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Loan Portfolio Performance Evaluation by Using Stochastic Recovery Rate

Shokouh Shahbeyk[1,*], **Shokoofeh Banihashemi**[1]

[1] Department of Statistics, Mathematics, and Computer Science, Allameh Tabataba'i University, Tehran, Iran; sh_shahbeyk@atu.ac.ir; shbanihashemi@atu.ac.ir.

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Abstract

One of the most critical aspects of credit risk management is determining the capital requirement to cover the credit risk in a bank loan portfolio. This paper discusses how the credit risk of a loan portfolio can be obtained by the stochastic recovery rate based on two approaches: beta distribution and short interest rates. The capital required to cover the credit risk is achieved through the Vasicek model. Also, the Black-Scholes Merton model for the European call option is utilized to quantify the Probability of Default (PD). Value at Risk (VaR) and Conditional Value at Risk (CVaR) are used as measures of risk to evaluate the level of risk obtained by the worst-case PD. A stochastic recovery rate calculates VaR related to the underlying intensity default. In addition, the intensity default process is assumed to be linear in the short-term interest rate, driven by a CIR process. The loan portfolio performance is evaluated by considering the relevant characteristics with the Data Envelopment Analysis (DEA) method. This study proposes the losses driven by the stochastic recovery rate and default probability. The empirical investigation uses the Black-Sholes-Merton model to measure the PD of eighth stocks from different industries of the Iran stock exchange market.

Keywords: Portfolio credit risk, Loan portfolio, Data envelopment analysis, Recovery rate, Default probability, Conditional value at risk.

1 | Introduction

Credit risk refers to the risk of default, non-payment, or non-adherence to contractual obligations by a borrower. Loans are the largest source of credit risk for most banks. Because of this, financial institutions must quantify credit risk in portfolios. On the other hand, an approach based on internal ratings, namely the

Corresponding Author: sh_shahbeyk@atu.ac.ir
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IRB-model proposed by Basel II and III in capital adequacy ratio [1], [2]. In the Internal Rating-Based (IRB) approach, the portfolio loss calculation and the portfolio's Value at Risk (VaR) are evaluated, in which the conditional PD is calculated under the Vasicek model. The Vasicek model is a popular one-factor model that derives the limiting form of portfolio loss. This model will allow the calculation of different risk measures, such as the Expected Loss (EL), the VaR, and the Conditional Value at Risk (CVaR). For this purpose, several risk measures based on the portfolio loss distributions will be presented.

VaR is a risk measure popular as an industry standard but not always sub-additive and convex. So, the concept of coherent risk measure that satisfies properties of translation invariance, homogeneity, subadditivity, and monotonicity is introduced. Conditional Value-at-Risk (CVaR) as an alternative and compatible risk measure is the weighted average of VaR and losses strictly greater than VaR for general distributions. The method of CVaR minimization has been employed for credit risk management of a portfolio of bonds [3] and portfolio hedging [4], [5]. VaR and CVaR can be obtained using Unexpected Loss (UL) and the EL in the Vasicek model [6]. Also, the credit risk of a loan portfolio can be calculated in terms of correlations, VaR, and CVaR using the Vasicek model and capital allocations obtained from the analytical framework. Afterward, outcomes are compared with the results found by implementing the IRB approach of Basel II and III [7]. As we know, the portfolio loss is a random variable in terms of Exposure at Default (EAD), recovery rate, and time until the obligor's default.

Moreover, this work considers the recovery rates stochastic, with beta distribution dependent on each other and the time until defaults [7]. Data Envelopment Analysis (DEA) is one approach to measure the efficiency of multiple Decision Making Units (DMUs) that is based on linear programming by considering the multiple inputs and multiple outputs and was introduced by [8]. DEA is originally designed for production theory as a quantitative, empirical, and nonparametric method. Comparison of banks and ranking of bank loan types have been described with DEA [9]. There have been a lot of theoretical and empirical models and approaches for decision-making on portfolio performance evaluation by DEA. One of these models is under the variance gamma process [10]. A DEA portfolio efficiency index was one of the seminal works of applying DEA in the context of portfolio performance evaluation [11]. Afterward, the classical DEA models have been widely applied to evaluate the performance of financial assets [12], [13]. By using DEA and goal programming, the most efficient companies have been listed on the Tehran Stock Exchange [14].

