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Optimizing the Routing Problem in the Vehicle Carrying Cash Considering the Route Risk (Case Study of Bank Shahr)

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Abstract

The process of transferring money from the treasury to the branches and returning it at specific and limited periods is one of the applications of the Vehicle Routing Problem (VRP). Many parameters affect it but choosing the right route is the most key parameter so that the money delivery process is carried out in a specific period with the least risk. In the present paper, new relationships are defined in the form of three concepts in order to minimize route risk. These concepts are: (i)the vehicle does not travel long routes in the first three movements. (ii)A branch is not served at the same hours on two consecutive days. (iii)An arc should not be repeated on two consecutive days. The proposed model with real information received from Bank Shahr has been performed for all branches in Tehran. Because the vehicle routing problem is an NP-Hard problem, a genetic algorithm was used to solve the problem .Different problems in various production dimensions were solved with GAMS and MATLAB software to show the algorithm solution quality. The results show that the difference between the genetic algorithm and the optimal solution is an average of 1.09% and a maximum of 1.75%.

Keywords: Genetic algorithm, Route risk, Vehicle routing, The problem of carrying cash.

1 Introduction

Due to modern and safe technologies, new banking tools such as mobile phones and electronic payments are increasing, reducing armed attacks and robberies. However, despite the economic crises, the amount of cash and physical money in the world economic cycle increases annually and plays a vital role compared to previous decades. For example, there were more than 1,979 million pounds of physical money and coins only in the UK economic cycle in 2019, a growth of about 28% compared to 2014 [1]. According to published statistics

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of the European Central Bank (ECB), the amount of physical money and coins in the European economic cycle was about 22,468 million euros in 2019, a growth of about 11% compared to 2016 [2]. The US Federal Reserve Bank has recorded $ 43 billion in the US economic cycle in 2018, a growth of about 19% compared to 2014. Accordingly, the amount of non-paper and physical money transactions in Europe is 7% of the total liquidity in the European cycle. Similar behaviour is observed in the US. On average, American consumers use cash in more than 40% of their transactions [3]. Therefore, routing problem in the vehicle carrying cash that means the physical transfer of banknotes, coins, and valuables things from one place to another place, is of great importance. Banks and companies carrying cash are always exposed to real risks such as theft and armed attack due to the essence of the portable commodity. Therefore, we have proposed a model in this paper that performs the process of money delivery with the least risk and in a specific period.

2 A research literature review

One of the basic and classic vehicle routing problems is the Travelling Salesman Problem (TSP), formulated in the 18th century by Hamilton and Kirkman. A salesman or distributor has to travel to several cities and service them in this state. The Vehicles Routing Problem (VRP) as a problem located in the heart of distribution management was represented by Danzig and Ramser in 1959. Several vehicles move simultaneously and return to the warehouse after meeting the demand nodes .On condition that, first, each demand node is met by only one of these vehicles, and second, each vehicle is not loaded more than its capacity along the route [4,5]. Of course, the goal of today models is to base the complexities of the real world such as transportation cost, the dependence of travel times on the route traffic volume, the Pickup and Delivery Problem with Time Windows (PDPTW), input information such as the demand amount that changes dynamically over time. All these features are considered for designing a suitable routing strategy [6-9]. Several investigations are done in the field of risk-based vehicle routing models .We mention some of the studies in continue. Soriano et al. represented the routing problem in the vehicle carrying cash considering the customer meeting times variety as the polygraph. Large Neighbourhood Search (LNS), including a linear penalty function for evaluating routes, optimal local searches, and an adaptive destruction rate, has been used in the present study in order to balance the shortcut and security [10]. An evolutionary multi-objective model based on the new Game Theory was proposed by Ghanbarpour and Zandieh in order to increase money transfer security and reduce transportation costs. They created the two-objective vehicle routing problem with a time window to minimize the money transfer risk and distance traveled by the vehicle .The possibility of ambush thieves is calculated using the game theory method to estimate the theft risk better.

Also, the possibility of a successful theft is estimated with a Multiple-Criteria Decision Analysis (MCDA) [11]. Hoggom et al. represented the time inequality to reach money centers through multiple time windows. In this paper, the proposed algorithm and four different penalty methods were solved by Tabu Search (TS) [4]. Locartalarico et al. proposed the risk as a ratio of portable money amount and the distance between demand points. Also, to reduce cost and increase security, they solved the routing problem in the security section of physical money transportation through the metaheuristic techniques and Local Search (LS) [12-15]. Jan et al. expressed different views on increasing the level of unpredictability. To solve the problem, they used a route-time network technique for routing carry money to reduce the cost and increase the money transfer security [16].

