E-ISSN: 2676-6167 | P-ISSN: 2538-5100

Journal of Applied Research on Industrial Engineering

www.journal-aprie.com

J. Appl. Res. Ind. Eng. Vol. x, No. x (xx) x–x.

Development of a multi-objective model for the routing problem of vehicles carrying valuable commodity under route risk conditions (Case study of Shahr Bank)

Mohammad, Shafiekhani; Alireza, Rashidi Komijan; Hassan, Javanshir

Citation:

Shafiekhani, M., & Rashidi Komijan, A., & *Javanshir, H.* (2022). Development of a multi-objective model for the routing problem of vehicles carrying valuable commodity under route risk conditions (Case study of Shahr Bank). *Journal of applied research on industrial engineering, Volume* (Issue), PP.

Received: Reviewed: Revised: Accept:

Abstract

In this paper, a new type of vehicle routing problem in the valuable commodity transportation industry is modeled considering the route risk constraint. The proposed model has two objective functions for risk minimization. In the first objective function, three concepts are presented, which are: 1) the vehicle does not travel long distances in the first three moves because it carries more money, 2) to serve the same branch on two consecutive days, at the same time 3) The bow should not be repeated in two consecutive days. This reduces the possibility of determining a fixed pattern for the service and increases the security of the service. In the second objective function, the risk is a function of the amount of money, the probability of theft and the probability of its success. Two different meta-heuristic algorithms have been used to solve the proposed model, including the genetic algorithm and an ant colony optimization algorithm. In computational testing, the best parameter settings are determined for each component and the resulting configurations are compared in the best possible settings. The validity of the answers of the algorithms has been investigated by generating different problems in various dimensions and using the real information of Shahr Bank. The results show that the genetic algorithm provides better results compared to the ant colony algorithm, with an average of 0.93% and a maximum difference of 1.87% with the optimal solution.

Keywords: Risk, valuable commodity, vehicle routing problem with a time window, genetic algorithm, ant colony optimization algorithm.

, Author: xxx

E-mail address: xxx

https://doi.org/10.22105/jarie.2023.391399.1540

1 Introduction

Vehicle Routing Problem with Time Windows (VRPTW) is one widely used Vehicle Routing Problem (VRP) that optimizes product distribution from the manufacturing location or distribution center to the market [5]. Each customer must receive service within a certain time interval in this problem. The VRPTW aims to optimize routes between the warehouse and customers and minimize waiting time and delay. However, contemporary models consider the complexities of the real world, including the cost of transportation, travel time dependence on the route traffic, the time window in the collection (pick up cash) and delivery, and input data, such as a demand rate that changes dynamically through time. All mentioned properties are used to design an appropriate routing strategy [6, 7, 8].

Hence, the design of a routing strategy based on the concepts and sophistication in today's world has become a critical case regarding the risk problem and reducing probable risks in distrusting products, hazardous, valuable commodities, and physical money. In other words, security objectives are superior to economic ones, so improper planning for the safe and secure distribution of commodities increases the time and operational costs of transportation in the system and causes human damage and loss for staff, customers, and security forces. Accordingly, CIT vehicle routing indicates that physical transportation of cash, coins, and valuable objects from one place is crucial. The nature of portable items always puts banks and money-carrying companies at real risk, such as robbery and armed attack. Hence, the present study proposes a CIT operation with minimum risk, provided the process is completed within a certain period.

2 Research Background

The present study examined vehicle routing problems with time windows (VRPTW). VRP is one of the important supply chain problems that address commodity distribution. Some studies have been conducted on this subject, such as Nasr et al., who studied risk-based vehicle routing models [1]. Soriano et al. examined the cash vehicle routing problem by consideration of customer visit time diversification based on a multigraph. This research used neighborhood search, which exploits linear penalty function for insertion evaluations, efficient local searches, and adaptive destruction rate to balance short routes and security [9]. Ghanbarpoor and Zandieh introduced a multi-objective evolutionary model based on the new game theory to maximize cash transit security and minimize transportation costs. They generated a bi-objective routing problem with time windows that can minimize the risk of cash transit and the route traveled by the CIT vehicle. The probability of a thieves' ambush is measured using game theory to better estimate robbery risk.

Moreover, the probability of successful robbery is estimated through multicriteria decision-making techniques [2]. Hoogboom et al. addressed the time misalignment of reaching money centers through multiple time windows. This study solved the proposed algorithm and four penalty methods through Tabu Search [3]. LucaTalarico et al. introduced risk as a proportion of portable money and distance between demand points and solved the routing problem is physical money transit security to minimize cost and increase security through metaheuristic techniques and local search [11, 12, 13, 14]. Chang Yu Yan et al. presented a different view on increased level unpredictability. They used the route-time network technique for cash transit routing to minimize cost and increase cash transit security [18].

2.1 Risk measurement approaches in the transportation network

According to studies conducted by Chen et al., Parsafard et al., Toumazis and Kwon, Androutsopoulos and Zografos, and Kazantzi, the risk is measured based on the Equation (1) in failure mode and effects analysis (FMEA), which indicates the probability in effect rate [16, 19, 20].

Equation (1) $R_{ij} = P_{ij} C_{ij}$

If the risk arises from cash transit, R_{ij} equals the risk of armed attack and money robbery in node ij, and if the risk is related to transiting dangerous substances, then R_{ij} equals the risk of dangerous substances being released into nature and damaging the environment and population around the incident in node ij. The P_{ij} represents the probability of an incident, which is the probability of an armed attack in node ij in a cash transit case and the probability of an accident and release of dangerous substances in node ij in dangerous substances transportation case. The term C_{ij} indicates the effect rate of an incident, which is money robbery in node ij in a cash transit case, while representing the population rate exposed to the incident in node ij when dangerous substances are transported.