Moreover, when ambiguities are applied in the model, the results of fuzzy DEA in portfolio optimization will be much more accurate and consistent with the facts [15]. In addition, possibilities of diagnosing credit risk through DEA and designing an appropriate model for diagnosing credit risk, which can be used in different sectors of the national economy, have been explored [16]. Also, DEA is applied to calculating the Probability of Default (PD) for high-rated portfolios [17]. Therefore, we apply the DEA method to find the inefficiency of the loan portfolio or each stock in the loan portfolio. In the losses, investment has a negative return rate over time, so an approach based on the directional distance function as the Range Directional Measure (RDM) model was presented [18]. This paper uses two methods to consider the stochastic recovery rate as a random variable. One of them is based on beta distribution. The other stochastic recovery rate is related to the underlying intensity default. Also, the intensity default process is assumed to be linear in the short-term interest rate driven by a CIR process. Also, PD is obtained by the Black-Sholes-Merton model [19]. The RDM inspires our model to deal with negative values for performance evaluation in loan portfolios, with risk measures as the input and mean return as the output. Risk measures can be obtained by the worst-case PD that can be obtained by using the Vasicek model and stochastic recovery rate. In this study, the risk measures VaR and CVaR are applied.

We have organized the paper as follows. Section 2 is devoted to the preliminary concepts of coherent risk measure, VaR, CVaR, loss given default, EAD, recovery rate, and the Vasicek model. Section 3 introduces the Credit Portfolio performance model for loans based on RDM. Section 4 is devoted to the empirical example of the Iran Stock Exchange market for calculating the PD. Section 5 is concluded.

2 | Preliminary

This section is devoted to some preliminaries which are needed in the sequel.

2.1 | Risk Measure

We present the properties of a coherent risk measure and the definitions of VaR and CVaR.

Definition 1. Assume (Ω, F, P) to be the probability space and $I(\Omega, F)$ to be the set of random variables of one dimensional on the space. The function $\rho: I(\Omega, F) \to R$ is a coherent risk measure whenever it satisfies the following axioms for random variables $X, Y \in I(\Omega, F)$:

I. Monotonicity: if $X \leq Y$, then $\rho(Y) \leq \rho(X)$.

II. Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

III. Translation invariance: for all $\alpha \in R$, $\rho(X + \alpha) = \rho(X) - \alpha$.

IV. Positive homogeneity: for all $\alpha \geq 0$, $\rho(\alpha X) = \alpha\rho(X)$.

VaR is a risk measure that is a benchmark standard for firm-wide measures of risk.

Definition 2. For a given time horizon and confidence level $\beta \in (0,1)$, we consider a decision vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ in $\mathbb{R}^n$ which represents the position of n financial assets in a portfolio.

The distribution of the vector of asset returns $Y = (Y^1, Y^2, \dots, Y^n)^T$ constitutes various returns, and its cumulative loss function is shown by $\psi(\lambda, \Gamma)$. In other words, the return on a portfolio is the weighted average of returns in the portfolio, so the loss function is the negative of this sum and is denoted by

$$f(\lambda, Y) = -(\lambda_1 Y^1 + \lambda_2 Y^2 + \cdots + \lambda_n Y^n) = -\lambda^T Y.$$

The VaR of a portfolio is the loss in the portfolio's market value over the horizon time that is exceeded with probability $1 - \beta$. Therefore, VaR is defined as follows:

$$\mathrm{VaR}_\beta(\lambda) = \inf\{\lambda: \psi(\lambda, \Gamma) \geq \beta\}.$$

Conditional Value at Risk

An alternative risk measure to VaR is CVaR, also known as an expected shortfall. CVaR is the conditional expectation of the losses exceeding VaR, and it is specified as the

$$\mathrm{CVaR}_\beta(\lambda) = E\big[f(\lambda, Y): f(\lambda, Y) \geq \mathrm{VaR}_\beta(\lambda)\big].$$