2.1 Research gaps and innovations

According to the literature review, the previous models have mainly focused on reducing risk and increasing security, whereas the solutions proposed in this paper for reducing the risk, formulated as mathematical models, are not observed in previous research .As an innovation, the present paper consists of new concepts and relationships to promote safety and reduce service risk to branches. In other words, three concepts are used in this paper for reducing route risk that is not mentioned in previous studies.

(i) The vehicle does not travel long routes and arcs in the first three movements when carries more cash.
(ii) A branch is not served at the same and similar hours on two consecutive days.
(iii) As far as possible, an arc is not repeated on two consecutive days.

Items (ii) and (iii) are considered to vary service time and sequence for various branches on different days. This technique greatly reduces the possibility of setting a consistent pattern to serve branches and increases service security.

3 Defining the problem

In most country banks, the planning and routing problem for vehicle carrying cash are carried out completely empirical by the treasury officials, and it is usually without any change, consistent, and predictable. Banks and cash transportation companies are always exposed to real risks such as theft and armed attack due to the essence of the portable commodity. Because most thieves monitor the money transfer route and time for a while to plan and set their theft ,these attacks may cause serious harm to branch staff, customers, and security personnel. Therefore, in the present paper, we have proposed a model that performs the money delivery process with the least risk and in a specific period. Accordingly, the assumptions of the problem are as follows:

- The problem model is single-treasury. It is assumed that the vehicle carrying cash should be located in the treasury place at the start of the operation. They should service all demand routes in a specific period and return to the treasury at the end.
- For each point "i", a specific and defined amount of physical money demand, which is predetermined, is considered. It is serviced by a vehicle.
- The physical money volume should not exceed the roof of the vehicle.
- The vehicles carrying cash should be all homogeneous and 15 vehicles. The capacity of each vehicle may indicate the maximum money amount or valuable commodity that the vehicle is allowed to carry in the form of a currency (15 billion Rials) and proportion to the vehicle characteristics. Due to the small volume of physical money, the main constraint is the risk along the route, not the amount of portable money.
- The stopping time of the vehicle carrying cash at the demand points for servicing is 25 min, and the maximum specific time for the first three movements of the vehicle should not be more than 30 min.
- The minimum time interval which a node meets on two consecutive days is 15 min.
- The maximum allowable time to reach the nodes is 360 min during the day.
- Minimum distance between treasury (origin) and branches (destination) is approximately 1 Km and maximum distance between treasury and branches is approximately 30 Km.

4 Materials and methods

In the present study, a model for money delivery operations to branches with the least risk and a specific period is represented .Accordingly, the proposed model is a one-objective model to minimize the route risk. It is formulated in the vehicle routing problem considering the possibility of the simultaneous pickup and delivery problem with time windows. Also, a genetic metaheuristic algorithm is used to solve this problem. The main advantage of using the meta-heuristic genetic algorithm compared to other algorithms (such as Tabu search and ants colony or bees, weeds, fireflies, etc.), according to the results of previous studies, is that this algorithm achieves shorter travel distances in vehicle routing problems.The data in this study were randomly selected for all Bank Shahr branches of Tehran (135 branches). In order to validate the metaheuristic algorithm proposed in this study, several experimental problems in different dimensions were randomly generated after setting the parameters and creating the initial solution. Then, algorithm results were compared regarding solution quality and their computation time. Also, how the representation of periods and distance are in the proposed model was set based on travel time, service time, minimum time interval, maximum allowable time to reach the node in a min on different days.

5 Modelling

In the present study, a mixed-integer model is represented for the single-treasury routing problem considering simultaneous pickup and delivery problems with time windows for carrying cash.

5.1 Indexes and sets

N: A set of all nodes
i,j: Node index
D: A Set of demand points nodes (branches)
o: Origin node index (treasury)
K: A set of all vehicles
k: Vehicle index
R: A set containing all movements counters
r: Movement counter index
T: A set of planning horizon days
t,t′: Day index

5.2 Parameters

$time_{ij}$: Travel time from node i to node j.
UL_i: Service time to node i.
Dem_{it}: Node i demand on day t for the money to be delivered.
$Pick_{it}$: Node i demand on day t for the money it has to deliver.
$\propto$: Minimum time interval which a node meets on two consecutive days.
MAX: Maximum amount of money that can be carried by each vehicle.
α_i: Maximum allowable time to reach node i.
M: The large number.

