k+1}}. \quad \textbf{(22)}$$

Eq. (23) is obtained according to *Eq. (22)* and the presumption of induction.

$$\frac{\sum_{i=1}^{k+1} a_i}{\sum_{i=1}^{k+1} b_i} \le \frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} b_i} + \frac{a_{k+1}}{b_{k+1}} \le \sum_{i=1}^{k} \frac{a_i}{b_i} + \frac{a_{k+1}}{b_{k+1}} = \sum_{i=1}^{k+1} \frac{a_i}{b_i}. \quad \textbf{(23)}$$

Hence, the lemma rule was verified in this order.

Lemma 2. If $a_1, a_2, \cdots, a_n$ are a sequence of real and positive integers, then, $\left(\sum_{i=1}^{n} a_i\right)^2 \le n\sum_{i=1}^{n} (a_i)^2$.

Proof: We utilize inductive reasoning to prove. The inductive basis for $k = 1$ is established in accordance with *Eq. (24)*.

$$P(1): k = 1 \Rightarrow (a_1)^2 \le (a_1)^2. \quad \textbf{(24)}$$

In assuming induction, presume that *Eq. (25)* is established.

$$P(k): \left(\sum_{i=1}^{k} a_i\right)^2 \le k\sum_{i=1}^{k} (a_i)^2. \quad \textbf{(25)}$$

We must prove that *Eq. (26)* holds as an inductive rule.

$$P(k+1): \left(\sum_{i=1}^{k+1} a_i\right)^2 \le (k+1)\sum_{i=1}^{k+1} (a_i)^2. \quad \textbf{(26)}$$

According to the property of real numbers, *Eq. (27)* is established.

$$\left(\sum_{i=1}^{k+1} a_i\right)^2 = \left(\sum_{i=1}^{k} a_i\right)^2 + a_{k+1}^2 + 2a_{k+1}\left(\sum_{i=1}^{k} a_i\right). \quad \textbf{(27)}$$

This signifies that *Eq. (28)* will also be verified.

$$\left(\sum_{i=1}^{k+1} a_i\right)^2 = \left(\sum_{i=1}^{k} a_i\right)^2 + a_{k+1}^2 + 2a_{k+1}a_1 + 2a_{k+1}a_2 + \cdots + 2a_{k+1}a_k. \quad \textbf{(28)}$$

We know that the equation is always established for the two real integers, A and B. Thus, on the fundaments of this equation and concerning assuming induction, *Eq. (29)* will be ascertained

$$\left(\sum_{i=1}^{k+1} a_i\right)^2 \le k\sum_{i=1}^{k} (a_i)^2 + a_{k+1}^2 + \left(a_{k+1}^2 + a_1^2\right) + \left(a_{k+1}^2 + a_2^2\right) + \cdots + \left(a_{k+1}^2 + a_k^2\right). \quad \textbf{(29)}$$

Eq. (29) will result in *Eq. (30)*

$$\left(\sum_{i=1}^{k+1} a_i\right)^2 \le (k+1)\sum_{i=1}^{k} (a_i)^2 + (k+1)a_{k+1}^2. \quad \textbf{(30)}$$

Eq. (30) is equivalent to the induction verdict; thus, the ruling was proved.

With due attention to *Lemma 1*, *Eq. (31)* is established

$$\frac{\sum_{p=1}^{n}\sum_{i\in I^-} w_i\hat{r}_{pi}}{\sum_{i\in I^-} w_i\hat{r}_{ji}\sum_{p=1}^{n}\sum_{i\in I^-}\frac{1}{w_i\hat{r}_{pi}}}=\frac{\sum_{p=1}^{n} S_p^-}{S_j^-\sum_{p=1}^{n}\frac{1}{S_p^-}}\le\frac{1}{S_j^-}\left(\sum_{p=1}^{n}\left(S_p^-\right)^2\right)=\frac{1}{\sum_{i\in I^-} w_i\hat{r}_{ji}}\left(\sum_{p=1}^{n}\left(\sum_{i\in I^-} w_i\hat{r}_{pi}\right)^2\right). \tag{31}$$