It is shown that CVaR can be reduced to a linear programming problem, so we have the following approximation function:

$$\tilde{F}_\beta(\lambda, \Gamma) = \Gamma + \frac{1}{(1-\beta)Q}\sum_{q=1}^{Q}\left(f(\lambda, Y_q) - \Gamma\right)^+ = \Gamma + \frac{1}{(1-\beta)Q}\sum_{q=1}^{Q}\left(-\lambda^T Y_q - \Gamma\right)^+,$$

where $(Z)^+ = \max\{Z, 0\}$ and Q represents the scenarios of log-returns of assets $Y_1, Y_2, \dots, Y_Q$, where each element $Y_q (q = 1,2, \dots, Q)$ is a vector in $\mathbb{R}^n$ and Γ is in $\mathbb{R}$. Therefore, $\mathrm{CVaR}_\beta(\lambda)$ has an equivalent definition as follows:

$$\min \mathrm{CVaR}_\beta(\lambda) = \min_{\Gamma \in \mathbb{R}} \tilde{F}_\beta(\lambda, \Gamma).$$

2.2 | Credit Risk

This section introduces the components of EL, the Vasicek model, and the Merton model.

Definition 3.

I. Loss-Given Default (LGD) is defined as the number of funds that are lost by a bank when a borrower defaults on a loan.

II. EAD is the predicted amount of loss a bank may face in the event of and at the time of, the borrower's default.

III. The recovery Rate (R) is defined by $1 - \text{LGD}$.

Banks can design internal models based on the IRB approach to calculate capital reserves in light of common characteristics identified by academic and industry studies. Apparently, LGD is not an exception in this regard.

2.3 | Vasicek Model

We state the credit risk model developed by Vasicek. This model has been used to measure the credit risk of a portfolio because it is the basis for the Basel regulatory capital requirements and is also widely used in the financial industry [6]. In this model, the value of A_i, is driven by its macroeconomic factor, η, and an idiosyncratic independent term, ε_i, in the following form:

$$A_i = \sqrt{r}\eta + \sqrt{1-r}\varepsilon_i,$$

where both η and ε_i follow independent standard Gaussian distributions. r represents correlations between A_i and A_j.

$$\rho(A_i, A_j) = E\left[(\sqrt{r}\eta + \sqrt{1-r}\varepsilon_i)(\sqrt{r}\eta + \sqrt{1-r}\varepsilon_j\right] = r.$$

By the assumption of a systematic factor η probability of conditional default is equal to the

$$PD_{c,i|\eta} = p(A_i \le L_i|\eta) = \text{prob}(\sqrt{1-r}\varepsilon_i \le L_i - \sqrt{r}\eta\,|\eta) = \text{prob}(\varepsilon_i \le \frac{L_i - \sqrt{r}\eta}{\sqrt{1-r}}|\eta) = \phi_N\left(\frac{L_i - \sqrt{r}\eta}{\sqrt{1-r}}\right).$$

And we have

$$PD_{c,i|\eta} = \phi_N\left(\frac{\phi_N^{-1}(PD_i) - \sqrt{r}\eta}{\sqrt{1-r}}\right).$$

We suppose that $\eta = \phi_N^{-1}(0.999)$. Then worst case PD is obtained as follows:

$$WCPD = PD_{c,i|\eta}(1-\alpha) = \phi_N\left(\frac{\phi_N^{-1}(PD_i) - \sqrt{r}\phi_N^{-1}(1-\alpha)}{\sqrt{1-r}}\right).$$

Therefore, we can conclude that for a large portfolio of loans with the same size and risk, the loss is 99% less than the following:

$$N * EAD * (1-R) * WCPD,$$

which N is the number of loans, EAD and R are EAD and recovery rates, respectively. The Vasicek model assumes that a Gaussian process drives the default, and this could generate a certain risk underestimation.

2.4 | Merton Model

Robert Merton developed a model using the European call option of BSM [19]. In this approach, Merton's proposed structural model is used to model the probability of credit risk default. Structural models for credit risk modeling are based on the borrower company's balance sheet structure. The balance sheet, on the one hand, represents the resources, and, on the other hand, represents the expenditures that are provided from the resources. Resources include liabilities and equity, and expenditures on the balance sheet are the same as assets. A firm is considered a risk asset portfolio consisting of shares owned by its shareholders and debts related to lenders. It is assumed that the firm provides both equity (E) and debt (D), and then by the main accounting equation, the value of the firm (V) is $D + E$. In this model, we have two possible scenarios:

I. The firm will default (the firm cannot repay its debt at maturity (T)); in this case, the debt holders have a higher priority than their stockholders to return on their investments. They receive the value of the remaining assets and incur a loss of $D - V$ while the shareholders will receive nothing.