5.3 Decision variables

X_{ijkrt}: Variable 0 and 1. It indicates that vehicle k travels from i to j on day t in its r^{th} movement or not.
L_{ikrt}: The amount of new money inside vehicle k when it reaches node i on day t in its r^{th} movement.
P_{ikrt}: The amount of money collected inside vehicle k when it reaches node i on day t in its r^{th} movement.
S_{ikrt}: The arrival time of vehicle k when it reaches node i on day t in its r^{th} movement.
$w'_{ijtt'}$, $w_{ijtt'}$: An ideal variable when a route of node i to node j is not repeated in two consecutive days.
$f'_{itt'}$, $f_{itt'}$: An ideal variable when time interval in entering a node is not the same on two consecutive days.
h'_{ijkrt}, h_{ijkrt}: An ideal variable when a vehicle travels a short route in its first three movements due to carrying large amounts of money.

5.4 Mathematical model

$$\sum_{\substack{i\in N\\ i\neq j}}\sum_{k\in K}\sum_{r\in R} X_{ijkrt} = 1 \qquad \forall j \in D\,, t \in T \qquad (1)$$

$$\sum_{\substack{i\in N\\ i\neq j}} X_{jik(r+1)t} \le \sum_{\substack{i\in N\\ i\neq j}} X_{ijkrt} \qquad \forall j\in N, k\in K, t\in T, r\in R \qquad (2)$$

$$\sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}} X_{ijkrt} \le 1 \qquad \forall k\in K, r\in R, t\in T \qquad (3)$$

$$\sum_{r\in R}\sum_{j\in D} X_{ijkrt} \le 1 \qquad \forall i\in o, k\in K, t\in T \qquad (4)$$

$$\sum_{i\in D}\sum_{\substack{j\in N\\ j\neq i}}\sum_{k\in K} X_{ijkrt} \le 0 \qquad \forall r=1\,, t\in T \qquad (5)$$

$$L_{jkrt} \le L_{ik(r-1)t} - Dem_{it} + M\left(1 - X_{ijkrt}\right) \quad \forall i,j\in N: i\neq j, i\neq o\,, k\in K, r\in R\,, t\in T \quad (6)$$

$$P_{jkrt} \ge P_{ik(r-1)t} + Pick_{it} - M\left(1 - X_{ijkrt}\right) \quad \forall i,j\in N: i\neq j, i\neq o\,, k\in K, r\in R\,, t\in T \quad (7)$$

$$S_{jkrt} \ge S_{ik(r-1)t} + time_{ij} + UL_i - M\left(1 - X_{ijkrt}\right) \; \forall i,j\in N: i\neq j, j\neq o\,, k\in K, t\in T, r\in R \qquad (8)$$

$$time_{ij} x_{ijkrt} \le LB \qquad \forall i,j\in N: i\neq j\,, k\in K, t\in T, r\le 3 \qquad (9)$$

$$\left|\sum_{k\in K}\sum_{r\in R} S_{ikrt} - \sum_{k\in K}\sum_{r\in R} S_{ikr(t+1)}\right| \ge \propto \qquad \forall i\in D: t\in T \qquad (10)$$

$$\sum_{r\in R}\sum_{k\in K} x_{ijkrt} + \sum_{r\in R}\sum_{k\in K} x_{ijkrt'} + w_{ijtt'} - w'_{ijtt'} = 1 \;\; \forall i,j\in N: i\neq j\,, t,t'\in T: t'=t+1 \quad (11)$$

$$\sum_{r\in R} L_{ikrt} \le MAX \qquad i\in D, k\in K\,, t\in T \qquad (12)$$

$$L_{jkrt} + P_{jkrt} \le M \sum_{\substack{i\in N\\ i\neq j}} x_{ijkrt} \qquad \forall j\in D, k\in K, r\in R\,, t\in T \qquad (13)$$

$$S_{jkrt} \le M \sum_{\substack{i\in N\\ i\neq j}} x_{ijkrt} \qquad \forall j\in D, k\in K, r\in R\,, t\in T \qquad (14)$$

$$\sum_{k\in K}\sum_{r\in R} S_{ikrt} \le a_i \qquad t\in T, i\in D \qquad (15)$$

$$\text{Min} = \sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}}\sum_{k\in K}\sum_{r=1}^{3}\sum_{t\in T} h'_{ijkrt} + \sum_{i\in D}\sum_{t\in T}\sum_{t'=t+1} f_{itt'} + \sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}}\sum_{t\in T}\sum_{t'=t+1} w'_{ijtt'} \qquad (16)$$