Similarly, based on *Lemma 2*, *Eq. (32)* is ascertained. *N* signifies the number of negative criteria (costs) for the problem (in fact $N=\sum_{i\in I^-}1$).

$$\frac{\sum_{p=1}^{n}\sum_{i\in I^-} w_i\hat{r}_{pi}}{\sum_{i\in I^-} w_i\hat{r}_{ji}\sum_{p=1}^{n}\sum_{i\in I^-}\frac{1}{w_i\hat{r}_{pi}}}\le\frac{1}{\sum_{i\in I^-} w_i\hat{r}_{ji}}\left(\sum_{p=1}^{n}\left(\sum_{i\in I^-} w_i\hat{r}_{pi}\right)^2\right)\le\frac{1}{\sum_{i\in I^-} w_i\hat{r}_{ji}}\left(\sum_{p=1}^{n}\sum_{i\in I^-} N\left(w_i\right)^2\left(\hat{r}_{pi}\right)^2\right). \tag{32}$$

Eq. (33) can be rendered by iteratively taking *Lemma 1* under consideration

$$\frac{\sum_{p=1}^{n}\sum_{i\in I^-} w_i\hat{r}_{pi}}{\sum_{i\in I^-} w_i\hat{r}_{ji}\sum_{p=1}^{n}\sum_{i\in I^-}\frac{1}{w_i\hat{r}_{pi}}}\le\frac{1}{\sum_{i\in I^-} w_i\hat{r}_{ji}}\left(\sum_{p=1}^{n}\sum_{i\in I^-} N\left(w_i\right)^2\left(\hat{r}_{pi}\right)^2\right)\le\sum_{i\in I^-} w_i\sum_{p=1}^{n}\frac{N\left(\hat{r}_{pi}\right)^2}{\hat{r}_{ji}}. \tag{33}$$

Theorem 1. The efficiency interval *Eq. (17)* is capable of being developed efficiency interval *Eq. (34)*

$$\left[\sum_{i\in I^+} w_i\hat{r}_{ji}-\sum_{i\in I^-} w_i\hat{r}_{ji},\sum_{i\in I^+} w_i\hat{r}_{ji}+\sum_{i\in I^-} w_i\sum_{p=1}^{n}\frac{N\left(\hat{r}_{pi}\right)^2}{\hat{r}_{ji}}\right]. \tag{34}$$

Proof: Given that the *Eqs. (31)-(33)* are fixed bylaws.

Remark 1. This study presented two versions of the MOORA method: "ratio system" and "reference point approach". In the second version, the final score on which the alternatives are ranked is non-negative. However, there is no such guarantee for the score provided by the first version, and this score may be negative for some alternatives. This is the difference between the two versions presented. To provide an interval, the beginning of the interval needs to be smaller than the end. Using the first version of the MOORA method, this was proved in *Theorem 1*. However, there is no guarantee that the second version of the MOORA method will create efficiency intervals for alternatives. Therefore, in this research, the first version of the method is necessarily used.

In the continuation of this step, and in contemplating that the weights of the criteria are not determined, it is only through the second to the fourth steps that the priority of criteria is attained from the reference expert. Primarily, an optimistic linear programming model is presented to determine the weight of each criterion. In the subsequent steps, the efficiency interval which has taken shape for each alternative provides the basis for ranking that specific alternative. The criterion weights are achieved for the assumed alternative by solving the multi-objective *Eq. (35)*.

$$\max \sum_{i \in I^+} (w)_i^{k^*} (\hat{r})_{ji}^{k^*},$$
$$\min \sum_{i \in I^-} (w)_i^{k^*} (\hat{r})_{ji}^{k^*},$$
$$\max \sum_{i \in I^-} (w)_i^{k^*} \sum_{p=1}^{n} \frac{N\left((\hat{r})_{pi}^{k^*}\right)^2}{(\hat{r})_{ji}^{k^*}}, \tag{35}$$
s.t.
$$\sum_{i=1}^{m} (w)_i^{k^*} = 1,$$
$$(w)_i^{k^*} - (w)_{i+1}^{k^*} \geq d^{k^*}(i, \varepsilon), \quad i = 1, 2, \ldots, m-1,$$
$$(w)_m^{k^*} \geq d^{k^*}(m, \varepsilon).$$