Bola et al. presented equation (2) to calculate the risk of dangerous substances. This equation is the same as formula 1, while Equation (2) measures the effect of different parameters on the risk. The term $R(\sigma)^r$ represents the risk of transporting dangerous substances after the customer visits σ^r in the route r and depends on the accident rate $TTAR^r$, probability of dangerous substances released after the accidents $P_{release}$, explosiveness, and flammability of dangerous substances (α, β), the volume of dangerous substances transported by vehicle y_{ij}^k, length of arc al_{ij}, and the accident's consequences (PD_{ij}) [21].

Equation (2) $$R(\sigma)^r = TTAR^r \times P_{release} \times \beta \times \sum (i,j) \in \sigma^{r(y_{ij}^k)^{\alpha(al_{ij}) \times PD_{ij}}}$$

LucaTalarico et al. designed Equation (3) to measure the CIT risk. This equation is similar to Equation (1), while the probability of successful robbery v_{ij} has been multiplied by it to make it more precise and accurate. The term D_i^r shows the cash at the risk of robbery in the route r, and p_{ij} represents the probability of an armed attack in the node ij.

Equation (3) $R_j^r = \sum_{(i,j)\in r} p_{ij} v_{ij} D_i^r$

Talarico et al. (2015) replaced the probability of accident p_{ij} with the distance between two nodes c_{ij}. They made this change for two reasons: 1) node length is a given parameter in each vehicle routing and is simply accessible. 2) no information exists about the accident probability in the node [11, 12, 13, 14].

Bozkaya et al. introduced a new model for CIT risk measurement using two factors social and economic level of the node and the node's use rate.

Equation (4) $R_{ijkd} = w_U c_U UBRI_{ijkd} + w_S c_S SERI_{ijkd}$

R_{ijkd} Represents the risk of the node in the day d with vehicle k; $UBRI_{ijkd}$ shows the risk index based on the node used in the node ij in the day d with vehicle k. The term $SERI_{ijkd}$ indicates the social-economic risk index of node ij with vehicle k; w_U and w_S are weights of risk elements and c_U and c_S show the cost of risk elements. According to the mentioned points, some risk equations are inserted into the objective function, while others are entered to constraints of vehicle routing problems under the risk conditions [4].

3 Research gaps and innovations

According to the literature review, the models used in the research background have focused on risk reduction and security increase. In contrast, the present study has proposed some solutions and points to alleviate risk in the frame of a formulated mathematical model that was not considered in previous studies. The initiative aspect of this study is the consideration of new concepts and relations to improve safety and reduce the risk of providing service for branches. In other words, the present study uses a bi-objective function to decrease route risk. Three concepts used in the first objective function have not been applied in previous studies:

The vehicle should not travel long routes and arcs in the first three moves when more cash is transported;

A branch must not receive a similar service at the same time in two consecutive days;

An arc must not be repeated in two consecutive days if possible.

The second and third options are considered to avoid the same time and service sequence for branches on different days. This case decreases the probability of a fixed pattern providing branches with services while increasing service security.

Moreover, the second objective function considers money robbery in the transit process. This function indicates the probability of thieves' attacks and the amount of stolen money. Since all attacks of thieves may not be successful, a probability is considered for it, and the risk function is multiplied by the successful robbery.

4 Statement of Problem

This problem aims to measure optimal routes between treasury and demand points (bank branches) so that objective functions (total risk in service route) are minimized. Therefore, the problem assumptions are explained as follows:

- The model of the problem is single-treasury (depot), assuming that CIT vehicles must be settled in the treasury location to provide service in all demand routes within a certain time and return to the treasury at the end;
- Each point i is considered as a specific amount of predetermined cash served by a vehicle;
- Cash volume must not exceed the vehicle roof;
- Cash-in-transit vehicles are homogenous (15 vehicles), and the capacity of each vehicle may indicate the maximum amount of cash or valuable commodity the vehicle is allowed to transit in the frame of monetary unit (15 billion Rls) based on the vehicle features. The primary constraint is not the amount of cash-in-transit due to the low volume of cash or physical money, while the constraint is the risk that may occur in the route;
- CIT vehicle's stop time in demand points for service providing is 25 minutes, and the maximum time for the first three moves of the vehicle must not exceed 30min;
- The minimum time interval in which a node is visited in two consecutive days is 15min;
- The allowed time limit to reach nodes equals 360min throughout the day;

The minimum distance between the treasury (origin) and branches (destination) is almost 1km, and the maximum distance between the treasury and branches is almost 30km.

5 Method

The present study aims to provide a model for branches' cash-in-transit operations with the lowest risk, provided that the process is ended within a certain interval. Therefore, the proposed model was bi-objective and included minimizing the route risk. The model was formulated as a vehicle routing problem by consideration of simultaneous pickup and delivery through a specific time window. Moreover, metaheuristic GA and ACO algorithms were used to solve the problem. The relevant data were randomly selected for all Shahr Bank branches (n=135) in Tehran, Iran. to validate the proposed metaheuristic algorithms, parameters were adjusted, and initial solutions were created; then, some trial problems in different sizes were randomly generated. The results of these algorithms were compared in terms of their solution quality and computation time. Furthermore, the proposed model displays periods based on the travel time, service time, minimum time interval, and maximum time limit based on the minute to reach the node on different days.

6 Modelling

This study presents a mixed integer model for single-treasury routing problems considering simultaneous delivery and pickup and observing the time window for cash transit.

