E-ISSN: 2676-6167 ǀ P-ISSN: 2538-5100

Journal of Applied Research on Industrial Engineering

www.journal-aprie.com

J. Appl. Res. Ind. Eng. Vol. x, No. x (2023) x–x.

Paper Type: Original Article

Bi-Level Non-Radial Network DEA Model for Evaluating Performance of Bank Branches

Kh, Ghaziyani [1,*]; Farhad, Hosseinzadeh Lotfi; Sohrab, Kordrostami; Alireza Amirteimoori

[1] Affiliation; Email Address;

[2] Affiliation; Email Address;

Citation:

Received: Reviewed: Revised: 09/ 21 Accepted:

Abstract

A significant part of each system is determining system efficiency to conduct future planning-associated operations. Data Envelopment Analysis (DEA) is often employed to measure system efficiency. The paper considers a bi-level structure and proposes a new non-radial method by generalizing Russell's Model to measure system efficiency. Furthermore, the data from 33 branches of an Iranian state bank in 2021 are investigated to present the application example. The results indicate that among 33 branches, only two branches are regarded as efficient at both leader and follower levels.

Keywords: Data envelopment analysis, Bi-level programming, Undesirable data, Non-radial Model, Bank.

1 | Introduction

The bank's production process typically includes multiple inputs and outputs; therefore, studies on bank efficiency use Data Envelopment Analysis (DEA) for evaluation purposes. DEA is a nonparametric technique that imposes no limitations on the form of the input-output relationship's function. Additionally, the modern view of performance evaluation is mainly focused on the growth, development, and improvement of the item's capacity under evaluation, and by introducing models for inefficient units, DEA brings about performance enhancement of such units. DEA is based on mathematical programming for the performance evaluation of a set of homogeneous Decision-Making Units (DMUs). The idea of using multiple inputs and outputs was first introduced by Charnes et al. [1], and a host of generalized DEA models were later employed in various fields. Without considering the internal structure of DMUs, early DEA methods only used external inputs and outputs, which led them to regard the system as a black box and calculate efficiency accordingly. Hence, such systems would disregard internal structures, i.e., intermediate measures. In the real world, most systems comprise interconnected stages; in other words, one stage's output is regarded as the next stage's input. These types of inputs/outputs are called intermediate measures. In determining efficiency, it is impossible to investigate intermediate measures using classic DEA models. Therefore, network-structured models were developed to resolve the issue. Färe et al. [2] pioneered the field. The network's internal systems can be divided into two simple stages rather than complex systems

Corresponding Author:

https://doi.org/10.22105/jarie.2023.293475.1351

with numerous stages. Bi-stage systems are of great significance because network systems can often be converted into bi-stage systems with results that can be extended to problems of higher complexity. The present paper studies a mode of bi-level systems, i.e., a bi-level system, in which one system's objective function is another system's limitation. The paper also introduces a non-radial model based on Russell's idea to evaluate the system.

Sherman and Gold [3] used the idea to evaluate the performance of bank branches in the US. Vassilogou and Giokas [4], Oral and Yolalan [5], and Sherman and Ladino [6] employed DEA to study the performance of bank branches in Greece, Turkey, and the US, respectively. Wang et al. [7] concluded that the operations of banks and similar industries are a two-stage process that includes capital collection and profitability. Seiford and Zhu [8] examined the performance of 55 American commercial banks through a bi-stage (profitability and marketability) process. They used conventional models independently and individually to calculate the efficiency of profitability, marketability, and overall efficiency and concluded that large banks show superior performance in profitability while smaller banks are better in marketability. Luo [9] employed Seiford and Zhu's Model to evaluate the performance of a greater number of banks (354 large banks in the US), which provides higher generalizability in the author's results compared to those of Seiford and Zhu. Luo [9] also studied the relationship between geographical location and bank efficiency and between bank efficiency and bankruptcy.

Yu et al. [10] used the concept of cross-efficiency in DEA to solve fixed cost allocation problems in a two-stage system. Lin and Chiu [11] employed the Independent Component Analysis (ICA) method and network DEA to evaluate the performance of four dimensions of top Taiwanese banks. Hakim et al. [12] introduced a bilevel model for centralized resource allocation. Shafiee et al. [13] utilized a mixed integer bi-level DEA model to evaluate the performance of 15 bank branches. Zha et al. [14] and Kong et al. [15] used consecutive bi-stage systems for bank performance evaluation.