In *Eq. (35)*, $(w)_i^{(k^*)}, i = 1, 2, ..., m$ illustrates the re-indexing of weights (for the criteria) according to the viewpoints of the k^* expert. This model permits the assumed alternative j, $(j = 1, 2, ..., n)$, to select from amongst the normalized weights, the criteria, which have been rendered in the prioritization, provided by the reference expert and are factual for the weight vector, which evaluates that alternative in its optimum optimistic condition. $d^{k^*}(., \varepsilon)$ is a monotone increasing function and is non-negative, is called the discrimination intensity function. This shows the amount of difference between the weights of criteria, and after collaborating or interacting with expert k^*, it is determined. In the case, which equivalent weights are acceptable for the criteria, $d^{k^*}(., \varepsilon)$ can have a value equal to zero.

Thereby, in *Eq. (35)*, not only is it possible to distinguish between varied criteria based on the viewpoint of the reference expert, but it is also possible to determine this amount of difference in numerous issues, along with the opinion of that expert. Cook & Kress [60] were initially inspired by an article by Thompson et al. [61] and employed the conversion of weight constraints to a linear form, from these types of constraints to control weights to create discrimination in polling stations. Likewise, Noguchi et al. [62], including Llamazares & Pena [63], have also executed valuable surveys on weight control constraints of this nature, and each of the studies proposed in this survey can be utilized to modify *Eq. (35)*. Other applications of these weight control constraints can be found in articles presented by [64]–[71] and such papers can be mentioned.

Eq. (35) is a tri-linear objective model capable of being solved by methods, for example, the epsilon-constraint method, weighting method, absolute priority method, the Goal Programming (GP) method, and like them. Each of these methods solves the problem by determining the importance of objective functions or setting goals for them. The importance of objective functions in *Eq. (35),* in fact, determines the importance of COPRAS and MOORA methods in determining the weights of attributes. Since these two methods are equally important in determining the weights of the attributes, the weighting method with equal weights has been used. By employing the weighting method for the objective functions and placing identical weights for them, the linear programming *Eq. (36)* will be achieved.

After solving *Eq. (36)* and attaining the optimum weights, an efficiency interval such as interval *Eq. (34)* shall be accomplished for each alternative.

$$\max \frac{1}{3}\left(\sum_{i\in I^{+}}(w)_{i}^{k^{*}}(\hat{r})_{ji}^{k^{*}}\right)-\frac{1}{3}\left(\sum_{i\in I^{-}}(w)_{i}^{k^{*}}(\hat{r})_{ji}^{k^{*}}\right)+\frac{1}{3}\left(\sum_{i\in I^{-}}(w)_{i}^{k^{*}}\sum_{p=1}^{n}\frac{N\left(\left((\hat{r})_{pi}^{k^{*}}\right)^{2}\right)}{(\hat{r})_{ji}^{k^{*}}}\right),$$

s.t.

$$\sum_{i=1}^{m}(w)_{i}^{k^{*}}=1,$$
$$(w)_{i}^{k^{*}}-(w)_{i+1}^{k^{*}}\geq d^{k^{*}}(i,\varepsilon),\ i=1,2,\ldots,m-1, \tag{36}$$
$$(w)_{m}^{k^{*}}\geq d^{k^{*}}(m,\varepsilon).$$

Step 6. The alternatives' degrees of priority are calculated according to their efficiency interval obtained, in relevance to each other. Let us assume that $[a_1, a_2]$ and $[b_1, b_2]$ are efficiency intervals for alternatives A and B, respectively. Thereby, the extent of priority of alternative A in relative to alternative B will be computed using *Eq. (37)* [72]

$$P(A>B)=\frac{\max\{0,a_2-b_1\}-\max\{0,a_1-b_2\}}{(a_2-a_1)+(b_2-b_1)}. \tag{37}$$

It is apparent that the priority of alternatives with respect to each other is numerical and within a range of $[0,1]$ and that $P(A>B)+P(B>A)=1$. It is certain that an alternative surpasses and is superior to another when the priority of the first alternative over the second alternative is more than 0.5. Hence, the score of each alternative can be considered equivalent to the total degree of priority of that alternative compared to the other alternatives.