6.1 Indexes and sets

N: total number of nodes

i,j: node index

D: total number of nods of demand points (branches)

O: index of origin node (treasury)

K: total number of vehicles

k: vehicle index

R: a set that includes all move counters

T: total number of planning horizon days
t,t': day index

6.2 Parameters

$time_{ij}$: the time spent traveling from i to j
UL_i: time of providing service for node i
Dem_{it}: demand of node i in day t for the cash it must receive
$Pick_{it}$: demand of node i in day t for the cash it must deliver
α: minimum time interval when a node is visited in two consecutive days
MAX: maximum cash-in-transit transported by each vehicle
α_i: maximum time limit to reach node i
p_{ij}: the probability of money robbery in the distance traveled from node i to j
d_{ij}: the amount of robbed money in the distance traveled from node i to j
v_{ij}: the probability of successful robbery in the distance traveled from node i to j
M: the large number

6.3 Decision variables

X_{ijkrt}: binary variable that determines whether vehicle k travels the distance from i to j in its move r and day t
L_{ikrt}: the amount of money added in vehicle k when reaches node i in its move r and day t
P_{ikrt}: the amount of money collected in vehicle k when reaches node i in its move r and day
S_{ikrt}: the time when vehicle k reaches node i in its move r and day
$w'_{ijtt'}$, $w_{ijtt'}$: ideal variable in which one route from node i to j is not repeated in two consecutive days
$f''_{itt'}$, $f_{itt'}$: the ideal variable in which the time interval of reaching a node is not the same in two consecutive days
h'_{ijkr}, h_{ijkrt}: the ideal variable in which a vehicle moves in a short route in the first three moves when carrying much money.

7 Model of Problem

7.1 Risk calculation

To measure the probability of successful robbery (v_{ij}) in the objective function, a fixed risk is not considered for all nodes but is measured by using SAW[1] , the method from the perspective of thieves. This method calculates the utility of nodes based on the weight of criteria (w_c) and the superiority degree of nodes based on different criteria (r_{ij}^c). The equation below indicates how the probability of successful robbery in each node is calculated:

Equation (5) $v_{ij} = \sum_c w_c r_{ij}^c$

Decision-making criteria (c) include traffic and congestion, number of cameras, escape route, main or side street, one-way or two-way street, and street width. To measure robbery probability, the probability of selecting different strategies by thief equals the probability of armed robbery in each node p_{ij}. To calculate this probability, the linear programming model is written from the perspective of the attacker:

Equation (6)

$$Min = \sum_{i=0}^{N} \sum_{\substack{j=0 \\ j \neq i}}^{N} p_{ij}$$

[1] Simple Additive Weighting Method

Subject to:

$$\sum_{k}^{K}\sum_{i=0}^{N}\sum_{\substack{j=0\\j\neq i}}^{N} d_{ij}^{k(r)} p_{ij} \geq 1 \quad \forall r \in R$$

This equation represents the objective function and minimizes the probability of robbery in each node. The constraint term maximizes the thief's gain.

7.2 Mathematical Modeling

(1) $\sum_{\substack{i\in N\\i\neq j}}\sum_{k\in K}\sum_{r\in R} X_{ijkrt} = 1 \qquad \forall j \in D\,, t \in T$

(2) $\sum_{\substack{i\in N\\i\neq j}} X_{jik(r+1)t} \leq \sum_{\substack{i\in N\\i\neq j}} X_{ijkrt} \qquad \forall j \in N, k \in K, t \in T, r \in R$

(3) $\sum_{i\in N}\sum_{\substack{j\in N\\j\neq i}} X_{ijkrt} \leq 1 \qquad \forall k \in K, r \in R, t \in T$

(4) $\sum_{r\in R}\sum_{j\in D} X_{ijkrt} \leq 1 \qquad \forall i \in o, k \in K, t \in T$

(5) $\sum_{i\in D}\sum_{\substack{j\in N\\j\neq i}}\sum_{k\in K} X_{ijkrt} \leq 0 \qquad \forall r = 1\,, t \in T$

(6) $L_{jkrt} \leq L_{ik(r-1)t} - Dem_{it} + M\left(1 - X_{ijkrt}\right) \quad \forall i,j \in N: i \neq j, i \neq o\,, k \in K, r \in R\,, t \in T$

(7) $P_{jkrt} \geq P_{ik(r-1)t} + Pick_{it} - M\left(1 - X_{ijkrt}\right) \quad \forall i,j \in N: i \neq j, i \neq o\,, k \in K, r \in R\,, t \in T$

(8) $S_{jkrt} \geq S_{ik(r-1)t} + time_{ij} + UL_i - M\left(1 - X_{ijkrt}\right) \; \forall i,j \in N: i \neq j, j \neq o\,, k \in K, t \in T, r \in R$

(9) $time_{ij} x_{ijkrt} \leq LB \qquad \forall i,j \in N: i \neq j\,, k \in K, t \in T, r \leq 3$

(10) $\left|\sum_{k\in K}\sum_{r\in R} S_{ikrt} - \sum_{k\in K}\sum_{r\in R} S_{ikr(t+1)}\right| \geq \propto \qquad \forall i \in D: t \in T$

(11) $\sum_{r\in R}\sum_{k\in K} x_{ijkrt} + \sum_{r\in R}\sum_{k\in K} x_{ijkrt'} + w_{ijtt'} - w'_{ijtt'} = 1 \quad \forall i,j \in N: i \neq j\,, t, t' \in T: t' = t+1$

