E-ISSN: 2676-6167 | P-ISSN: 2538-5100

Journal of Applied Research on Industrial Engineering

www.journal-aprie.com

J. Appl. Res. Ind. Eng. Vol. x, No. x (xx) x–x.

Paper Type: Original Article

Loan Portfolio Performance Evaluation by Using Stochastic Recovery Rate

Shokouh Shahbeyk*; Shokoofe Banihashemi

Allameh Tabataba'i University

Citation:

Received: Reviewed: Revised: Accept:

Abstract

One of the most important aspects of credit risk management is determining the capital requirement to cover credit risk in a bank loan portfolio. In this paper, credit risk of loan portfolio can be obtained by stochastic recovery rate based on two approaches: beta distribution and short interest rate. The capital required to cover credit risk is achieved through the Vasicek model. Also, the Black-Scholes Merton model for European Call Option is utilized to quantify the probability of default. Value-at-Risk (VaR) and conditional value at risk (CVaR) are utilized as measures of risk to evaluate the level of risk and obtained by worst case probability of default (PD) .We have used stochastic recovery rate for calculating VaR which is related to the underlying intensity default. Also, intensity default process is assumed to be linear in the short term interest rate which is driven by a CIR process. Therefore, by considering these characteristics to evaluate the loan portfolio performance with data envelopment analysis (DEA) method, we propose our model where the losses are driven by stochastic recovery rate and default probability. We present an empirical investigation to measure PD of eighth stocks from different industries of Iran stock exchange market by using Black-Sholes-Merton model.

Keywords: Portfolio credit risk, Loan portfolio, Data Envelopment Analysis, Recovery rate, Default probability, Conditional value at risk.

1 | Introduction

Credit risk refers to the risk of default or non-payment or non-adherence to contractual obligations by a borrower. The loans are the largest source of credit risk for most banks. Because of this, financial institutions must quantify credit risk at portfolio. On the other hand, approach based on internal ratings, namely the IRB-model proposed by Basel II and III in capital adequacy ratio [6,7]. in the IRB approach, the calculation of the portfolio loss and the value at risk (VaR) of

https://doi.org/10.22105/jarie.2023.346023.147810

the portfolio is evaluated, in which the conditional PD is calculated under the Vasicek model. The Vasicek model is a popular one-factor model that derives the limiting form of the portfolio loss. This model will allow calculating different risk measures such as the expected loss (EL), the value at risk (VaR) and the Conditional Value at risk .For this purpose several risk measures based on the portfolio loss distributions will be presented. Value-at-Risk (VaR) is a risk measure that is popular as an industry standard but it is not always sub-additive and convex. So, the concept of coherent risk measure is introduced that satisfies in properties of translation invariance, homogeneity, subadditivity and monotonicity. Conditional Value-at-Risk (CVaR) as an alternative and compatible risk measure is the weighted average of VaR and losses strictly greater than VaR for general distributions. The method of CVaR minimization has been employed for credit risk management of portfolio of bonds [2], portfolio hedging [1]. VaR and CVaR can be obtained using unexpected loss and expected loss in Vasicek model [11]. Also, the credit risk of loan portfolio can be calculated in terms of correlations, VaR and CVaR by using Vasicek model, and capital allocations obtained from analytical framework. Afterward, outcomes are compared with the results found by implementing the Internal Rating Based (IRB) approach of Basel II and III[8]. As we know, the portfolio loss is a random variable, that is in terms of exposure at default, recovery rate and time until the default of obligor. Moreover, this work considers the recovery rates as stochastic with beta distribution and dependent on each other and on the time until defaults[8]. Data Envelopment Analysis (DEA) is one approach to measure the efficiency of multiple Decision Making Units (DMUs) that is based on the linear programming by considering the multiple inputs and multiple outputs and was introduced by Charnes et al. [5]. DEA is originally designed for production theory as a quantitative, empirical and non-parametric method. Comparison of Banks and Ranking of Bank Loans Types have been described with DEA [10]. There have been a lot of theoretical and empirical models and approaches for decision-making on portfolio performance evaluation by DEA. A DEA portfolio efficiency index was one of the seminal work of applying DEA in the context of portfolio performance evaluating [17]. Afterward, the classical DEA models have been widely applied to evaluate the financial assets performance [4,7]. By using, DEA and goal programming, the most efficient companies have been listed on the Tehran Stock Exchange [9]. Moreover, when the ambiguities are applied in the model, the results of fuzzy data envelopment analysis in portfolio optimization will be much more accurate and consistent with the facts[19]. In addition, possibilities of diagnosis credit risk through DEA and design an appropriate model for diagnosis of credit risk has been explored, which can be used in different sectors of national economy[15]. Also, DEA is applied to Calculating Probability of Default for High Rated Portfolio [13]. Therefore, in order to find inefficiency of loan portfolio or each stock in the loan portfolio, we apply DEA method. In the losses, an investment has a negative rate of return over a time period, so an approach based on the directional distance function as Range Directional Measure (RDM) model was presented[10]. In this paper, the stochastic recovery rate can be considered as a random variable with two approaches. One of them is based on beta distribution. The other stochastic recovery rate is related to the underlying intensity default. Also the intensity default process is assumed to be linear in the short term interest rate which is driven by a CIR process. Also, PD is obtained by Black-Sholes-Merton model[9]. Our model is inspired by the RDM to deal with negative values for performance evaluate in loan portfolio with risk measures as the input and mean return as the output. Risk measures can be obtained by worst case probability of default that can be obtained by using Vasicek model and stochastic recovery rate. In this study, the risk measures VaR and CVaR are applied.