II. The firm will not default; in this case, shareholders will receive more $V - D$.

Considering both conditions, the value of the equity at maturity T is as follows:

$$\max(V - D, 0).$$

We can estimate the value of the equity by using the Black-Scholes formula:

$$E = V * N(d_1) - D * e^{-rT} * N(d_2), \quad (1)$$

where

$$d_1 = \frac{\ln\left(\frac{V}{D}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\ln\left(\frac{V}{D}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

r is the free risk rate interest, σ is the volatility of the firm's value, and N is the cumulative standard function for a standard normal distribution.

If the volatility σ replace by the volatility of the firm's value σ_V, then the distance to default is given by

$$d_2 = \frac{\ln\left(\frac{V}{D}\right)+\left(r-\frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}},$$

where μ_V is the expected rate of return of the firm's assets, and the expected growth of assets is equal to $\left(\mu_V - \frac{\sigma_V{}^2}{2}\right)$.

After evaluating the distance to the default, we can measure the PD. The PD under the risk-neutral measure as per the Black Scholes Merton model is given by

$$PD = \phi_N(-d_2) = \phi_N\left(-\frac{\ln\left(\frac{V}{D}\right)+\left(r-\frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}}\right). \quad (2)$$

Eq. (2) is the PD with three unknowns V, μ_V, and σ_V. The value of the firm V is equal to the sum of the debt and the equity of the firm, so the debt D is known, and we need to find equity E. The equity value E is a continuous time stochastic process that is the Weiner process.

$$dE = \mu_E E dt + \sigma_E E dW,$$

where dW is the continuous-time stochastic process, μ_E is the expected continuously compounded return on E, and σ_E is the volatility of the equity value. By Ito's lemma

$$\sigma_E E = \sigma_V V \frac{dE}{dV} = \sigma_V \phi_N(d_1). \quad (3)$$

Eqs. (1) and *(3)* have two unknowns one is σ_V and another is V. We can easily estimate the two parameters by solving *Eqs. (1)* and *(3)*, and $\mu_V = \frac{V_t - V_{t-1}}{V_{t-1}}$ [20].

2.5 | Range Directional Measure Model

In the conventional DEA models, each DMUj $(j = 1, \dots, n)$ is specified by a pair of non-negative input and output vectors $(x_{ij}, y_{rj}) \in \mathbb{R}_+^{(m+s)}$ in which inputs, $x_{ij}(i = 1, \dots, m)$, are utilized to produce outputs: $y_{rj}(r = 1, \dots, s)$. These models cannot be used for DMUs with negative and positive inputs and/or outputs. Portela et al. [18] considered a DEA model that can be applied in cases where input/output data take positive and negative values. Then, the generic directional distance model can be represented as

$$\begin{aligned} \max \quad & \alpha \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \alpha R_{ro}, \quad r = 1,2,\dots,s, \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ro} - \alpha R_{io}, \quad i = 1,2,\dots,m, \\ & \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j, \alpha, R_{ro}, R_{io} \geq 0, \end{aligned}$$

where $o \in \{1,2,\dots,n\}$ is the unit under assessment. For negative values of the given data set, an ideal point is defined as $I = (\max_j y_{rj},\ r = 1,2,\dots,s,\ \min_j x_{ij},\ i = 1,2,\dots m)$, and the vectors R_{ro} and R_{io} as the range of possible improvement DMU_o are defined as

$$R_{io} = x_{io} - \min_j\{x_{ij}\},\ i = 1,2,\dots,m,$$

$$R_{ro} = \max_j\{y_{rj}\} - y_{ro},\ r = 1,2,\dots,s.$$

3 | DEA-Based Loan Portfolio Model

We assume a loan portfolio with N obligors and a time horizon equal to the longest maturity among the credit assets in the portfolio. The random variable L, representing the portfolio loss, is defined following the notation used in Jouanin et al. [21]:

$$\begin{aligned} L &= \sum_i EAD_i * LGD_i * 1_{\{D\}}, \qquad 1_{\{D\}} = \begin{cases} 1 & PD \\ 0 & 1 - PD' \end{cases} \\ EL &= \sum_i EAD_i * LGD_i * PD_i = \sum_i EAD_i * (1 - R_i) * PD_i. \end{aligned} \tag{4}$$