(12) $\sum_{r\in R} L_{ikrt} \leq MAX \qquad i \in D, k \in K\,, t \in T$

(13) $L_{jkrt} + P_{jkrt} \leq M \sum_{\substack{i\in N\\i\neq j}} x_{ijkrt} \qquad \forall j \in D, k \in K, r \in R\,, t \in T$

(14) $S_{jkrt} \leq M \sum_{\substack{i\in N\\i\neq j}} x_{ijkrt} \qquad \forall j \in D, k \in K, r \in R\,, t \in T$

(15) $\sum_{k\in K}\sum_{r\in R} S_{ikrt} \leq a_i \qquad t \in T, i \in D$

(16) Min $z_1 = \sum_{i\in N}\sum_{\substack{j\in N\\j\neq i}}\sum_{k\in K}\sum_{r=1}^{3}\sum_{t\in T} h'_{ijkrt} + \sum_{i\in D}\sum_{t\in T}\sum_{t'=t+1} f_{itt'} + \sum_{i\in N}\sum_{\substack{j\in N\\j\neq i}}\sum_{t\in T}\sum_{t'=t+1} w'_{ijtt'}$

(17) Min $z_1 = p_{ij}\, v_{ij}\, d_{ij}$

Constraint (1) ensures that each branch receives service from only one vehicle. Constraint (2) indicates that a vehicle makes the move r if it makes move r+1. Constraint (3) ensures that each vehicle travels at most one route (distance between two nodes) in each move. Constraint (4) ensures that each vehicle exits the treasury at most once each day. Constraint (5) ensures that the vehicle does not start its travel from the branch. Constraints (6) and (7) indicate the association between the cash carried in the vehicle in the distance between two consecutive nodes; it also deletes sub-tours. Constraint (8) represents the time spent to reach two consecutive nodes. Constraint (9) is one risk-control equation that ensures the vehicles travel short routes in their first three moves when they transit a large volume of cash.

This equation is written as follows:

$$time_{ij} x_{ijkrt} + h_{ijkrt} - h'_{ijkrt} = LB \quad \forall i,j \in N: i \neq j, k \in K, t \in T, r \leq 3 \quad (18)$$

Constraint (10) is the second risk controller that ensures an unequal minimum interval that the vehicle visits a node in two consecutive days. This equation is designed as follows:

$$(19) \left| \sum_{k\in K} \sum_{r\in R} S_{ikrt} - \sum_{k\in K} \sum_{r\in R} S_{ikrt'} \right| + f_{itt'} - f'_{itt'} = \propto \quad \forall i \in D: t, t' \in T: t' = t+1$$

Linearizing Equation (19):

$$(19-1) \sum_{k\in K} \sum_{r\in R} S_{ikrt} - \sum_{k\in K} \sum_{r\in R} S_{ikrt'} + f_{itt'} - f'_{itt'} \geq \propto - MZ_{tt'} \quad \forall i \in D, t, t' \in T: t' = t+1$$

$$(19-2) \sum_{k\in K} \sum_{r\in R} S_{ikrt} - \sum_{k\in K} \sum_{r\in R} S_{ikrt'} + f_{itt'} - f'_{itt'} \leq -\propto + M(1 - Z_{tt'}) \ \forall i \in D, t, t' \in T: t' = t+1$$

Constraint (11) is the third risk controller that ensures not repeating a route from i to j within two consecutive days. This Equation is formulated as follows:

$$(20) \sum_{r\in R} \sum_{k\in K} x_{ijkrt} + \sum_{r\in R} \sum_{k\in K} x_{ijkrt'} \leq 1 \qquad \forall i,j \in N: i \neq j, t, t' \in T: t' = t+1$$

To prevent a no-solution equation, this constraint is considered a soft constraint (Equation 9). Variable h_{ijkrt} must become zero to achieve equation (18). To prevent a no-solution equation, this constraint should be considered a soft constraint (Equation 10). It is obvious that variable $f_{itt'}$ should become zero to achieve equation (19). To prevent a no-solution equation, this constraint should be considered a soft constraint (Equation 10). It is obvious that variable $w'_{ijtt'}$ should become zero to achieve equation (20). The mentioned options are observed in objective functions. Constraint (12) ensures that no vehicle carries money over the determined amount. This point contributes to a lower risk level. Constraints (13) and (14) indicate the relationship between decision variables. Constraint (15) ensures that the time the vehicle reaches the customer is shorter than the time limit. The objective function (16) minimizes route risk in a way that a vehicle travels a short route in the first three moves, the time through which the vehicle reaches a node in two consecutive days is not similar, and a route from i to j is not repeated in two consecutive days. The objective function (17) minimizes the route risk.

8 Proposed solution approaches

This study used GA and ACO algorithms to solve the proposed model. According to previous studies, GA provides more advantages than other algorithms (e.g., tabu search, bees, weeds, fireflies, etc.) because GA achieves shorter travel distances, can be implemented simply, and has an efficient function to solve bi-objective problems. Moreover, GA pursues the solution based on population and generates numerous solutions in each iteration. The notable feature of the ACO algorithm in routing problems is saving and transforming the status of the ant colony system with the more aggressive and active choice command. Hence, this study used the mentioned algorithms to solve the problem.