Constraint 1 ensures that each branch is serviced from only one vehicle. Constraint 2 indicates that if the vehicle does its $(r+1)^{th}$ movement, it has certainly done its r^{th} movement. Constraint 3 guarantees that each vehicle travels at most one route (the distance between two nodes). Constraint 4 ensures that each vehicle leaves the treasury maximum once every day. Constraint 5 ensures that the vehicle does not start its travel from the branch .Constraints 6 and 7 show the relationship of the money amounts inside the vehicle between two consecutive nodes and remove subtours. Constraint 8 shows the relationship between the times of reaching two consecutive nodes. Constraint 9 is one of the risk control relationships. It ensures a vehicle travels short routes in its first three movements due to carrying large amounts of money.
This relationship was originally as follows:

$$time_{ij} x_{ijkrt} + h_{ijkrt} - h'_{ijkrt} = LB \;\; \forall i,j\in N: i\neq j\,, k\in K, t\in T, r\le 3 \; (17)$$

Constraint 10 is the sec risk control relationship that ensures that the minimum time interval in which a vehicle meets a node is not the same on two consecutive days. This relationship was originally as follows:

$$\left|\sum_{k\in K}\sum_{r\in R} S_{ikrt} - \sum_{k\in K}\sum_{r\in R} S_{ikrt'}\right| + f_{itt'} - f'_{itt'} = \propto \;\; \forall i \in D: t,t' \in T: t' = t+1 \quad (18)$$

Linearization of the above equation is as follows:

$$\sum_{k\in K}\sum_{r\in R} S_{ikrt} - \sum_{k\in K}\sum_{r\in R} S_{ikrt'} + f_{itt'} - f'_{itt'} \geq \propto - MZ_{tt'} \quad \forall\, i \in D\,, t\,, t' \in T: t' = t+1 \quad (18-1)$$

$$\sum_{k\in K}\sum_{r\in R} S_{ikrt} - \sum_{k\in K}\sum_{r\in R} S_{ikrt'} + f_{itt'} - f'_{itt'} \leq -\propto + M(1 - Z_{tt'})\; \forall\, i \in D\,, t, t' \in T: t' = t+1 \; (18-2)$$

Constraint 11 is the third risk-controlling relationship that ensures that a route from i to j is not repeated on two consecutive days. This equation was originally as follows:

$$\sum_{r\in R}\sum_{k\in K} x_{ijkrt} + \sum_{r\in R}\sum_{k\in K} x_{ijkrt'} \leq 1 \qquad \forall i,j \in N: i \neq j\,, t, t' \in T: t' = t+1 \qquad (19)$$

In order to the problem has a solution always, this constraint is considered a soft constraint (equation 9). The variable of h_{ijkrt} must be 0 to establish equation 17. In order to the problem has a solution always, this constraint is considered a soft constraint (equation 10). The variable $f_{itt'}$ must be 0 to establish equation 18. In order to the problem has a solution always, this constraint is considered a soft constraint (equation 11). The variable $w'_{ijtt'}$ must be 0 to establish equation 19. These cases are observed in the objective function. Constraint 12 ensures that no vehicle will carry cash more than the specified amount. This also helps reduce the risk level. Constraints 13 and 14 show the relationship between decision variables. Constraint 15 ensures that the arrival time of the vehicle to the customer is less than the maximum allowable time for arrival. The objective function 16 represents the route risk minimization so that the vehicle moves in a short route in the first three movements, the time interval that a vehicle reaches a node is not the same in two consecutive days, and a route from i to j is not repeated on two consecutive days.

6 Proposed solution approach

In general, any genetic algorithm for solving a problem has the following components:

6.1 How to the representation the solution

The obligation to use a sequence of definite numbers represents routing problems. There are several methods available in numerous studies to represent it in this field. Due to the model essence and the algorithm type (continuous), the solution can be represented as a vector with a fixed length. A better expression is a matrix whose number of rows is equal to the number of periods (T) and whose number of columns is equal to the number of demand points (D). Numbers inside the matrix are real numbers between 1 and (number of vehicles +1) that are randomly generated. The notable point is the allocation of the vehicle carrying cash and the sequence of demand points. The integer indicates the vehicle carrying cash, and the decimal point indicates the sequence of demand points. The number with the lowest decimal point is served first .For example, as shown in Fig. 1, the number 1.97 in the following vector means the demand point No. 1 is serviced by vehicle No. 1, the number 2.99 means the demand point No. 2 is serviced by vehicle No. 2, and the number 1.22 means the demand point No. 3 is serviced by vehicle No. 1, etc. Demand points of 1.03, 2.18, 1.22, etc., have the lowest decimal points.

1.97	2.99	1.22	1.26	1.73	2.57	2.18	1.03

Fig. 1. A solution random vector

6.2 the setting parameters and initialization of the population

In setting the parameters of the genetic algorithm, the response surface methodology can be used .The response surface test design method includes a set of mathematical and statistical methods for modeling and problem analysis. This method is used when the problem solution (goal) is affected by an individual factor (input), and the goal is to optimize this solution. The determining of initial parameters amounts for algorithm includes initial population size (nPop = 100), probability of crossover or recombination (Pc = 0.82), probability of mutation (Pm = 0.36), maximum repetition number of the algorithm (Max-It = 250), type of selection operator and type of mutation and crossover operators.