Unlike EL, the UL is not an aggregate of individual loss but depends on loss correlations between all loans in the portfolio. The standard deviation of the loss variable usually measures the deviation of losses from the EL. The portfolio standard deviation of credit losses can be decomposed into the contribution from each credit facility. The capital requirement to cover credit risk in a bank loan portfolio (UL) can be obtained with the following relationship:

$$\begin{aligned} UL &= VaR - E = EAD * LGD * \left(\phi_N\left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}}\right) - PD\right) \\ &= EAD * (1 - R_i) * \left(\phi_N\left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}}\right) - PD\right), \end{aligned} \tag{5}$$

where

$$VaR = EAD * (1 - R_i) * \phi_N\left(\frac{\phi_N^{-1}(PD_i) - \sqrt{\rho}\phi_N^{-1}(.999)}{\sqrt{1-\rho}}\right). \tag{6}$$

VaR is considered a risk measure. The recovery rate R_i may be assumed to be deterministic or stochastic with mean m_i and standard deviation s_i. In this paper, the recovery rate R_i can be obtained by two stochastic approaches. One of them is related to the underlying intensity default $\lambda(t)$ via

$$R_i(t) = a_R + b_R e^{-\lambda(t)},$$

with $a_R \geq 0$, $b_R \geq 0$ and $0 \leq a_R + b_R \leq 1$. The intensity default process is assumed to be linear in the short-term interest rate, which is driven by a CIR process, i.e.

$$\lambda(t) + \Lambda_0 + \Lambda_1 r(t),$$

$$dr(t) = (\theta_r - a_r r(t))dt + \sigma_r\sqrt{r(t)}dW(t), \quad r(0) = 0,$$

with $\Lambda_0 \geq 0$, $r_0 \geq 0$ [22].

And the other is related to the most common distributional form of R_i that is the Beta (a_i, b_i) distribution, with the parameters a_i and b_i estimated by the method of moments, knowing the values of m_i and s_i analytically [7]:

$$a_i = \frac{m_i^2(1-m_i)}{s_i^2} - m_i, \quad b_i = \frac{m_i^2(1-m_i)}{s_i^2} - (1-m_i).$$

It is well known that asset correlations play a critical role in measuring portfolio credit risk and in determining both economic and regulatory capital. In a credit portfolio, having many components does not assure good diversification because the components may be highly correlated, and the default of one may lead to the default of the rest of the portfolio. This concept is called concentration risk in credit risk management.

We introduce a DEA model based on the assumption that loss follows *Eq. (6)* for loan portfolio performance evaluation. Since losses can be negative, we apply an RDM-based model on the appropriate underlying distribution for calculating efficiency. Also, risk VaR and EL are considered the only input and output, respectively.

Let's assume $L^1, L^2, \dots, L^n$ be the loss of the n loans in the loan portfolio. For a specific loan L^o where $o \in \{1,2,\dots,n\}$ and regarding the negative loss value, the vector $g^T = \left(R_{VaR_\beta^o}, R_{E(L^o)}\right)$, where

$$R_{VaR_\beta^o} = \left(VaR_\beta^0 - \min\left(VaR_\beta^j : j = 1, \dots, n\right)\right),$$
$$R_{E(L^o)} = \left(\max\left(E(L^j) : j = 1, \dots, n\right) - E(L^0)\right).$$

This vector is a range of possible input and output improvements. The VaR_β^o is the value of risk and $E(L^o)$ is the EL of the loan under evaluation. According to *Eq. (4)*, we solve the following linear model:

$$\begin{aligned} \max \quad & \alpha \\ \text{s.t} \quad & E\left(L(\lambda)\right) \geq E(L^o) + \alpha R_{E(L^o)}, \\ & VaR_\beta\left(L(\lambda)\right) \leq VaR_\beta^o - \alpha R_{VaR_\beta^o}, \\ & e^T\lambda = 1, \qquad \lambda \geq 0, \alpha \geq 0. \end{aligned}$$

The optimal value of α, which is shown by α^*, indicates the distance between the loan under evaluation and the efficient frontier. In other words, α^* represents the inefficiency score of the loan under evaluation, $1-\alpha^*$ is the amount of efficiency. The vector $\lambda^T = (\lambda_1, \lambda_2, \dots, \lambda_n)$ is the proportions of the initial capital of n loan in a portfolio, and e denotes the n-dimensional vector of ones. The above model is obtained under the normal distribution. For future work, proper underlying distribution should be considered. Also, we can apply CVaR as another risk measure.