Step 7. The prioritization of the alternatives will be accomplished according to their scores in relevance to each other. In the previous step, the alternatives shall be ranked according to the score secured for each alternative. It is evident that the higher the score of an alternative, the better its ranking.

So far, the methodology of the proposed method has been discussed in detail, and the necessary mathematical proofs have been provided at each step. In the following, the step-by-step algorithm of the proposed method is summarized. After forming the decision matrix, the following steps are followed to rank the alternatives:

Step 1. Normalize the decision matrix (*Eq. 7*).

Step 2. Determine the experts' prioritization matrices (*Eq. 14*).

Step 3. Determine the prioritization distance of each expert to other experts (*Eq. 15*).

Step 4. Determine the reference expert (*Eq. 16*).

Step 5. Determine the efficiency interval of each alternative (*Eq. 34*) by solving a linear programming problem (*Eq. 36*).

Step 6. Determine the score of each alternative (*Eq. 37*).

Step 7. Rank the alternatives.

The flowchart of the proposed method is presented in *Fig. 1*.

Note 1. The aspect crucial in the abovementioned method is the utilization of the term "efficiency interval" for the final interval gained for each alternative. Usually, the value of "efficiency" is in the range $[0,1]$. It is clear, though, that there is no assurance that the interval secured from this method complies with the conditions for every alternative. Hence, we must note that the range attained from this method is for the scoring and ranking of alternatives and is not essentially a subset of the range $[0,1]$.