2 | Enhanced Russell Measure

The non-radial Russell model was first introduced by Färe and Lovell [16]. The Model, later developed by Färe et al. [2], was enhanced by Pastor et al. [17] and called the Enhanced Russell Model (ERM), and it is presented as follows:

$$
\begin{aligned}
Re^{*} = \min \quad & Re = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r}, \\
\text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io}, \quad i=1,\dots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \varphi_r y_{ro}, \quad r=1,\dots,s, \\
& 0 < \theta_i \le 1, \quad i=1,\dots,m, \\
& \varphi_r \ge 1, \quad r=1,\dots,s, \\
& \lambda_j \ge 0, \quad j=1,\dots,n.
\end{aligned}
\tag{1}
$$

In the ERM, the given DMU is efficient if and only if $Re^{*}=1$. The above Model can be linearized using Charnes-Cooper transformations.

3 | Bi-Level Models and Their Solution Methods

Decentralized planning has been long identified as the most crucial decision-making problem. Many strategies and solutions that are based on the concept of systemic decomposition with broad scopes cannot model samples of independent subsystems that often exist in practice. Standard mathematical programming problems include finding the optimal solution for only an individual decision-maker.

However, many problems hold a hierarchical structure in which each level has an objective function that is either independent or in conflict with other levels. Such problems can be modeled using the multi-level mathematical programming method. Various objectives can be optimized through multi-level programming models in a hierarchical structure. Such problems have multiple decision-making levels, and each of these levels controls a part of the current decision variables in the decision space. In such problems, each level has its specific objective function, and each objective function in each hierarchical level has its specific limitations. At the same time, there also may be common limitations for the problem as a whole.

A bi-level programming model, introduced by Von Stackelberg [18] , refers to a structure in which there are two DMUs connected based on a non-cooperative structure. In each structure, the DMU that makes the first decision is the leader, and the other unit that attempts to improve its performance using the leader's decision is the follower. The two given DMUs have independent or perhaps conflicting objectives. In this sense of bi-level systems, the leader first specifies a decision; then, the follower attempts to optimize its objective function with knowledge of the leader's decision [19].

A bi-level programming problem refers to a series of mathematical problems in which one objective function's limitations are the objective function of another level. Such problems have a variety of applications in different industries, such as economics, industrial engineering, construction engineering, and chemical engineering, in which there are various groups in the form of a hierarchical structure.

A bi-level linear programming problem by Stackelberg [18] is expressed as follows:

$$\begin{aligned} &\operatorname{Min} z_1(x,y)=c_1x+d_1y,\\ &\text{where } y \text{ solves}\\ &\operatorname{Min} z_2(x,y)=c_2x+d_2y,\\ &\text{subject to } Ax+By\le b:\\ &x\ge 0, y\ge 0. \end{aligned} \tag{2}$$

where c1 and c2 are n1-dimensional row vectors of coefficients, d1 and d2 are n2-dimensional row vectors of coefficients, A is an m × n1 coefficient matrix, B is an m × n2 coefficient matrix, b is an m-dimensional column constant vector, z1 (x, y) and z2 (x, y) are the objective functions of leader and follower levels, and x and y are a set of decision variables controlled by the leader and the follower, respectively [19].

4 | Bi-Level DEA Model

A bi-level DEA model is as follows:

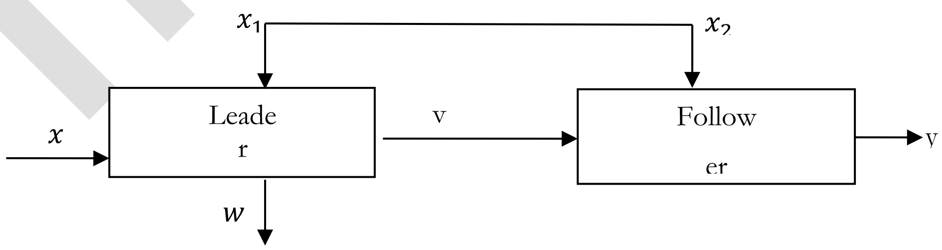

Fig. 1. The bi-level DEA model with common inputs [20].