8.1 Genetic Algorithm

In general, each GA consists of the following components to solve a problem:

How to display solution: solution display is a complex process in routing problems that must use a sequence of specific numbers. Various methods exist in studies to display solutions. This research displays the solution as a vector with constant length due to the nature of the model and type of algorithm (continuous). In other words, the solution is shown as a matrix where time (T) and demand points (D) are shown in its rows and columns, respectively. The numbers inserted in the matrix include real numbers between 1 and (number of vehicles+1) generated stochastically. The considerable point is assigning the CIT vehicle and sequence of demand points, so the integer represents the CIT vehicle, and the decimal value indicates the sequence of demand points. The number with less decimal receives service as the first one. For instance, 1.97 in the vector below indicates the demand point 1 with vehicle 1; number 2.99 indicates demand point 2 with vehicle 2, and 1.22 shows demand point 3 with vehicle 1. In this case, demand points 1.03, 2.18, and 1.22 with fewer decimals receive service in a row in Figure 1.

1.97	2.99	1.22	1.26	1.73	2.57	2.18	1.03

Fig. 1. One random solution vector

Setting parameters and initializing population: Response surface methodology is used to set GA parameters. Response surface design includes a set of mathematical and statistical methods for problem modeling and analysis. This method is used when the problem solution (objective) is affected by an independent factor (input) that aims to optimize this solution. The initial parameters of the algorithm comprise some initial population ($nPop = 100$), probability of crossover ($Pc = 0.82$), probability of mutation ($Pm = 0.36$), maximum algorithm iteration ($Max - It = 250$) type of section operator, and type of crossover and mutation operators.

Generate initial solution: the population is a subset of solutions in the current generation. Moreover, the population can be defined as a set of chromosomes. Therefore, stochastic initialization is used for population initialization—the random solutions guide population to optimization.

Fitness Function: the variable value of the problem is entered into the fitness function to find the optimality of each solution. The objective function is a fitness function in optimization problems (Sadeghi Moghadam et al., 2009). The objective function is used to find how individuals play a role in the problem scope, and the fitness function is usually used to convert the value of the objective function to a fitness value depending on it. In other words, we have:

$F(n) = g(f(x))$

where f represents the objective function, function g converts the objective function to a non-negative value, and F indicates its corresponding fitness value. Solution optimality is assessed based on the value obtained from the fitness function. The fitness function equals the objective function because this is an optimization problem. The objective function minimizes risk.

Selection operator: various methods exist for genetic algorithms that can be used to select genomes. Roulette wheel selection (RWS) and Tournament Software (TS) have been used in GA.

Crossover operator: recombination or crossover operator is done by selecting two (parent) chromosomes on the second part of the chromosome, a sequence of activities resulting in two new chromosomes (child or offspring). It is expected that desired characteristics of parents are combined to achieve better children. The uniform crossover- used in this algorithm- is the best in continuous solutions. In other words, two patterns are needed to do the crossover operator. The patterns are randomly selected from the initial population and multiplied by stochastic numbers (between 0 and 1) that are called mask numbers, and new chromosomes are generated as shown below:

Parent 1: $x_1 = (x_{11,}x_{12}, x_{13}, \dots, x_{1n})$

Parent 2 $\mathrm{x}_2 = (\mathrm{x}_{21,}\mathrm{x}_{22}, \mathrm{x}_{23}, \dots, \mathrm{x}_{2n})$

Mask: $\alpha = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n) \quad 0 \leq \alpha \leq 1$

Offspring 1: $\mathrm{y}_1 = (\mathrm{y}_{11,}\mathrm{y}_{12}, \mathrm{y}_{13}, \dots, \mathrm{y}_{1n}) \rightarrow \mathrm{y}_{1i} = \alpha_i \mathrm{x}_{1i} + (1 - \alpha_i)\mathrm{x}_{2i}$

Offspring 2: $\mathrm{y}_2 = \left(\mathrm{y}_{21,}\mathrm{y}_{22}, \mathrm{y}_{23}, \dots, \mathrm{y}_{2n}\right) \rightarrow \mathrm{y}_{2i} = (1 - \alpha_i)\mathrm{x}_{1i} + \alpha_i \mathrm{x}_{2i}$

In other words, one mask is used for recombination in Figure 2. To do this, an array with elements as many as genes is created. Elements of this array can take values 0 or 1. Elements of the mask array are initialized stochastically. Now, this mask is used to recombine two chromosomes. Value 1 in the mask array indicates that the gene must be selected from the first chromosome. The value 0 in the mask array shows that the gene must be selected from the second chromosome. The opposite is done for the chromosome of the second offspring.

Parent 1	0.72	0.35	0.21	0.88	0.10	0.19
Parent 2	0.25	0.61	0.18	0.36	0.94	0.49
Mask	1	1	0	0	1	0
Offspring 1	0.72	0.35	0.18	0.36	0.10	0.49

Fig. 2. Mask operator

After the crossover operator is done on each generated chromosome, a stochastic number in the interval [0,1] is generated, and the mutation operator is applied to it if this number is less than 0.3.

Mutation operator: mutation operator is used to achieve an excellent probable point in the solution space. The nature of mutation shows a kind of altering current solutions and does not lead to good solutions in most cases. However, the possible successful mutation can considerably affect the objective function and opens a new space in the solution scope. In GA, the probability of mutation in chromosomes equals around 0.01-0.001. this operator can be used to revive those optimal chromosomes that have been removed in the selection or iteration stages. This operator also ensures that the search probability of each point of problem space does not equal zero without paying attention to the dispersion of the initial population. In this algorithm, a mutation in real continuous space has been done using normal distribution:

$$:x_i^{new} \sim N(x_i, \sigma^2) \text{Equation (7)}$$

$$x_i^{new} = x_i + \sigma N(0,1)$$

where $\sigma N(0,1)$ equals the step length. Since mutation rarely occurs in nature, the mutation is GA is done with a probability rate less than 0.05. As mentioned, the mutation operator allows us access to the search space.