We have organized the paper as follows. Section 2 is devoted to the preliminary concepts of coherent risk measure, VaR, CVaR, loss given default, exposure at default, recovery rate and Vasicek model. In section 3, we introduce Credit Portfolio performance Model for loans based on RDM. Section 4 is devoted to the empirical example of Iran Stock Exchange market for calculating probability of default. Section 5 is concluded.

2 | Preliminary

This section is devoted to some preliminaries which are needed in the sequel.

2.1 | Risk measure

We present the properties of a coherent risk measure and the definitions of VaR and CVaR.

Definition 2.1.1

Assume $(\Omega, \mathrm{F}, \mathrm{P})$ to be the probability space and $I(\Omega, \mathrm{F})$ to be the set of random variables of one dimensional on the space. The function $\rho : I(\Omega, \mathrm{F}) \to R$ is a coherent risk measure whenever it satisfies following axioms for random variables $X, Y \in I(\Omega, \mathrm{F})$,

a) Monotonicity: If $X \leq Y$, then $\rho(\mathrm{Y}) \leq \rho(X)$;

b) Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(\mathrm{Y})$;

c) Translation Invariance: For all $\alpha \in \mathrm{R}$, $\rho(X+\alpha) = \rho(X) - \alpha$;

d) Positive homogeneity: For all $\alpha \geq 0$, $\rho(\alpha X) = \alpha \rho(X)$.

Value at Risk (VaR) as one of the risk measures which is a benchmark standard for firm-wide measures of risk.

Definition 2.1.2

- **Value at Risk (Var):** For a given time horizon and confidence level $\beta \in (0,1)$, we consider decision vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ in $\mathbb{R}^n$ which represents the position of n financial assets in a portfolio.

The distribution of the vector of asset returns $Y = (Y^1, Y^2, \ldots, Y^n)^T$ constitutes various returns and its cumulative loss function is shown by $\psi(\lambda, \Gamma)$. In other words, the return on a portfolio is the weighted average of returns in the portfolio, so the loss function is being the negative of this sum and is denoted by

$$f(\lambda, Y) = -(\lambda_1 Y^1 + \lambda_2 Y^2 + \ldots + \lambda_n Y^n) = -\lambda^T Y.$$

The Value at Risk of a portfolio is the loss in the portfolio's market value over the horizon time that is exceeded with probability $1-\beta$. Therefore, VaR is defined as follows

$$VaR_{\beta}(\lambda)=\inf\{\lambda:\psi(\lambda,\Gamma)\geq\beta\}.$$

- **Conditional Value at Risk (CVaR)**: An alternative risk measure to VaR is Conditional VaR (CVaR) which is also known as expected shortfall. CVaR is the conditional expectation of the losses exceeding VaR and it is specified as the

$$CVaR_{\beta}(\lambda)=E\left[f(\lambda,Y)\middle|f(\lambda,Y)\geq \mathrm{VaR}_{\beta}(\lambda)\right].$$