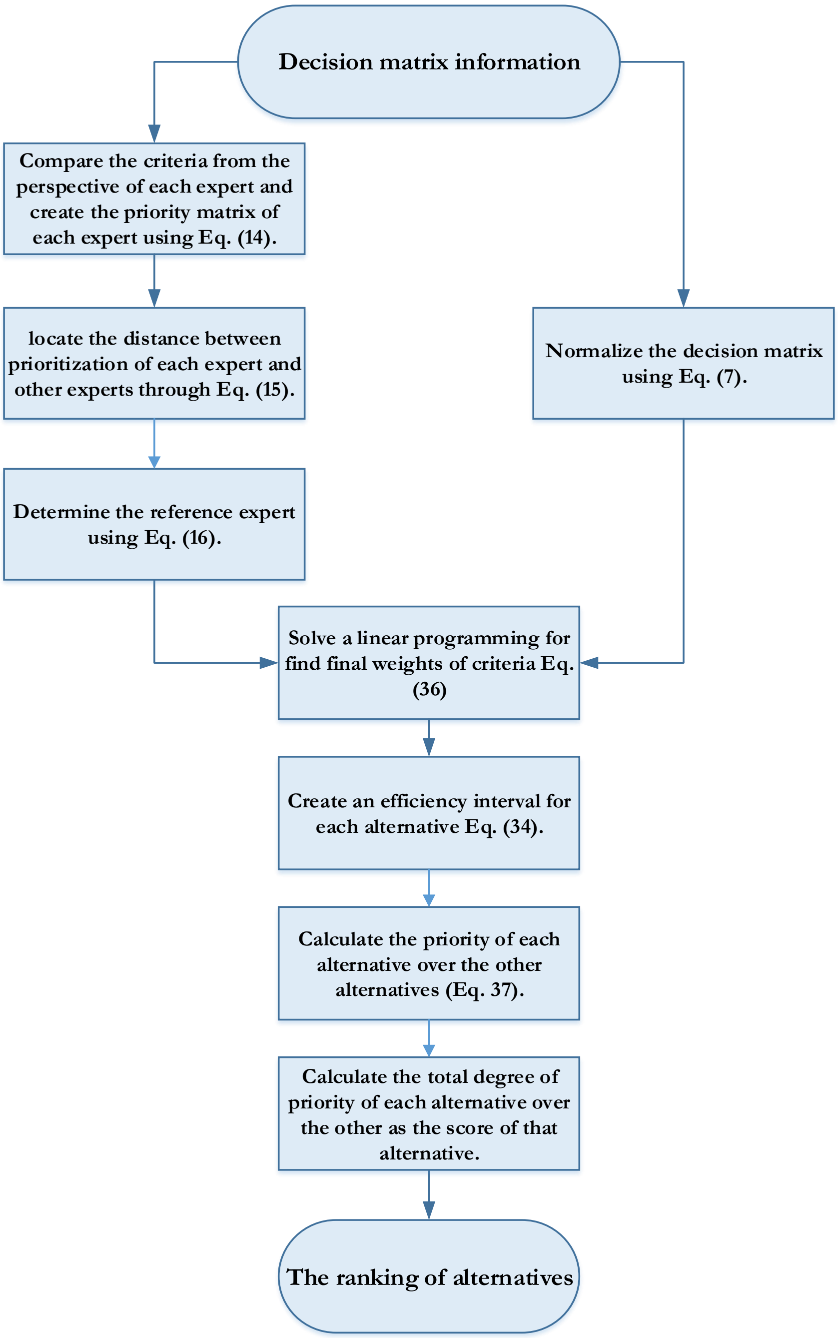

Fig. 1. The flowchart of the proposed method.

4 | Case-Study

Alinezhad and Khalili [13], in a book consisting of 27 MADM methods, surveyed the EDAS method, explaining this method by mentioning a problem. In continuation, we shall review their case study. In addition to implementing the proposed method for this problem, we shall draw a comparison between the results of various other MADM methods.

A road construction company is considering an excavator from the models A_1, A_2, and A_3, as suggested by experts. Experts provided attributes such as annual maintenance cost (C_1), price (C_2), working weight (C_3), fuel consumption rate (C_4), the complexity level of working with the excavator by the operator (C_5), and bucket capacity (C_6). The decision matrix is shown in *Table 1*. The purpose is to choose the best excavator model.

Table 1. The decision matrix.

	Cost (C1)	Cost (C2)	Benefit (C3)	Cost (C4)	Cost (C5)	Benefit (C6)
A_1	0.710	4.100	0.180	0.720	0.990	0.250
A_2	1.330	5.900	0.740	0.310	0.420	0.830
A_3	1.450	4.900	0.270	0.650	0.420	0.440

If the weights of the criteria rendered by the DM are $(w_1, w_2, \cdots, w_6) = (0.171, 0.185, 0.177, 0.225, 0.157, 0.085)$, the results of the implementation methods, such as the EDAS, TOPSIS, VIKOR, MOORA, and COPRAS approaches relative to this issue, can be observed in *Table 2*.

Table 2. The TOPSIS, VIKOR, EDAS, MOORA and COPRAS scores and the rank of each alternative.

Methods		A1	A2	A3
TOPSIS	Score	0.286	0.766	0.393
	Rank	3	1	2
VIKOR	Score	0.998	0.000	0.520
	Rank	3	1	2
EDAS	Score	0.139	0.901	0.330
	Rank	3	1	2
MOORA (ratio system)	Score	-0.378	-0.125	-0.326
	Rank	3	1	2
MOORA (reference point approach)	Score	0.123	0.051	0.103
	Rank	3	1	2
COPRAS	Score	0.265	0.434	0.300
	Rank	3	1	2

However, suppose the weights of the criteria are not distinctly at hand, and only the priority of the criteria is specified hereunder. In that case, we intend to employ the proposed method.

First expert: $C_6 \prec C_5 \prec C_1 \prec C_3 \prec C_2 \prec C_4$.

Second expert: $C_6 \prec C_5 \prec C_3 \prec C_1 \prec C_2 \prec C_4$.

Third expert: $C_6 \prec C_5 \prec C_1 \prec C_3 \prec C_4 \prec C_2$.

According to the first step of the proposed method, the normalized matrix of this problem is in the form of *Table 3*.

Table 3. The normalized decision matrix.