Termination condition in GA: the procedure is iterated until reaching termination conditions. The metaheuristic method is terminated when it reaches the maximum default iterations ($Max - It$). In other words, the maximum time limit of implementation can be used as a termination metric, and a new solution is obtained for the model.

8.2 Ant Colony Optimization Algorithm

How to display solution: because the selected algorithm is discrete, a random matrix is used in the permutation method. Matrix' row represents the service day (T), and its columns indicate $D + K - 1$ where D indicates the number of demand points and K shows the number of vehicles. Moreover, $K - 1$ stars are required to generate a possible solution. The sign * is used to separate the routes of each vehicle. For example, the following process is done to find the separation point (*) for eight demand points and 3 CIT vehicles, and eight demand points and 2 CIT vehicles:

Separation point or star: 8+3-1=10

Separation point or star: 2+8-1=9

<table>
<tr><td>5</td><td>4</td><td>7</td><td>8</td><td>10</td><td>2</td><td>9</td><td>3</td><td>1</td><td>6</td></tr>
<tr><td colspan="4" rowspan="2">CIT vehicle 1</td><td>*</td><td rowspan="2">CIT vehicle 2</td><td>*</td><td colspan="3" rowspan="2">CIT vehicle 1</td></tr>
<tr><td>Separation point</td><td>Separation point</td></tr>
</table>

Fig. 3. Matrix display of solution

As seen in this in Figure 3, points 5, 4, 7, and 8 receive service from CIT vehicle 1 in a row, CIT vehicle 2 provides service for point 2, and points 3, 1, and 6 receive service from CIT vehicle 3.

Setting parameter and initializing pheromone: following parameters are required to generate a solution: number of ants/CIT vehicle (nAnt: 50), ability of pheromone effect rate (α: 1), ability of problem's heuristic information (β: 1), pheromone evaporation coefficient (ρ: 0.0 5). It must be explained that the value 0.95 was considered for parameter ρ (pheromone evaporation coefficient) to propose better solutions. The maximum number of iterations equals $Max - It = 300$.

Generating initial solution: a random pheromone value is taken to start the algorithm. Moreover, the traveling salesman problem has been used for demand point routing for each vehicle. In this problem, ants cooperate through an indirect communicational method using a Pheromone and move through graphic arcs when generating solutions. The number of ants that have been predetermined (parameter of n ants) is randomly placed on the selected network node.

Update pheromone: pheromone is updated after creating all routes by ants. This process is done after the first reduction in pheromones on all arcs with a constant factor; ants pass through their routes by adding pheromones to arcs. After selecting the next point and before starting the next stage, the pheromone function is updated because an amount of pheromone is evaporated gradually to avoid premature convergence in the algorithm. For this purpose, parameter ρ is used to avoid the unlimited accumulation of pheromones, and it allows the algorithm to forget the wrong decision made in the past. After evaporation, all ants release pheromones on the arcs that have moved in their route. It must be explained that the variable neighborhood search method is used to prevent the local optimality trap. To do this, the Intra-route Relocate Operator measures the route to find whether the move is optimal (in terms of shortest solution) and feasible (in terms of risk constraint).

Algorithm termination condition: metaheuristic method is terminated when it reaches the maximum number of iterations ($MaxIt$) or maximum time limit. A new solution is obtained for the model in each iteration.

9 Results

To validate the proposed algorithms and quality of generated solutions rather than the optimal solutions, 30 problems were generated in the frame of small, medium, and significant problems that were solved through GAMS and MATLAB R2015b software.

- Small samples: include 12 problems in a way that 4-15 branches receive service from 2-3 vehicles in three workdays.
- Medium samples: include ten problems in a way that 25-70 branches receive service from 4-8 vehicles in 4 workdays.

Large sample: include eight problems in a way that 80-135 branches receive service from 9-15 vehicles in 4-6 workdays.

9.1 Comparison between solutions and relative deviation

All 12 small and 18 medium-large problems are implemented by proposed metaheuristic algorithms five times in the next step, and the obtained results (including the worse generated solution, average solutions, and the obtained solution) with their computation time (per second) are presented. To assess the validity of algorithms and compare with them, the RPD index or relative deviation percent has been defined in the equation below for the proposed algorithm:

Equation (8) $\mathrm{RPD} = (\mathrm{ALG_S} - \mathrm{ALG_{BS}})/\mathrm{ALG_{BS}}$

where $\mathrm{ALG_S}$ and $\mathrm{ALG_{BS}}$ respectively indicate the model and best/worst solutions obtained by the proposed algorithm within five implementations of the sample problem. In other words, the maximum distance is obtained from the difference between the model solution and the worst solution divided by the model solution. To calculate the minimum distance, the difference between the model solution and an average solution is divided by the model solution and presented with computation time.

9.2 Comparing GA with the ACO algorithm

As seen in Table 1, algorithms generate similar solutions for small samples, while the computation time of the ACO algorithm is shorter than GA in small samples 1 and 2. For medium and large samples, all solutions of GA and computation time are better than the ACO algorithm. Moreover, the solution presented in large sample 25 has less deviation rather than the solution generated by GA. However, the deviation rate of the ACO algorithm was greater than GA in other medium and large samples.

9.3 Comparison with GAMS

As seen in Table 1, the GAMS method generates a solution for small samples, while it cannot generate a solution for medium and large samples. Moreover, the presented solutions have lower deviations compared to the model solution. However, solutions have been obtained within a shorter computation time than GAMS. Accordingly, the computation time of GAMS in small sample 12 is about 27.350s.