6.3 Production of primary solution

The population is a subset of solutions in the current generation. Also, the population can be defined as a set of chromosomes. Therefore, we used random initialization to quantify the population. Random solutions are the solutions that propel the population to optimization.

6.4 Fitness function

The problem variable value is placed in the fitness function, and in this way, each solution desirability will be determined. The objective function is used as a fitness function in optimization problems (Sadeghi Moghadam et al., 2009). The objective function determines how persons play a role in the problem field. The fitness function is usually used to convert the objective function value to a fitness value dependent on it. In other words, F(n) = g(f(x)).

So that f is an objective function. The function g converts the objective function value to a non-negative number, and F is its fitness value .The value obtained from the fitness function determines whether the solution is appropriate or not. Since the problem type is optimization, the fitness function is the same as the objective problem function. The aim of the objective problem function is risk minimization.

6.5 Selection operator

There are different methods for genetic algorithms that can be used to select genomes. This algorithm uses Roulette Wheel Selection (RWS) and Tournament Selection (TS) methods.

6.6 Crossover operator

The crossover or recombination operator is performed by selecting two chromosomes (parents) on the second part of the chromosome, which is the activities sequences, and its result is two new chromosomes (children). Here, it is expected that the desirable features of the parents will be combined, and better children will be obtained. The best crossover in continuous solutions is a uniform crossover used in this algorithm .In other words, we need two patterns to perform the crossover operator. These patterns are randomly selected from the original population and multiplied by random numbers between 0 and 1 (called mask), and new chromosomes are produced as follows.

$x_1 = (x_{11,}x_{12}, x_{13}, \dots, x_{1n})$: Parent 1
$x_2 = (x_{21,}x_{22}, x_{23}, \dots, x_{2n})$: Parent 2
$\alpha = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n) \quad 0 \le \alpha \le 1$: Mask
$y_1 = (y_{11,}y_{12}, y_{13}, \dots, y_{1n}) \rightarrow y_{1i} = \alpha_i x_{1i} + (1-\alpha_i)x_{2i}$: Child 1
$y_2 = (y_{21,}y_{22}, y_{23}, \dots, y_{2n}) \rightarrow y_{2i} = (1-\alpha_i)x_{1i} + \alpha_i x_{2i}$: Child 2

In other words, as shown in Fig. 2, we use a mask in this method for integration. In this way, we create an array equal to the number of genes containing the element. The elements of this array can only take values of 0 or 1. We randomly assign values to mask array elements. Now, we integrate two chromosomes using this mask. Values 1 and 0 in the mask array indicate that the gene should be selected from the first and second chromosomes, respectively. For the chromosome of the second child, the trend is reversed.

Parent 1	0.72	0.35	0.21	0.88	0.10	0.19

Parent 2	0.25	0.61	0.18	0.36	0.94	0.49
Mask	1	1	0	0	1	0
Child 1	0.72	0.35	0.18	0.36	0.10	0.49

Fig. 2. Mask operator

A random number is generated in the range of [0, 1] after performing the crossover operator for each generated chromosome. If this random number is less than 0.3, a mutation operator will perform on it. The position of two genes in the selected chromosome is changed to perform the mutation operator.

6.7 Mutation operator

We use the mutation operator to achieve possible good points in the solution space. The mutation essence is somehow the transformation of current solutions. In most cases, it will not lead to good solutions. However, if successful, it can significantly affect the objective function and open a new space for solutions. The genetic algorithm considers the probability of mutation in chromosomes between 0.001 and 0.01. We hope that good chromosomes lost or deleted during the selection or reproduction stages will be revived using this operator. In addition, this operator ensures that regardless of the dispersion of the initial population, the probability of searching anywhere in the problem space never becomes 0. In this algorithm, the mutation operator in real continuous space is performed using a normal distribution as follows:

$$x_i^{new} \sim N(x_i, \sigma^2)$$
$$x_i^{new} = x_i + \sigma N(0,1)$$

Where, σN (0,1) is the step length.

We perform mutation operation with a very low probability (less than 0.05) in the genetic algorithm because mutation rarely occurs in nature. As mentioned, the advantage of the mutation operator is that it gives us access to all search space.

6.8 Stop condition for genetic algorithm

The genetic algorithm is repeated until the stop condition is met. The metaheuristic method stops when it reaches the default maximum repetition (Max-It). In other words, maximum allowable time for performance can be applied as a stop criterion in each repetition, and a new solution is obtained for the model.