	Cost (C_1)	Cost (C_2)	Benefit (C_3)	Cost (C_4)	Cost (C_5)	Benefit (C_6)
A1	0.339	0.471	0.223	0.707	0.858	0.257
A2	0.636	0.678	0.916	0.304	0.364	0.854
A3	0.693	0.563	0.334	0.638	0.364	0.453

Similarly, the priority matrix of experts, based on the second step of the proposed method, is given below:

$$R_1=\begin{bmatrix}000011\\101011\\100011\\111011\\000001\\000000\end{bmatrix}. \quad (34)$$

$$R_2=\begin{bmatrix}001011\\101011\\000011\\111011\\000001\\000000\end{bmatrix}. \quad (35)$$

$$R_3=\begin{bmatrix}000011\\101111\\100011\\101011\\000001\\000000\end{bmatrix}. \quad (36)$$

The distance between the prioritization of each expert and other experts based on the third step of the proposed method is as $\rho_1 = 4,\ \rho_2 = 6,\ \rho_3 = 6$ .As $\rho_1 = \min_{1\le k\le 3} \rho_k$, thus, the opinion of the first expert is utilized as a reference expert to offer linear programming *Eqs. (41)-(43)*, which are based on the fifth step of the proposed method.

$$\begin{aligned}
&\max \frac{1}{3}\left(0.22277(w)_3 + 0.25717(w)_6\right)\\
&-\frac{1}{3}\left(0.33943(w)_4 + 0.47145(w)_2 + 0.70704(w)_1 + 0.85751(w)_5\right)\\
&+\frac{1}{3}\left(11.78462(w)_4 + 8.48444(w)_2 + 5.65740(w)_1 + 15.55378(w)_5\right),\\
&\text{s.t.}\\
&\quad (w)_1 + (w)_2 + (w)_3 + (w)_4 + (w)_5 + (w)_6 = 1,\\
&\quad (w)_1 \ge (w)_2 + \varepsilon,\\
&\quad (w)_2 \ge (w)_3 + \varepsilon,\\
&\quad (w)_3 \ge (w)_4 + \varepsilon,\\
&\quad (w)_4 \ge (w)_5 + \varepsilon,\\
&\quad (w)_5 \ge (w)_6 + \varepsilon,\\
&\quad (w)_6 \ge \varepsilon.
\end{aligned} \quad (37)$$

$$
\begin{aligned}
&\max \frac{1}{3}\left(0.91582(w)_3+0.85381(w)_6\right)\\
&-\frac{1}{3}\left(0.63583(w)_4+0.67843(w)_2+0.30442(w)_1+0.36379(w)_5\right)\\
&+\frac{1}{3}\left(6.29104(w)_4+5.89597(w)_2+13.13977(w)_1+4.68487(w)_5\right),\\
&\text{s.t.}\\
&\quad (w)_1+(w)_2+(w)_3+(w)_4+(w)_5+(w)_6=1,\\
&\quad (w)_1\ge (w)_2+\varepsilon,\\
&\quad (w)_2\ge (w)_3+\varepsilon,\\
&\quad (w)_3\ge (w)_4+\varepsilon,\\
&\quad (w)_4\ge (w)_5+\varepsilon,\\
&\quad (w)_5\ge (w)_6+\varepsilon,\\
&\quad (w)_6\ge \varepsilon.
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&\max \frac{1}{3}\left(0.33415(w)_3+0.45262(w)_6\right)\\
&-\frac{1}{3}\left(0.69319(w)_4+0.56344(w)_2+0.63830(w)_1+0.36379(w)_5\right)\\
&+\frac{1}{3}\left(5.77040(w)_4+7.09923(w)_2+6.26666(w)_1+8.83737(w)_5\right),\\
&\text{s.t.}\\
&\quad (w)_1+(w)_2+(w)_3+(w)_4+(w)_5+(w)_6=1,\\
&\quad (w)_1\ge (w)_2+\varepsilon,\\
&\quad (w)_2\ge (w)_3+\varepsilon,\\
&\quad (w)_3\ge (w)_4+\varepsilon,\\
&\quad (w)_4\ge (w)_5+\varepsilon,\\
&\quad (w)_5\ge (w)_6+\varepsilon,\\
&\quad (w)_6\ge \varepsilon.
\end{aligned}
\tag{39}
$$

In *Eqs. (41)-(43)*, $(w)_1,(w)_2,(w)_3,(w)_4,(w)_5,(w)_6$ are, respectively, the weights of criteria C_4,C_2,C_3,C_1,C_5,C_6. In order to simplify the task, the discrimination intensity function ε has been selected. However, this function can be extracted in interaction with experts and DMs. *Eqs. (41)-(43)* will be utilized to extract the weights for criteria relevant to the first, second and third alternatives, respectively. After solving these models for the various values ε, the criteria weights are illustrated in *Table 4*, whereas the efficiency interval for each alternative is given in *Table 5*. Also, a summary of the ranking of alternatives can be seen in *Fig. 2*.