It is shown that, CVaR can be reduced to a linear programming problem, so we have the following approximation function

$$\tilde{F}_{\beta}(\lambda,\Gamma)=\Gamma+\frac{1}{(1-\beta)Q}\sum_{q=1}^{Q}(f(\lambda,Y_q)-\Gamma)^{+}=\Gamma+\frac{1}{(1-\beta)Q}\sum_{q=1}^{Q}(-\lambda^{T}Y_q-\Gamma)^{+},$$

where $(Z)^{+}=\max\{Z,0\}$ and Q represents the scenarios of log-returns of assets $Y_1,Y_2,\dots,Y_Q$, where each elements $Y_q(q=1,2,\dots,Q)$ is a vector in R^n and Γ is in $\mathbb{R}$. Therefore, $CVaR_{\beta}(\lambda)$ has an equivalent definition as follows

$$\min CVaR_{\beta}(\lambda)=\min_{\Gamma\in R}\tilde{F}_{\beta}(\lambda,\Gamma).$$

2.2 | Credit risk

In this section, The component of expected loss, Vasicek model and Merton model are introduced.

Definition 2.2.1

- **Loss given default** (LGD) is defined as the amount of funds that is lost by a bank when a borrower defaults on a loan.
- **Exposure at default** (EAD) is the predicted amount of loss a bank may face in the event of, and at the time of, the borrower's default.
- **Recovery rate**(R) is defined by $1-LGD$.

Based on IRB approach banks are able to design internal models to calculate capital reserve in light of common characteristics identified by studies of academics and industry, and apparently, LGD is not an exception in this regard.

2.3 | Vasicek Model

We state the credit risk model developed in Vasicek (1987). This model have been used to measure the credit risk of a portfolio because it is the base for the Basel regulatory capital requirements and it is also widely used in the financial industry [11].In this model the value of A_i , is driven by a own macroeconomic factor, η, and an idiosyncratic independent term, ε_i, in following form

$$A_i = \sqrt{r}\eta + \sqrt{1-r}\varepsilon_i ,$$

where both η and ε_i follow independent standard Gaussian distributions. r shows correlations between A_i and A_j .

$$\rho(A_i, A_j) = E[(\sqrt{r}\eta + \sqrt{1-r}\varepsilon_i)(\sqrt{r}\eta + \sqrt{1-r}\varepsilon_j)] = r$$

By the assumption of a systematic factor η Probability of conditional default is equal to:

$$\begin{aligned} PD_{c,i|\eta} &= p(A_i \le L_i \mid \eta) = prob(\sqrt{1-r}\varepsilon_i \le L_i - \sqrt{r}\eta \mid \eta) \\ &= prob(\varepsilon_i \le \frac{L_i - \sqrt{r}\eta}{\sqrt{1-r}} \mid \eta) = \phi_N(\frac{L_i - \sqrt{r}\eta}{\sqrt{1-r}}). \end{aligned}$$

And we have

$$PD_{c,i|\eta} = \phi_N(\frac{\phi_N^{-1}(PD_i) - \sqrt{r}\eta}{\sqrt{1-r}})$$

We suppose that $\eta = \phi_N^{-1}(.999)$. Then worst case PD is obtained as follows:

$$WCPD = PD_{c,i|\eta}(1-\alpha) = \phi_N(\frac{\phi_N^{-1}(PD_i) - \sqrt{r}\phi_N^{-1}(1-\alpha)}{\sqrt{1-r}})$$

Therefore, we can conclude that for a large portfolio of loans with the same size and risk, the loss is 99% less than the following

$$N * EAD * (1-R) * WCPD$$

Which N is number of loans, EAD and R are exposure at default and recovery rate respectively. The Vasicek model assumes that the default is driven by a Gaussian process and this could generate a certain risk underestimation.

2.4 | Merton Model

Robert Merton (1974) developed a model by using the European call option of BSM. In this approach, Merton's proposed structural model is used to model the probability of credit risk default. Structural models for credit risk modeling are based on the structure of the borrower company's balance sheet. The balance sheet on the one hand represents the resources and on the other hand represents the expenditures that are provided from the resources. Resources include liabilities and equity, and expenditures on the balance sheet are the same as assets. A firm is considered a risk asset portfolio consisting of shares owned by its shareholders and debts related to lenders. It is assumed that the firm provides both equity (E) and debt (D), then by main accounting equation, the value of the firm (V) is $D+E$. In this model, we have two possible scenarios.