The production possibility set of the diagram in *Fig. 1* is as follows:

$$
\begin{aligned}
T_1 = \Big\{ (x, x^1, v, w) \;\Big|\; & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le x_i^o, \; i = 1,\dots,M, \\
& \sum_{j=1}^{n} \lambda_j^1 x_{tj}^1 \le x_{ot}^1, \quad t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^1 v_{rj} \ge v_{or}, \; r = 1,\dots,R, \\
& \sum_{j=1}^{n} \lambda_j^1 w_{sj} \ge w_{so}, \; s = 1,\dots,S, \\
& \sum_{j=1}^{n} \lambda_j^1 = 1, \\
& \lambda_j^1 \ge 0, j = 1,\dots,n \Big\}.
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
T_2 = \Big\{ (v, x^2, y) \;\Big|\; & \sum_{j=1}^{n} \lambda_j^2 v_{rj} \le v_{ro}, r = 1,\dots,R, \\
& \sum_{j=1}^{n} \lambda_j^2 x_{tj}^2 \le x_{ot}^2, \quad t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^2 y_{kj} \ge y_{ok}, \; k = 1,\dots,K, \\
& \sum_{j=1}^{n} \lambda_j^2 = 1, \\
& \lambda_j^2 \ge 0, \\
& j = 1,\dots,n \Big\}.
\end{aligned}
\tag{4}
$$

We attempt to improve the performance using the ERM with three minimization and two maximization functions.

$$
\begin{aligned}
& Min \{\theta_1,\dots,\theta_M\}, \\
& Min \{\theta_1^1,\dots,\theta_t^1\}, \\
& Min \{\theta_1^2,\dots,\theta_f^2\}, \\
& Max \{\varphi_1^1,\dots,\varphi_s^1\}, \\
& Max \{\varphi_1^2,\dots,\varphi_b^2\}, \\
& s.t. \begin{pmatrix}
\theta_i x_{io} & : i = 1,\dots,M \\
\theta_t^1 x_{to}^1 & : t = 1,\dots,T \\
\theta_f^2 x_{fo}^2 & : f = 1,\dots,F \\
V_o & \\
\varphi_s^1 w_{so} & : s = 1,\dots,S \\
Y_o^{ud} & \\
\varphi_b^2 y_{bo} & : b = 1,\dots,B
\end{pmatrix} \in T_v^*, \\
& \theta_i \le 1, \quad i = 1,\dots,M, \\
& \theta_t^1 \le 1, \quad t = 1,\dots,T, \\
& \theta_f^2 \le 1, \quad f = 1,\dots,F, \\
& \varphi_s^1 \ge 1, \; s = 1,\dots,S, \\
& \varphi_b^2 \ge 1, \; b = 1,\dots,B.
\end{aligned}
\tag{5}
$$

Model (4) can be rewritten for both leader and follower levels as follows:

$$
\begin{aligned}
& Min\,\{\theta_1,\dots,\theta_M\},\\
& Min\,\{\theta_1^1,\dots,\theta_T^1\},\\
& Max\,\{\varphi_1^1,\dots,\varphi_s^1\},\\
& s.t.: \sum_{j=1}^{n}\lambda_j^1 x_{ij} \le \theta_i x_{io},\ i=1,\dots,M,\\
& \sum_{j=1}^{n}\lambda_j^1 x_{tj}^1 \le \theta_t^1 x_{to}^1,\ t=1,\dots,T,\\
& \sum_{j=1}^{n}\lambda_j^1 v_{rj} \ge v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^1 w_{sj} \ge \varphi_s^1 w_{so},\ s=1,\dots,S,\\
& \sum_{j=1}^{n}\lambda_j^1 = 1,\\
& \lambda_j^1 \ge 0,\\
& \theta_i \le 1,\ i=1,\dots,M,\\
& \theta_t^1 \le 1,\ t=1,\dots,T,\\
& \tilde{\varphi}_s^1 \ge 1,\ s=1,\dots,S.
\end{aligned}
\tag{6}
$$

Model (5) denotes the leader level where the performance enhancement is attempted to be conducted properly by minimizing two inputs and maximizing the output.