7 Discussion and computational results

The 30 problems in the form of small, medium, and large problems were generated as follows, and all problems were solved by GAMS and MATLAB R2015b software to validate the proposed algorithm and the generated solutions quality compared to the optimal solutions.

-Small samples: They include 12 problems so that 4-15 branches are serviced by 2 or 3 vehicles in 3 working days.
-Medium samples: They include ten problems so that 25-70 branches are serviced by 4-8 vehicles in 3 or 4 working days.
Large samples include eight problems so that 80-135 branches are serviced by 9-15 vehicles in 4-6 working days.

7.1 Comparison of solutions and Relative Standard Deviation (RSD)

The proposed metaheuristic algorithm performed all 12 small problems and 18 medium to large problems five times in the next stage. Fig. 3 shows the results obtained from solving each problem, including the worst generated solution, average solutions, and the obtained model solution, along with their computational time in seconds. To validate the algorithm and compare them with each other, the relative percent deviation index or relative standard deviation is defined for the proposed algorithm with the following equation:

$$RPD = (ALG_S - ALG_{BS})/ALG_{BS}$$

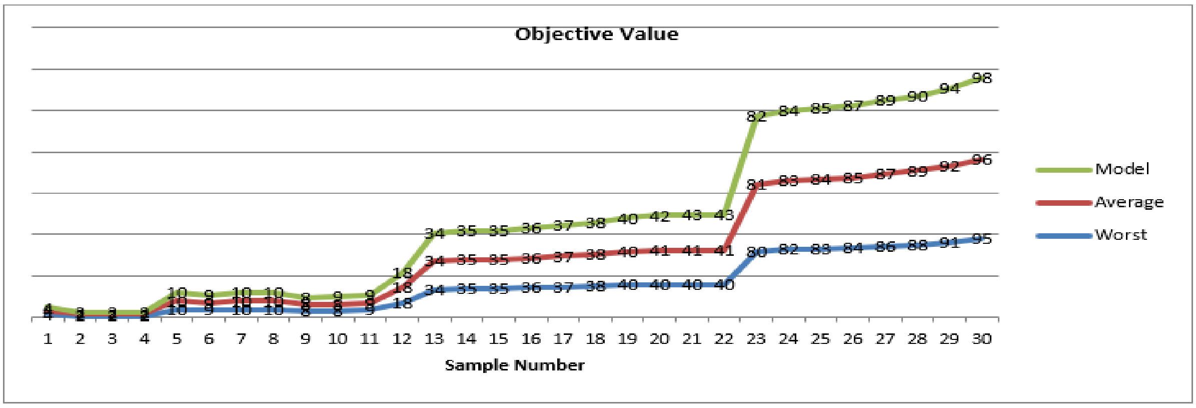

Fig. 3. The solutions values

ALG_S and ALG_{BS} in this equation are the model solution and the best solution/worst solution obtained from the proposed algorithm for five times performing the sample problem, respectively. In other words, the greatest difference is formulated as the model solution minus the worst solution, divided by the model solution. The least difference is formulated as the model solution minus the average solution, divided by the model solution. (Figs 4)

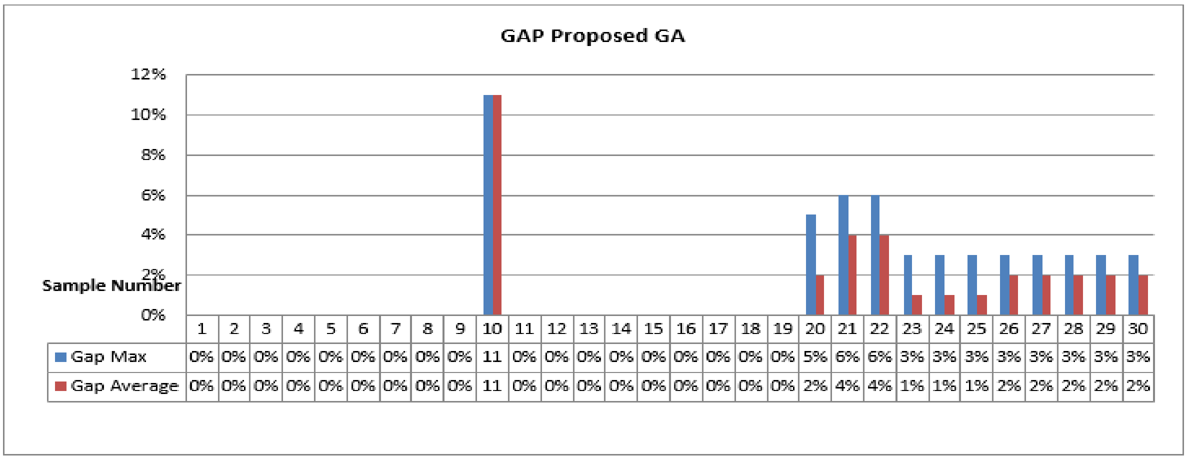

Fig. 4. The solutions gaps

These cases with the computation time are shown in Figs 5. The maximum solution time for a problem among the most complex samples was recorded 24 sec and minimum time is 0.