- The firm will default (the firm is unable to repay its debt at maturity (T), in this case the debt holders have a higher priority than their stockholders to return on their investments. They receive the value of the remaining assets and incur a loss of $D-V$ while the shareholders will receive nothing.
- The firm will not default, in this case, shareholders will receive more $V-D$.

Consider the both the conditions, the value of the equity at maturity T is as follows

$$\max(V-D,0).$$

We can estimate the value of the equity by using the Black Scholes Formula

$$E = V * N(d_1) - D * e^{-rT} * N(d_2) \qquad (1)$$

where

$$d_1 = \frac{\mathrm{Ln}(\frac{V}{D}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\mathrm{Ln}(\frac{V}{D}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

r is the free risk rate interest, T is the time period, σ is the volatility of the firms value, and N is the cumulative standard function for a standard normal distribution.

If the volatility σ replace by the volatility of the firms value σ_V, then the distance to default is given by

$$d_2 = \frac{\mathrm{Ln}(\frac{V}{D}) + (\mu_V - \frac{\sigma_V{}^2}{2})T}{\sigma_V \sqrt{T}}$$

Where μ_V is expected rate of return of the firm's asset and expected growth of assets is equal to

$$(\mu_V - \frac{\sigma_V{}^2}{2}).$$

After evaluating the distance to the default, we can measure the probability of default. The PD under the risk neutral measure as per the Black Scholes Merton model is given by:

$$PD = \phi_N(-d_2) = \phi_N(-\frac{\mathrm{Ln}(\frac{V}{D}) + (\mu_V - \frac{\sigma_V{}^2}{2})T}{\sigma_V \sqrt{T}}) \quad (2)$$

Equation (2) is the PD with three unknowns V, μ_V, and σ_V. The value of the firm V is equal to the sum of the debt and the equity of the firm, so the debt D is known and we need to find equity E. The equity value E is a continuous time stochastic process that is Weiner process.

$$dE = \mu_E E dt + \sigma_E E dW$$

where dW is the continuous time stochastic process, μ_E is the expected continuously compounded return on E, and σ_E is the volatility of the equity value. By Ito's lemma,

$$\sigma_E E = \sigma_V V \frac{dE}{dV} = \sigma_V \phi_N(d_1). \quad (3)$$

Equations 1 and 3 having two unknowns one is σ_V and another is V. We can easily estimate the two parameters by solving equations 1 and 3, and $\mu_V = \frac{V_t - V_{t-1}}{V_{t-1}}$ [4].

2.5 | Range Directional Measure Model (RDM)

In the conventional DEA models, each $\mathrm{DMU}_j (j = 1, \dots, n)$ is specified by a pair of non-negative input and output vector, $(x_{ij}, y_{rj}) \in R_+^{(m+s)}$, in which inputs, x_{ij} $(i = 1, \dots, m)$, are utilized to produce outputs, y_{rj} $(r = 1, \dots, s)$. These models cannot be used for cases where DMUs include both negative and positive inputs and/or outputs. Portela et al. (2004) considered a DEA model which can be applied in cases where input/output data take positive and negative values[18]. then the generic directional distance model can be represented as

$$
\begin{aligned}
\max \quad & \alpha \\
s.t. \quad & \sum \lambda_j y_{rj} \geq y_{ro} + \alpha R_{ro}, r = 1, 2, \dots, s, \\
& \sum \lambda_j x_{ij} \leq x_{io} - \alpha R_{io}, i = 1, 2, \dots, m, \\
& \sum \lambda_j = 1, \lambda_j, \alpha, R_{ro}, R_{io} \geq 0
\end{aligned}
$$

where $o \in \{1, 2, \dots, n\}$ is the unit under assessment. For negative values of given data set, an ideal point is defined as $I = \left(\max_j y_j, r = 1, 2, \dots, s, \quad \min_j x_j = i = 1, 2, \dots, m \right)$ and the vectors R_{ro} and R_{io} as the range of possible improvement of DMU_o are defined as $R_{io} = x_{io} - \min_j \{x_{ij}\}, i = 1, 2, \dots, m$ and $R_{ro} = \max_j \{y_{rj}\} - y_{ro}, r = 1, 2, \dots, s.$