Table 4. Weights of criteria for the varied alternatives in lieu of different Ɛ values.

Ɛ	Alternatives	Cost (C1)	Cost (C2)	Benefit (C3)	Cost (C4)	Cost (C5)	Benefit (C6)
0.00001	A1	0.199988	0.200008	0.199998	0.200018	0.199978	0.000010
	A2	0.000030	0.000050	0.000040	0.999850	0.000020	0.000010
	A3	0.000030	0.499945	0.000040	0.499955	0.000020	0.000010
0.00010	A1	0.199880	0.200080	0.199980	0.200180	0.199780	0.000100
	A2	0.000300	0.000500	0.000400	0.998500	0.000200	0.000100
	A3	0.000300	0.499450	0.000400	0.499550	0.000200	0.000100
0.00100	A1	0.198800	0.200800	0.199800	0.201800	0.197800	0.001000
	A2	0.003000	0.005000	0.004000	0.985000	0.002000	0.001000
	A3	0.003000	0.494500	0.004000	0.495500	0.002000	0.001000
0.01000	A1	0.188000	0.208000	0.198000	0.218000	0.178000	0.010000
	A2	0.030000	0.050000	0.040000	0.850000	0.020000	0.010000
	A3	0.030000	0.445000	0.040000	0.455000	0.020000	0.010000
0.02000	A1	0.176000	0.216000	0.196000	0.236000	0.156000	0.020000
	A2	0.060000	0.100000	0.080000	0.700000	0.040000	0.020000
	A3	0.060000	0.390000	0.080000	0.410000	0.040000	0.020000
0.04500	A1	0.146000	0.236000	0.191000	0.281000	0.101000	0.045000
	A2	0.135000	0.225000	0.180000	0.325000	0.090000	0.045000
	A3	0.135000	0.252500	0.180000	0.297500	0.090000	0.045000

Table 5. The efficiency interval, score and final ranking for various alternatives in lieu of different Ɛ.

Ɛ	Alternatives	Interval	Score	Rank
0.00001	A1	[-0.431,8.77]	0.944	2
	A2	[-0.304,13.443]	1.254	1
	A3	[-0.601,7.283]	0.802	3
0.00010	A1	[-0.430,8.768]	0.944	2
	A2	[-0.304,13.430]	1.254	1
	A3	[-0.600,7.280]	0.802	3
0.00100	A1	[-0.430,8.739]	0.946	2
	A2	[-0.301,13.306]	1.251	1
	A3	[-0.596,7.248]	0.803	3
0.01000	A1	[-0.422,8.451]	0.962	2
	A2	[-0.274,12.065]	1.225	1
	A3	[-0.551,6.930]	0.813	3
0.02000	A1	[-0.413,8.130]	0.983	2
	A2	[-0.243,10.686]	1.191	1
	A3	[-0.502,6.575]	0.826	3
0.04500	A1	[-0.392,7.330]	1.055	2
	A2	[-0.167,7.238]	1.070	1
	A3	[-0.378,5.690]	0.876	3