4 | Application

Since the Merton model has been used as a standard for estimating the PD of firms and banks, we use the Black-Scholes-Merton model described in Section 2.4 in this section. In this paper, according to the need of our country to conduct financial research in the field of banking, we have used data related to companies listed on the Tehran Stock Exchange that can obtain loans from a hypothetical bank. This research includes the symbol firms listed on the stock exchange operating for at least 5 years. Their financial statements have been published, and have reached an activity capable of receiving bank facilities such as loans from a hypothetical bank. In selecting the statistical sample, the selected companies have a fiscal year ending on 12/29 or 12/30. According to this procedure, 7 firms have been selected. Information on the value of equity and its variability, as well as the amount of debt to be paid by next year, has been collected through the official website Codal.ir software.

It should be noted that in these calculations, the loan maturity for all companies is equal to $T = 1$. Also, the risk-free rate of return is considered to be $r = 0.15$. Merton model calculations are coded with Maple software. *Table 1* collected data for parameters E, D, and σ_E are reported for each firm. A smaller difference between a company's asset and debt values would lead to a higher probability of PD. In this example, *Eqs. (1)* and *(3)* are used to estimate the PD. *Table 1* presents the results of firms' value, volatility of firms' value, and estimated PD with the Merton model, respectively. Looking at *Table 1*, we find that in stocks such as GEMS and BENN, the difference between the value of the assets and the value of the companies' debts are smaller, their PD is more, and BENN is near one.

Table 1. List of symbols, equity, volatility of equity, debt, the value of firms, volatility of the value of firms, and estimated PD with the Merton model (Values in a million rials).

Symbol	PRKT	APPE	EPRS	GEMS	ATIR	BENN	GARN
Name	Behpardakht Mellat	Asan Pardakht Pers	Parsian E-commerce	General Mechanic	Iran Tractor F	Behnoush Iran	Padide Shimi Gharn
E	7346040	8153293	10234784	27354086	13890539	1316005	5423310
σ_E	0.15	0.19	0.26	0.97	0.47	0.156	0.49
D	6371888	6180984	12758244	19806948	3424171	5920235	12435557
V	1216424.79	1316424.79	2032849.57	4381271079	1558212.99	6411598.48	6493649.652
σ_V	0.086318	0.11546	0.127899	0.698499825	0.39211808	0.032019	0.4092393941
PD	$1.287*10^{-13}$	$4.892*10^{-10}$	0.000426	0.460034	0.000195	0.867999	0.000388

5 | Conclusion

DEA is increasingly used in various areas of finance, health, agriculture, and others for performance evaluation. This paper dealt with its use in the performance evaluation of the credit risk of the loan portfolio. To the best of our knowledge, PD is an essential parameter in the analysis of credit risk in the finance world. In addition, it is one of the components of credit risk that the Merton model can obtain. Also, the stochastic recovery rate has been obtained by two approaches. One approach is Beta distribution-based, and the other is underlying intensity default. Therefore, the expected UL and the capital requirement to cover credit risk in the loan portfolio were obtained using PD and stochastic recovery rate. This paper focuses on the DEA method as a nonparametric approach that relates the produced outputs to assigned inputs and then determines an efficiency score. This score can be interpreted as a performance measurement in investment analysis. In this regard, we introduced a model in the DEA framework for loan portfolios inspired by the RDM model where risk VaR and EL are considered as the only input and output, respectively. VaR and CVaR can be obtained using UL and the EL in the Vasicek model. Because of the PD, stochastic recovery rate, VaR, and CVaR using the Vasicek and RDM models, the bank management can construct profitable loan portfolios. In this work, PD is calculated with the Merton model.

Regarding this purpose, we have applied data related to companies listed on the Tehran Stock Exchange that can obtain loans from a bank. As seen in the example, a smaller difference between a company's asset value and debt value would lead to a higher probability of PD. Other approaches for default probability and the recovery rate can be used for future works.

Conflicts of Interest

All co-authors have seen and agreed with the manuscript's contents, and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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