3 | DEA-Based Loan Portfolio Model

We assume a loan portfolio with N obligors and a time horizon equal to the longest maturity among the credit assets in the portfolio . The random variable L, representing the portfolio loss, is defined following the notation used in Jouanin et al. [8].

$$
L = \sum_i EAD_i * LGD_i * 1_{\{D\}}, \qquad 1_{\{D\}} = \begin{cases} 1 & PD \\ 0 & 1 - PD \end{cases},
$$

$$
EL = \sum_i EAD_i * LGD_i * PD_i = \sum_i EAD_i * (1 - R_i) * PD_i. \tag{4}
$$

Unlike Expected Loss, the Unexpected Loss (UL) is not an aggregate of individual loss but rather depends on loss correlations between all loans in the portfolio. The deviation of losses from the EL is usually measured by the standard deviation of the loss variable. The portfolio standard deviation of credit losses can be decomposed into the contribution from each of the individual credit facilities . The capital requirement to cover credit risk in a bank loan portfolio (UL) can be obtained with the following relationship

$$UL = VaR - EL = EAD * LGD * \left(\phi_N \left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}} \right) - PD \right)$$

$$= EAD * (1 - R_i) * \left(\phi_N \left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}} \right) - PD \right). \qquad (5)$$

Where:

$$VaR = EAD * (1 - R_i) * \left(\phi_N \left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}} \right) \right). \qquad (6)$$

VaR is considered as risk measure. The recovery rate R_i may be assumed to be deterministic or stochastic with mean m_i and standard deviation s_i. In this paper, the recovery rate R_i can be obtained by two stochastic approach. One of them is related to the underlying intensity default $\lambda(t)$ via

$$R_i(t) = a_R + b_R e^{-\lambda(t)}$$

with $a_R \geq 0$, $b_R \geq 0$ and $0 \leq a_R + b_R \leq 1$.The intensity default process is assumed to be linear in the short term interest rate which is driven by a CIR process, i.e.

$$\lambda(t) = \Lambda_0 + \Lambda_1 r(t)$$

$$dr(t) = (\theta_r - a_r r(t))dt + \sigma_r \sqrt{r(t)} dW_r(t) \quad r(0) = r_0$$

with $\Lambda_0 \geq 0$, $r_0 \geq 0$.[3]

And the other, is related the most common distributional form of R_i that is the Beta (a_i, b_i) distribution, with the parameters a_i and b_i estimated by the method of moments, knowing the values of m_i and s_i analytically [5]:

$$a_i = \frac{m_i^2(1-m_i)}{s_i^2} - m_i, \qquad b_i = \frac{m_i^2(1-m_i)}{s_i^2} - (1-m_i).$$

It is well known that asset correlations play a critical role in measuring portfolio credit risk, and in determining both economic and regulatory capital. In a credit portfolio, having many components does not assure good diversification, because the components may be highly correlated to each other, and the default of one may lead to default of the rest of the portfolio. This concept is called concentration risk in credit risk management.

Now, we introduce a DEA model by the assumption that loss follow equation (6) for loan portfolio performance evaluation. Since losses can be negative, we apply RDM-based model based on appropriate underlying distribution for calculating efficiency. Also, risk (VaR) and expected loss are considered as the only input and output, respectively.

Let's assume $L_1, L_2, ..., L_n$ be the loss of the n loans in loan portfolio. For a specific loan L_0 where $0 \in \{1, 2, ..., n\}$ and regarding to the negative loss value, the vector $g^T = \left(R_{VaR_\beta^0}, R_{E(L^0)} \right)$, where

$$R_{VaR_\beta^0} = \left(VaR_\beta^0 - \min(VaR_\beta^j : j = 1, ..., n) \right),$$

$$R_{E(L^0)} = \left(m\text{ax}\,(E(L^j) : j = 1, ..., n) - E(L^0) \right).$$

This vector is a range of possible improvement in the input and the output. The VaR_β^0 is the value of risk, and $E(L_0)$ is the expected loss of loan under evaluation, $0 \in \{1, 2, ..., n\}$. According equation (4), we solve the following linear model

$$\begin{aligned} \max \quad & \alpha \\ s.t \quad & E(L(\lambda)) \geq E(L_o) + \alpha \, \mathrm{R}_{E(L_o)} \\ & VaR_\beta(L(\lambda)) \leq VaR_\beta^o - \alpha \, \mathrm{R}_{VaR_\beta^o} \\ & e^T \lambda = 1 \\ & \lambda \geq 0, \alpha \geq 0. \end{aligned}$$