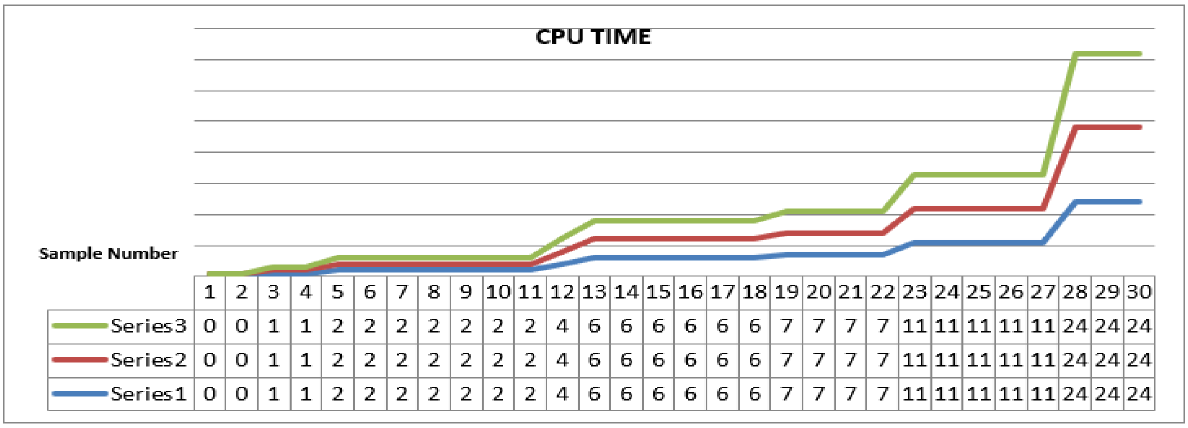

Fig. 5. The computation time of solutions

7.2 Comparison with the GAMS method

According to Table 1, the GAMS method had the solution for small samples but could not provide medium or large samples. Also, the solutions provided had a slight deviation from the model solution. However, compared to the GAMS method, the solutions were obtained in much less computational time . The 11 out of 12 problems of small samples had a deviation percentage of 0. Also, 7 out of 10 problems of medium samples had a deviation percentage of 0, and 3 problems had a deviation more than 3. All large problems had a deviation of more than 1. In small problems, 11 solutions were equal to the average solution. In medium problems, seven solutions were equal to the average solution. As known, for a problem with larger dimensions, the deviation of problem solutions from the average solution will be greater. However, this deviation is acceptable, and the proposed solution algorithm can be trusted for larger problems.GAMS software needed 26,860 sec to solve the most complex samples.

7.3 Comparison with genetic algorithm

We used the model solution method, average solutions, the worst solution, maximum deviation, and average deviations to evaluate the genetic algorithm performance. Table 1 shows the performing results of the proposed algorithm for 30 problems considering the algorithm performing time as a stop criterion. As shown in Table 1, the model solution is better than the average solutions in problem No. 10 of the small problems and problems No. 20, 21, and 22 of the medium problems. Also, all model solutions are better than the average solutions in large problems. After performing the above stages for the genetic algorithm, an average deviation of 1.09% and a maximum deviation of 1.75% from the optimal solution were observed.

7.4 Comparison with other research

A deviation percentage of 1.3% was reported in a similar investigation conducted by Ghannadpour et al. in 2018, entitled “A game theory-based vehicle routing problem with risk-minimizing of valuable commodity transportation.” Also, a deviation percentage of 1.3% was observed in a similar investigation conducted by Tavakkoli-Moghaddam et al. in 2014, entitled “Solving a multi-depot vehicle routing problem based on reduction risk by a multi-objective bat algorithm.” The deviation percentage was 1.9% in the research conducted by Atabaki et al. in 2018, entitled “A priority-based differential evolution algorithm for redesigning a closed-loop supply chain using robust fuzzy optimization.” In the study conducted by Stack et al. in 2014, entitled “Multi depots capacitated location-routing problem with simultaneous pickup and delivery and split loads,” the deviation percentage was reported 0.2%. Accordingly, the proposed algorithm performance is quite defensible by increasing the problem dimensions because it generates acceptable solutions relatively quickly for problems with medium and large dimensions.