Model (6) is designed for the follower level. In this Model, it is attempted again to model the follower level using the ERM.

$$
\begin{aligned}
& Min\left\{\theta_1^2,\dots,\theta_f^2\right\},\\
& Max\left\{\varphi_1^2,\dots,\varphi_K^2\right\},\\
& s.t.: \sum_{j=1}^{n}\lambda_j^1 x_{ij} \le x_{io},\ i=1,\dots,M,\\
& \sum_{j=1}^{n}\lambda_j^1 x_{tj}^1 \le x_{to}^1,\ t=1,\dots,T,\\
& \sum_{j=1}^{n}\lambda_j^1 v_{rj} \ge v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^1 w_{sj} \ge w_{so},\ s=1,\dots,S,\\
& \sum_{j=1}^{n}\lambda_j^2 v_{rj} \le v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^2 x_{fj}^2 \le \theta_f^2 x_{fo}^2,\ f=1,\dots,F,\\
& \sum_{j=1}^{n}\lambda_j^2 y_{kj} \ge \varphi_k^2 y_{ko},\ k=1,\dots,K,\\
& \sum_{j=1}^{n}\lambda_j^2 y_{bj}^{ud} = y_{bo}^{ud},\ b=1,\dots,B,\\
& \sum_{j=1}^{n}\lambda_j^1 = 1,\ j=1,\dots,n,\\
& \sum_{j=1}^{n}\lambda_j^2 = 1,\ j=1,\dots,n,\\
& \lambda_j^1 \ge 0,\ j=1,\dots,n,\\
& \lambda_j^2 \ge 0,\ j=1,\dots,n,\\
& \theta_f^2 \le 1,\ f=1,\dots,F,\\
& \varphi_k^2 \ge 1,\ k=1,\dots,K.
\end{aligned}
\tag{7}
$$

Furthermore, combining the inputs on one side and the outputs on the other side at the leader level leads to a bi-objective model in *Eq. (7)*.

$$
\begin{aligned}
& Min \frac{1}{M+T}\left(\sum_{i=1}^{M}\theta_i+\sum_{t=1}^{T}\theta_t^1\right),\\
& Max \frac{1}{S}\sum_{s=1}^{S}\varphi_s^1,\\
& s.t.\sum_{j=1}^{n}\lambda_j^1 x_{ij}\le \theta_i x_{io},\ i=1,\dots,M,\\
& \sum_{j=1}^{n}\lambda_j^1 x_{tj}^1\le \theta_t^1 x_{to}^1,\ t=1,\dots,T,\\
& \sum_{j=1}^{n}\lambda_j^1 v_{rj}\ge v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^1 w_{sj}\ge \varphi_s^1 w_{so},\ s=1,\dots,S,\\
& \sum_{j=1}^{n}\lambda_j^1 = 1,\ j=1,\dots,n,\\
& \lambda_j^1\ge 0,\ j=1,\dots,n,\\
& \theta_i\le 1,\ i=1,\dots,M,\\
& \theta_t^1\le 1,\ t=1,\dots,T,\\
& \varphi_s^1\ge 1,\ s=1,\dots,S.
\end{aligned}
\tag{8}
$$

In addition, combining the inputs on one side and the outputs on the other side at the follower level results in a bi-objective model in *Eq. (8)*.

$$
\begin{aligned}
& Max \frac{1}{K}\sum_{k=1}^{K}\varphi_k^2,\\
& Min \frac{1}{F}\sum_{f=1}^{F}\theta_f^2,\\
& s.t.\sum_{j=1}^{n}\lambda_j^1 x_{ij}\le \mu^* x_{io},\ i=1,\dots,M,\\
& \sum_{j=1}^{n}\lambda_j^1 x_{tj}^1\le \mu^* x_{to}^1,\ t=1,\dots,T,\\
& \sum_{j=1}^{n}\lambda_j^1 v_{rj}\ge v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^1 w_{sj}\ge w_{so},\ s=1,\dots,S,\\
& \sum_{j=1}^{n}\lambda_j^2 v_{rj}\le v_{ro},\ r=1,\dots,R,\\
& \sum_{j=1}^{n}\lambda_j^2 x_{fj}^2\le \theta_f^2 x_{fo}^2,\ f=1,\dots,F,\\
& \sum_{j=1}^{n}\lambda_j^2 y_{kj}\ge \varphi_k^2 y_{ko},\ k=1,\dots,K,\\
& \sum_{j=1}^{n}\lambda_j^2 y_{bj}^{ud}= y_{bo}^{ud},\ b=1,\dots,B,\\
& \sum_{j=1}^{n}\lambda_j^1 = 1,\ j=1,\dots,n,\\
& \sum_{j=1}^{n}\lambda_j^2 = 1,\ j=1,\dots,n,\\
& \lambda_j^1\ge 0,\ j=1,\dots,n,\\
& \lambda_j^2\ge 0,\ j=1,\dots,n,\\
& \theta_f^2\le 1,\ f=1,\dots,F,\\
& \varphi_k^2\ge 1,\ k=1,\dots,K.
\end{aligned}
\tag{9}
$$

$$
Min\ \mu = Max\left\{\frac{1}{M+T}\left(\sum_{i=1}^{M}\theta_i+\sum_{t=1}^{T}\theta_t^1\right), -\frac{1}{S}\sum_{s=1}^{S}\varphi_s^1\right\}. \tag{10}
$$