The optimal value of α which is shown by α^* indicates the distance between the loan under evaluation and the efficient frontier. In other words, α^* represents the inefficiency score of the loan under evaluation, $1 - \alpha^*$ is the amount of efficiency. The vector $\lambda^T = (\lambda_1, \lambda_2, ..., \lambda_n)$ is the proportions of initial capital of n loan in a portfolio and e denotes the n-dimensional vector of ones. The above model is obtained under normal distribution. For future work, it is better that proper underlying distribution is considered. Also, we can apply CVaR as another risk measure.

4 | Application

Since the Merton model has been used as a standard for estimating the PD of firms and banks, in this section we use the Black-Scholes-Merton model described in Section 2.4. In this paper, according to the need of our country to conduct financial research in the field of banking, we have used data related to companies listed on the Tehran Stock Exchange that are able to obtain loans from a hypothetical bank. This research includes the symbol firms listed on the stock exchange that have been operating for at least 5 years and their financial statements have been published, and have reached a level of activity that is capable of receiving bank facilities such as loans from a hypothetical bank. In the selection of the statistical sample, the selected companies have a fiscal year ending at 12/29 or 12/30. According to this procedure, 7 firms have been selected. Collection of information on the value of equity and its variability, as well as the amount of debt to be paid by next year has been collected through the software of the official website **Codal.ir**.

It should be noted that in these calculations, the loan maturity for all companies is equal to $T=1$. Also, the risk-free rate of return is considered to be $r=0.15$. Merton model calculations are coded with Maple software. In table 1 collected data for parameters E, D, and σ_E are reported for each firm. All the entered values are in million Rials. Smaller difference between a company's asset value and its debt value would lead to higher probability of PD. In this example, the equations 1 and 3 are used to estimate the PD. In Table1, we present the results of value of firms, volatility of value of firms and estimated PD with Merton model respectively. Looking at Table1, we find that the stocks such as GEMS and BENN which the difference between value of the assets and the value of the debts of the companies are smaller ,their PD are more and for BENN is near one.

Table 1. List of symbols, equity, volatility of equity, debt, value of firms, volatility of value of firms and estimated PD with Merton model.

Symbol	PRKT	APPE	EPRS	GEMS	ATIR	BENN	GARN
Name	Behpardakht Mellat	Asan Pardakht Pers	Parsian Ecommerc	General Mechanic	Iran Tractor F	Behnoush Iran	Padide Shimi Gharn
E	7346040	8153293	10234784	27354086	13890539	1316005	5423310
σ_E	0.15	0.19	0.26	0.97	0.47	0.156	0.49
D	6371888	6180984	12758244	19806948	3424171	5920235	12435557
V	1216424.79	1316424.79	2032849.57	4381271079	1558212.99	6411598.48	6493649.652
σ_V	0.086318	0.11546	0.127899	0.698499825	0.39211808	0.032019	0.4092393941
PD	$1.287*10^{-13}$	$4.892*10^{-10}$	0.000426	0.460034	0.000195	0.867999	0.000388

5 | Conclusion

Data Envelopment Analysis is increasingly being used in various areas of finance, health, agricultural and others for evaluation of performance. This paper dealt with its use in the performance evaluation of the credit risk of loan portfolio. . As we Know, PD is an important parameter in analysis of credit risk in the finance world and it is One of the components of credit risk which can be obtained by the Merton model. Also, stochastic recovery rate has been obtained with two approaches. One of them is based on beta distribution and the other is related to the underlying intensity default. So, By using PD and stochastic recovery rate we can obtain expected loss, unexpected loss and capital requirement to cover credit risk in the loan portfolio. In this paper, we focused on DEA method as a non-parametric approach which relates the produced outputs to assigned inputs, then determines an efficiency score. This score can be interpreted as a performance measurement in investment analysis. In this regard, we introduced model in DEA framework for loan portfolio inspired by RDM-model where risk (VaR) and expected loss are considered as the only input and output, respectively. VaR and CVaR can be obtained using unexpected loss and expected loss in Vasicek model. Because of the PD, stochastic recovery rate, VaR and CVaR by using Vasicek model and RDM model the bank management can construct profitable loan portfolio. In this work, PD is calculated with Merton model. For this purpose, we have applied data related to companies listed on the Tehran Stock Exchange that are able to obtain loans from a bank. As seen in the example, smaller difference between the assets value and the debt value of a company would lead to higher probability of PD. For future works, other approaches for default probability and recovery rate can e used.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

References

[1] Alexander, S., Coleman, TF. (2003). Derivative portfolio hedging based on CVaR. In: Szego, *Risk Measures for the 21st century*, (pp.339-363) Wiley, London.