Table 1. Computational results for the proposed algorithm

Row	Trial	D	K	T	GAMS		MATLAB							
					Mathematical Model		Proposed GA							
					OBJECTIVE VALUE	CPUs	OBJECTIVE VALUE			CPUw	CPUa	CPUm	Gap Max	Gap Average
							Worst	Average	Model					

1	4	2	3	4	0.6	4	4	4	0.3	0.3	0.3	0%	0%

Small Sample	2	5	2	3	2	1	2	2	2	0.3	0.3	0.3	0%	0%
	3	6	2	3	2	2	2	2	2	1	1	1	0%	0%
	4	7	2	3	2	3	2	2	2	1	1	1	0%	0%
	5	8	2	3	9	6	10	10	10	2	2	2	0%	0%
	6	9	2	3	9	15	9	9	9	2	2	2	0%	0%
	7	10	2	3	9	67	10	10	10	2	2	2	0%	0%
	8	11	2	3	9	156	10	10	10	2	2	2	0%	0%
	9	12	2	3	8	25	8	8	8	2	2	2	0%	0%
	10	13	3	3	8	244	8	8	9	2	2	2	11%	11%
	11	14	3	3	8	158	9	9	9	2	2	2	0%	0%
	12	15	3	3	17	26,860	18	18	18	4	4	4	0%	0%
Medium Sample	13	25	4	3	19	10,802	34	34	34	6	6	6	0%	0%
	14	30	4	3	NA	NA	35	35	35	6	6	6	0%	0%
	15	35	5	3	NA	NA	35	35	35	6	6	6	0%	0%
	16	40	5	3	NA	NA	36	36	36	6	6	6	0%	0%
	17	45	6	3	NA	NA	37	37	37	6	6	6	0%	0%
	18	50	6	3	NA	NA	38	38	38	6	6	6	0%	0%
	19	55	7	3	NA	NA	40	40	40	7	7	7	0%	0%
	20	60	7	4	NA	NA	40	41	42	7	7	7	5%	2%
	21	65	8	4	NA	NA	40	41	43	7	7	7	6%	4%
	22	70	8	4	NA	NA	40	41	43	7	7	7	6%	4%
Large Sample	23	80	9	4	NA	NA	80	81	82	11	11	11	3%	1%
	24	88	10	4	NA	NA	82	83	84	11	11	11	3%	1%
	25	96	11	4	NA	NA	83	84	85	11	11	11	3%	1%
	26	104	12	5	NA	NA	84	85	87	11	11	11	3%	2%
	27	112	13	5	NA	NA	86	87	89	11	11	11	3%	2%
	28	120	14	5	NA	NA	88	89	90	24	24	24	3%	2%
	29	128	15	6	NA	NA	91	92	94	24	24	24	3%	2%
	30	135	15	6	NA	NA	95	96	98	24	24	24	3%	2%
	Mean				-	-	-	-	-	-	-	-	1.75%	1.09%

D = demand points (branches), K = number of vehicles carrying cash, T = number of operating days

8 Conclusions and suggestions

In this study, we develop a model, which is formulated as a mixed-integer model, to handle the cash transportation vehicle routing problem. Because most thieves monitor and check the route and time of transportation money for a while to plan their theft. These attacks may cause serious harm to branch staff, customers, and security personnel. Therfore, three concepts are used in this paper for reducing route risk. This technique greatly reduces the possibility of setting a consistent pattern to serve branches and increases service security

(i) The vehicle does not travel long routes in the first three movements when carries more cash.
(ii) A branch is not served at the same and similar hours on two consecutive days.
(iii) As far as possible, an arc is not repeated on two consecutive days.

The 30 problems in the form of small, medium, and large problems were generated and all problems were solved by GAMS and MATLAB software to validate the proposed algorithm and the generated solutions quality compared to the optimal solutions. With attention to the inefficiency of GAMS software in solving the model in large dimensions, a genetic metaheuristic algorithm was used in this study. The model solution was equal to the average solution in 18 out of 30 problems. Finally, after performing the genetic algorithm, an average deviation of 1.09% and a maximum deviation of 1.75% from the optimal solution were observed.Although the proposed model could help security carriers efficiently formulate the vehicle routes, more considerations for road conditions and demand points can be incorporated into the model to make it more useful in the future. Including:

1- The limit of the distance between the source node (treasury) and the destination node (branch) should be added to the constraints of the problem.

2- To solve the problem, other heuristic algorithms are also used and the results are compared with the genetic algorithm and prioritized.

Finally, how to modify the model, to make it suitable for solving other similar problems, e.g., prison vehicle routing, could be pursued in the future.

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