Model (7), with the same constraints, can be rewritten as follows:

Accordingly, the constraints of *Model (8)* provide the following Model:

$$Max(\gamma = Min\left\{\frac{1}{K}\sum_{k=1}^{K}\varphi_k^2, \frac{1}{F}\sum_{f=1}^{F}\theta_f^2\right\}). \quad (11)$$

4.1 | Application Example

Applied research requires studying and identifying parameters that contribute to the research areas. To this end, meetings were held with bank directors and VPs, and extensive library and field studies were conducted on the banking system to acknowledge and extract the contributing input, output, and transfer parameters (links) from one fiscal year to another for evaluating the relative efficiency of the corresponding bank branches. Due to a large number of indices because of vast bank activity and high service load, questionnaires were distributed among branch managers to ask their opinions about the significance of a list of indices; accordingly, the most significant indices were selected by managers and employed in the Model. *Table 1* lists the given indices and measuring scales. The example examined in this section contains data from 33 commercial state banks of Iran in 2021 is shown in *Table 2*. Most research papers regard the process of banking operations as a bi-level process. In the first level (the leader level), the leader receives the data, i.e., total resource balance, IRR resource balance on the report's date, the balance of other resources without spent funds, foreign exchange resource balance on the report's date, the total IRR and foreign exchange resource balance on the report's date, as inputs and personnel expenses as the common input, generating the factors including bank charge received, interest received, types of granted facilities in the previous year-end, cases of granted grace period in the current year up to the report's date, the balance of non-demand loans on the report's date, the balance of non-demand loans in the previous year-end, non-performing loans, overdue loans, and bad debts. It should be noted that the desired intermediate measures are the demand deposit balance, the savings account balance, the short-term and long-term deposit balance, the 1% deposit, and Saba's 1% deposit, which are the first stage's outputs and the second stage's inputs.

Table 1. The inserted values in the system.

Factor	Title of Index	Unit of Index
Leader		
Common input	Personnel expenses	IRR
Direct input	Total resource balance	IRR
	IRR resource balance on the report's date	
	Balance of other resources without managed and unspent funds	
	Foreign exchange resource balance on the report's date	
	Total IRR and foreign exchange resource balance on the report's date	
Intermediate measure		
	Demand deposit balance	IRR
	Savings account balance	IRR
	Short-term deposit balance	IRR
	Long-term deposit balance	IRR
	The 1% deposit	IRR
	Saba's 100% deposit	IRR
	Deposit without the national development	IRR
First level's output		
	Interest received	
Follower		
Common input	Other expenses expect interest received	IRR
Desired output	Collection of granted loans up to the report's date	IRR
	Collection of non-demand loans up to the report's date	IRR
	Interest received	IRR
	Bank charge received	
Undesirable output	Bad debts	IRR
	Non-performing loans	IRR
	The non-demand loan balance in the previous year-end	
	The non-demand loan balance on the report's date	
	Granted facilities without the current year conversion	
	Granted facilities of the current year up to the report's date	
	Demand facilities	

Table 2. Displays the information on 33 branches of an Iranian state bank in 2021.