9.4 Comparison with other studies

In similar studies, Ghannadpour et al. (2018) conducted a study entitled “a game theory-based vehicle routing problem with risk-minimizing of valuable commodity transportation" and found a deviation rate of 1.3%. Tavakkoli-Moghaddam (2014) conducted a study entitled “solving a multi-depot vehicle routing problem based on reduction risk by a multi-objective bat algorithm" and obtained a 1.3% deviation rate. Atabaki (2018) conducted a study entitled “a priority-based differential evolution algorithm for redesigning a closed-loop supply chain using robust fuzzy optimization" and obtained a 1.9% deviation rate. Sattak et al. (2014) conducted a study entitled “capacity multi-depot routing problem with simultaneous pickup and delivery and cut loads" and obtained a deviation rate of 0.2%. Therefore, the performance of the proposed algorithms is acceptable by extending the problem's sizes because these algorithms can generate feasible solutions for medium and large sizes within a short time.

Table 1. Computational results of proposed algorithms

	Trial	D	K	T	GAMS		MATLAB									
					Mathematical Model		Proposed GA									
					OBJECTIVE VALUE	CPUs	OBJECTIVE VALUE			CPUw	CPUa	CPUm	Gap Max	Gap Average	OBJECTIVE V	
							Worst	Average	Model						Worst	Average
Small sample	1	4	2	3	4	0.7	4	4	4	0.3	0.3	0.3	0%	0%	4	4
	2	5	2	3	2	1	2	2	2	0.3	0.3	0.3	0%	0%	2	2
	3	6	2	3	2	2	2	2	2	1	1	1	0%	0%	2	2
	4	7	2	3	2	3	2	2	2	1	1	1	0%	0%	2	2
	5	8	2	3	9	6	10	10	10	2	2	2	0%	0%	10	10
	6	9	2	3	9	15	9	9	9	2	2	2	0%	0%	9	9
	7	10	2	3	9	67	10	10	10	2	2	2	0%	0%	10	10
	8	11	2	3	9	156	10	10	10	2	2	2	0%	0%	10	10
	9	12	2	3	8	25	8	8	8	2	2	2	0%	0%	7	8
	10	13	3	3	8	244	8	9	9	2	2	2	11%	6%	9	9
	11	14	3	3	8	158	9	9	9	2	2	2	0%	0%	9	9
	12	15	3	3	18	27,350	19	19	19	5	5	5	0%	0%	19	19
Medium sample	13	25	4	3	19	11,302	35	35	35	6	6	6	0%	0%	37	37
	14	30	4	3	NA	NA	36	36	36	7	7	7	0%	0%	38	38
	15	35	5	3	NA	NA	36	36	36	7	7	7	0%	0%	38	38
	16	40	5	3	NA	NA	37	37	37	7	7	7	0%	0%	39	39
	17	45	6	3	NA	NA	37	37	37	7	7	7	0%	0%	39	39
	18	50	6	3	NA	NA	38	38	38	7	7	7	0%	0%	40	40
	19	55	7	3	NA	NA	39	39	39	8	8	8	0%	0%	41	41
	20	60	7	4	NA	NA	40	41	42	8	8	8	5%	2%	42	43
	21	65	8	4	NA	NA	41	42	43	8	8	8	5%	2%	43	44
	22	70	8	4	NA	NA	42	43	43	8	8	8	2%	1%	44	45
Large sample	23	80	9	4	NA	NA	80	82	83	12	12	12	4%	2%	87	89
	24	88	10	4	NA	NA	82	84	85	12	12	12	4%	2%	90	92
	25	96	11	4	NA	NA	83	85	86	12	12	12	3%	2%	91	93
	26	104	12	5	NA	NA	85	87	88	12	12	12	3%	2%	94	95
	27	112	13	5	NA	NA	86	88	90	12	12	12	4%	2%	95	96
	28	120	14	5	NA	NA	87	89	91	24	24	24	4%	2%	96	98
	29	128	15	6	NA	NA	91	94	96	24	24	24	5%	3%	100	101
	30	135	15	6	NA	NA	94	97	99	24	24	24	5%	3%	103	106
	Mean				-	-	-	-	-	-	-	-	1.87%	0.93%	-	-

D: demand points (branches), K: number of CIT vehicles, T: number of activity days

Corresponding Author: xxx
E-mail address: xxx
https://doi.org/10.22105/jarie.2023.391399.1540

10 Conclusion and Recommendations

It is essential to consider risk problems and reduce possible risks in the distribution of commodities, especially hazardous and valuable commodities, and physical money in designing routing strategies with concepts and complexities in the current world. In other words, security objectives are more important than economic goals. If security and safety are not planned in the distribution of products, the time and operational costs of commodity transit in the system will be increased. The lack of such a plan also leads to human damage and loss for employees, customers, and security forces. Therefore, the present study presents a bi-objective mixed integer model for a single-treasury routing problem considering the simultaneous pickup and delivery and observing the time window. The first objective uses three concepts to reduce route risk. This technique alleviates a probably fixed pattern to provide service for branches and increases service security. The mentioned three concepts include:

I. The vehicle does not travel long routes in three first moves, which carries more money.
II. The vehicle does not service one branch on two consecutive days simultaneously.
III. An arc is not repeated in two consecutive days as much as possible.