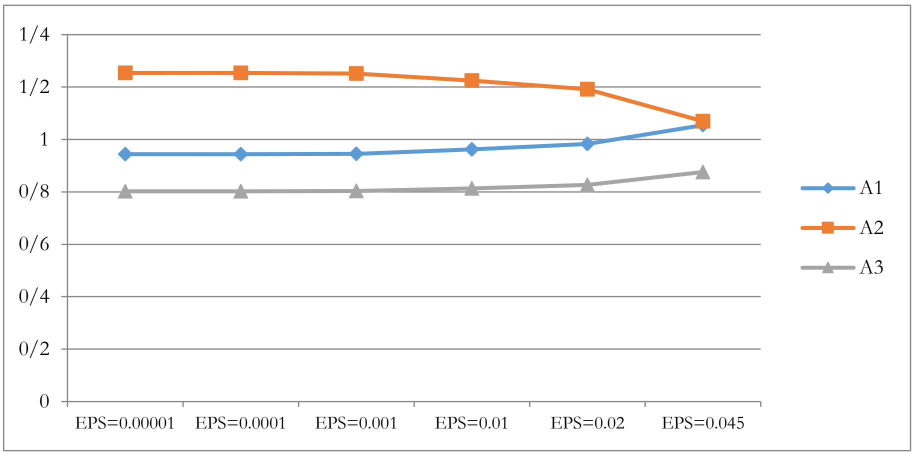

Fig. 2. A summary of the ranking of alternatives.

In order to show that the results of *Table 5* take precedence over the results of *Table 2*, we must show that the proposed model results are useful in application. First, *Table 5* results prioritize obtaining DM satisfaction because it is obtained by interacting more with them. To achieve the results of *Table 5*, DM opinions are considered in two ways. First, in determining the priority of the criteria and the reference expert, and second, in determining the discrimination intensity functions. However, in the results of *Table 2*, only DM may have interacted in determining the criteria weights. Second, the results in *Table 5* are more acceptable for alternatives than in *Table 2*. As can be seen, in all the methods used in *Table 2*, the weight of the criteria is $(w_1, w_2, \cdots, w_6) = (0.171, 0.185, 0.177, 0.225, 0.157, 0.085)$, which is either provided by the DM or determined by methods such as Shannon entropy because these weights have a significant impact on the final score of the alternatives, they can cause rejection by the alternatives that have been ranked worse, because they may achieve a better ranking by maintaining the priority of criteria and changing their weights. However, in the results of *Table 5,* there is no predetermined weight. The criteria are merely prioritized from a group decision-making process. Each alternative can calculate the weight of the criteria optimistically by solving the *Eq. (36)*; therefore, the final results will be more acceptable for that alternative. In addition, each MADM method considers details in the alternatives ranking process. In the results of *Table 5,* the details of the two methods, COPRAS and MOORA, are included, which makes the proposed method a more powerful tool for decision support.

Although the predetermined weights of the criteria in this case study are consistent with the opinion of the reference expert, the results of *Table 5* differ from all the methods used in *Table 2*. The reason for this difference is the use of more details in the proposed method and its flexibility in choosing the weight of the criteria in the prioritization framework provided by the reference expert. Hence, it is obvious that no objections will be accepted for the selected weights because the alternatives have chosen these weights in the most optimistic conditions, and this is one of the main advantages of the proposed method. Similarly, the proposed method has advantages over other methods in that by choosing the discrimination intensity functions after cooperating with the DM; it can use the opinions of DMs more intelligently in decision-making.

5 | Conclusion

What makes the MADM method popular is the acceptance of its results by DM and alternatives. More interaction with DM and more flexibility of the method to satisfy the alternatives can lead to more acceptance of the results of a method by DM and alternatives. This research presents a new hybrid method for ranking alternatives in a MADM problem. In the proposed method, interaction with DM in determining the priority of criteria and the discrimination intensity functions has caused their satisfaction with the final results. This method also allows each alternative to optimistically determine the weights of the criteria for solving a linear programming problem. This will maximize the acceptance of the results by the alternatives. Another advantage of the proposed method is the use of details provided by COPRAS and MOORA methods in determining the final ranking of alternatives, which makes the method a more powerful tool for decision support. The proposed method is implemented on a real-world problem, and its results are compared with the results of 6 other MADM methods. In this paper, two new MADM methods are used to determine performance intervals for alternatives. As a suggestion for future research, this idea could be utilized for other MADM methods. Of course, the need to form an interval will limit the choice of different MADM methods. It is also possible to use the logic of uncertainty to achieve more reasoned results and to present stochastic and fuzzy versions of the proposed method.

Conflicts of Interest

All co-authors have seen and agree with the manuscript's contents, and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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