[2] Andersson, F., Mausser, H., Rosen, D., Uryasev, S. (2001). Credit risk optimization with Conditional Value at Risk, *Journal of Mathematical Programming*, 89, 273-291.

[3] Bakshi, G., Madan, D., Zhang, F. (2001). Recovery in default risk modeling: Theoretical foundations and empirical applications. *Finance and Economics Discussion Series 2001-37*. Board of Governors of the Federal Reserve System (U. S).

[4] Basso, A., Funari, S. (2001). A data envelopment analysis approach to measure the mutual fund performance, *European Journal of Operational Research*, 135(3), 477-492.

[5] Charnes, A., Cooper, W.W. , Rhodes, E. (1978). Measuring efficiency of Decision Making Units, *European Journal of Operational Research*, 2, 429-444.

[6] Dar, A., Anuradha, N., Qadir, S. (2019). Estimating probabilities of default of different firms and the statistical tests. *Journal of Global Entrepreneurship Research*, 9, 27.

[7] Dario, C., Simar, L. (2006). A robust non-parametric approach to evaluate and explain the performance of mutual funds, *European Journal of Operational Research*, 175(1), 516-542.

[8] Di Clemente, A. (2020). Modeling Portfolio Credit Risk Taking into Account the Default Correlations Using a Copula Approach: Implementation to an Italian Loan Portfolio. *Journal of Risk and Financial Management 13*(6), 129.

[9] Edalatpanah, SA., Godarzi karim, R., Khalilian, B., Partouvi, S. (2020). Data envelopment analysis and efficiency of firms: a goal programing approach, *Innovation management and operational strategies*, 1, 1–16.

[10] Fallah, R., Kouchaki Tajani, M., Maranjory, M., Alikhani, R. (2021). Comparison of Banks and Ranking of Bank Loans Types on Based of Efficiency with DEA in Iran, *Big Data and Computing Visions*, 1, 36–51.

[11] Gordy, MB. (2000). A Comparative Anatomy of Credit Risk Models. *Journal of Banking and Finance* 24, 119–149.

[12] Gordy, MB. (2003). A Risk-Factor Model Foundation for Ratings-Based bank Capital Rules. *Journal of Financial Intermediations,* 12, 199–232.

[13]Grzybowska, U., Karwański, M. (2014). Application of Data Envelopment Analysis to Calculating Probability of Default for High Rated Portfolio, *Acta Physica Polonica Series a*, 3, 14-17.

[14] Jouanin, J. , Riboulet, G., & Roncalli, T. (2004). *Financial Applications of Copula Functions. In Risk Measures for the 21st Century*. Edited by Giorgio P. Szegö. Hoboken: John Wiley & Sons.

[15] Kavčáková, M., Kočišová, K. (2020). Using Data Envelopment Analysis in Credit Risk Evaluation of ICT Companies, *Agris on-line Papers in Economics and Informatics*, 4, 47-60.

[16] Merton, RC. (1974). On the pricing of corporate debt: the risk structure of interest rates. *The Journal of Finance* 29(2), 449–470.

[17] Murthi, B.P.S., Choi, P. K.Y., Desai, P. (1997). Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach, *European Journal of Operational Research*, 98(2), 408–418.

[18] Portela, MC., Thanassoulis, E., Simpson, G. (2004). A directional distance approach to deal with negative data in DEA: An application to bank branches. *Operational Research Society*, 55(10), 1111-1121.

[19] Rasoulzadeh, M., Fallah, M. (2020). An Overview of Portfolio Optimization Using Fuzzy Data Envelopment Analysis Models, *Journal of Fuzzy Extension and Applications*, , 1 (3), 180-188.

[20] Vasicek, OA. (2015). Probability of loss on loan portfolio. *In Handbook Finance, Economics and Mathematics.* Chapter 17. Hoboken: Wiley & Sons.