Moreover, the second objective function minimizes the cash-in-transit risk, probability of armed attack risk, and probability of successful robbery. The present study solved 30 small, medium, and significant problems through GAMS and MATLAB software to validate the proposed algorithms and quality of generated solutions compared to optimal solutions. According to the inefficiency of GAMS software in solving models of large sizes, this study used metaheuristic GA and ACO algorithms. Finally, the results indicated that GA could present better results with an average of 0.93% and a maximum of 1.87% difference with optimal solution compared to the ACO algorithm. Although the proposed model can effectively help banks and CIT companies to formulate the demand points routing, the following consideration must be taken:

1. Studying the dynamism of time spent by a vehicle to reach demand points
2. Examining the unpredictable consequences of robbery
3. Examining whether the decision maker has a neutral approach to risk

Ultimately, the model should be revised in a way that is used for solving other problems, such as prisoners' vehicle routing

References

[1] Nasr, N., Akhavan Niakib, T., Seifbarghyc, M., Husseinzadeh Kashand, A., (2022), An Agri-Fresh Food Supply Chain Network Design with RoutingOptimization: A Case Study of ETKA Company", Advances in Mathematical Finance & Applications, vol. 7, No 1, pp. 187-198.

[2] Ghanbarpour, F. and Zandiyeh, F., (2020), A new game-theoretical multiple-objective evolutionary approach for cash-in-transit vehicle routing problem with time windows, European Journal of Operational Research, vol.93, No 2, pp. 106-132.

[3] Hoogeboom, M. and Dullaert, W., (2019),Vehicle routing with arrival time diversification in cash distribution, European Journal of Operational Research, vol .275, No 1,pp. 93-107.

[4] Bozkaya, B., Salman, F. S. and Telciler, K. (2017) "An adaptive and diversified vehicle routing approach to reducing the security risk ofcash-in-transit operations", Networks, Vol. 69, No. 3, pp. 256-269.

[5] Braekers, K., Ramaekers, K. and Nieuwenhuyse, M., (2015), The vehicle routing problem: state of the art classification and review, Computers & Industrial Engineering, vol. 43, No.1, pp.238-247.

[6] Wang, J., Wang, C. and Zhang, Z., (2017), Dynamic route choice prediction model based on connected vehicle guidance characteristics, Journal of Advanced Transportation, vol. A, pp. 8-15.

[7] Davis, L., (2017), Dynamic origin-to-destination routing of wirelessly connected, autonomous vehicles on a congested network, Computers & Industrial Engineering, vol. 478, No.1, pp. 93–102.

[8] Davis, L, (2016), Improving traffic flow at a 2-to-1 lane reduction with wirelessly connected, adaptive cruise control vehicles, Computers & Industrial Engineeringvol. 451, No.2, pp.320–332.

[9] Soriano, A., Vidal, T., Gansterer, M. and Doerner, K., (2020), The vehicle routing problem with arrival time diversification on a multi-graph, European Journal of Operational Research, vol. 286, No 2, pp. 564-575.

[10] Talarico, L., Sorensen, K. and Springael, J., (2017), A bi-objective decision model to increase security and reduce travel costs in the cash-in-transit sector, International Transactions in Operational Research, vol.52, No. 2, pp. 24, 59.

[11] Chen, Y., Cowling, P., Polack, F., Remde, S. and Mourdjis, P. (2017) "Dynamic optimisation of preventative and corrective maintenance schedules for a large scale urban drainage system", European Journal of Operational Operations Research, Vol. 62, pp. 61-77.

[12] Talarico, L., Sorensen, K. and Springael, J., (2015), Metaheuristics for the risk-constrained cash-in-transit vehicle routing problem, European Journal of Operational Research, vol. 244, No.1, pp. 457–470.

[13] Talarico, L., Sorensen, K. and Springael, J., (2015), The k-dissimilar vehicle routing problem, European Journal of Operational Research, vol. 244, No.1, pp.129–140.

[14] Talarico, L., Springael, J., Sorensen, K. and Talarico,F., (2015), A large neighborhood metaheuristic for the risk-constrained cash-in-transit vehicle routing problem, Computers & Industrial Engineering, vol.78, Issue C,pp.547-556.

[15] Parsafard, M., Esmaeel, A., Masoud, K., Mohammadreza, N. and Li, X. (2015) "Practical approach for finding optimum routes for fuel delivery trucks in large cities", Transportation Research Record: Journal of the Transportation Research Board, Vol. 2478, pp. 66-74.

[16] Toumazis, I. and Kwon, C. (2015)"Worst-case conditional value-at-risk minimization for hazardous materials transportation", Transportation Science, Vol. 50, No. 4, pp. 1174-1187.

[17] Pradhananga, R., Taniguchi, E., Yamada, T. and Qureshi, A. G. (2014a) "Bi-objective decision support system for routing and scheduling of hazardous materials", Socio-Economic Planning Sciences, Vol. 48, No. 2, pp. 135-148.

[18] Shangyao, Y., Wang, S. and Wu, S., (2012), A model with a solution algorithm for the cash transportation vehicle routing and scheduling problem, Computers & Industrial Engineering, vol. 63, No.2, pp. 464–473.

[19] Androutsopoulos, K. N. and Zografos, K. G. (2012) "A bi-objective time-dependent vehicle routing and scheduling problem for hazardous materials distribution", EURO Journal on Transportation and Logistics, Vol. 1, No. 1-2, pp.157-183.

[20] Kazantzi, V., Kazantzis, N. and Gerogiannis, V.C. (2011) "Risk informed optimization of hazardous material multi-periodic transportation model", Journal of Loss Prevention in the Process Industries, Vol. 24, No. 6, pp. 773-767.

[21] Bula, G. A., Prodhon, C., Gonzalez, F. A., Afsar, H. M. and Velasco, N. (2017) "Variable neighborhood search to solve the vehicle routingproblem for hazardous materials transportation", Journal of Hazardous Materials, Vol. 324, No. 44, pp.472-480.

.