xs2	xs1	y3	yun3	yun2	y2	y1	vun	v	w	yun1	x4	x3	x2	x1
30	20	984,762	194,547	199,752	537,951	1,667,029	2,367,317	1,659,055	34,549	4,660,571		4,150,883	378	4,026,372
10	20	875,097	46,318	90,406	280,034	835,201	1,358,274	736,791	0	2,478,036	2,028,232	2,284,192	114	2,095,065
30	30	447,572	20,034	27,129	206,074	699,208	1,178,908	623,525	0	2,046,313	1,544,497	1,948,366	379	1,802,433
15	35	1,978,115	120,092	205,944	483,201	3,159,194	5,717,972	2,721,448	12,066	5,742,022	7,842,845	5,306,543	815	8,439,421
20	40	967,131	140,166	90,947	420,281	4,337,188	2,288,568	2,629,070	23,759	12,927,320	4,786,225	12,632,834	147	4,917,639
35	20	358,769	14,980	15,401	123,165	439,705	725,532	472,775	15,000	2,024,182	1,041,314	1,953,881	175	1,198,307
30	25	684,768	40,338	102,446	453,826	1,112,309	1,287,862	1,688,325	0	3,313,693	2,983,955	3,115,916	774	2,976,187
20	15	5,209,423	477,193	471,234	1,420,170	8,150,952	12,578,789	9,905,078	0	16,904,568	20,695,325	15,422,281	999	22,483,867
25	15	349,368	21,600	18,201	146,528	441,491	718,002	466,803	0	1,625,026	1,173,415	1,510,309	419	1,184,804
10	30	739,166	6,770	8,540	121,451	789,873	1,310,699	694,461	0	2,194,071	1,436,205	2,162,184	143	2,005,160
20	20	1,795,026	43,309	48,770	653,447	5,899,922	3,304,247	2,996,857	0	12,506,250	6,591,332	12,292,115	885	6,301,105
20	20	223,957	12,403	57,818	125,090	544,869	484,195	441,001	0	2,101,281	846,384	2,001,135	115	925,196
15	30	1,218,553	79,207	218,138	485,318	1,810,601	3,470,142	2,986,338	0	5,657,739	5,381,495	5,267,694	2,221	6,456,481
20	25	348,724	10,355	43,840	159,016	531,312	747,901	604,738	0	1,621,240	1,129,945	1,537,589	94	1,352,639
20	20	598,337	19,825	21,812	248,267	2,031,212	1,252,977	916,107	0	3,450,443	1,901,249	3,375,452	214	2,169,085
15	25	393,452	108,258	55,007	260,349	811,527	904,721	1,744,182	96,350	3,038,412	2,350,536	2,709,245	549	2,648,902
30	10	1,504,435	38,012	106,203	648,341	5,752,525	3,116,616	3,049,793	21,684	9,016,540	5,800,672	8,800,158	1,118	6,166,408
25	15	1,240,363	22,782	98,562	166,123	790,642	2,221,568	554,547	0	1,885,538	1,293,469	1,729,693	265	2,776,115
20	15	99,667	16,258	32,933	96,181	780,093	506,781	474,006	0	1,527,380	946,601	1,465,337	60	980,788
25	30	200,232	20,539	28,294	221,584	653,352	567,569	639,529	0	1,912,804	1,164,154	1,822,627	110	1,207,098
10	40	1,299,305	52,137	242,896	388,680	1,486,255	2,876,212	1,776,202	0	4,853,812	3,940,506	4,416,771	888	4,652,413
20	15	721,913	11,795	49,350	123,563	727,489	1,417,128	861,027	0	4,185,938	1,831,724	4,088,658	128	2,278,156
30	20	127,638	21,289	26,093	123,478	470,614	440,628	582,239	0	2,110,280	984,543	2,019,913	1,159	1,022,867
20	30	576,340	51,731	107,405	324,390	1,018,008	1,245,604	1,102,273	0	3,521,326	1,934,547	3,296,736	507	2,347,878
15	40	746,761	40,536	110,152	253,141	1,789,732	1,750,217	1,456,352	0	5,397,923	2,690,031	5,194,540	146	3,206,569
25	15	525,910	32,428	92,614	233,860	737,084	1,218,400	950,005	0	3,464,958	1,712,882	3,298,970	463	2,168,405
30	20	1,001,314	132,718	235,454	756,938	2,050,363	2,268,081	1,220,324	5,000	5,636,476	3,199,436	5,148,036	220	3,488,404
35	35	741,158	14,111	60,202	315,315	2,019,681	1,548,894	1,496,459	3,540	3,257,546	2,506,341	3,143,437	413	3,045,352
28	13	664,505	39,802	80,225	221,562	841,210	1,913,693	1,510,076	0	2,971,396	3,570,631	2,807,987	262	3,423,769
15	25	915,939	31,334	33,397	245,883	1,254,747	2,033,204	1,179,120	0	3,449,011	2,452,424	3,332,959	580	3,212,324
20	50	606,226	28,307	355,833	239,004	1,519,810	1,154,929	1,352,000	0	4,303,644	1,800,736	3,769,198	112	2,506,929
15	40	33,601,030	369,172	2,306,748	1,831,742	3,989,384	35,987,044	2,227,462	0	11,157,509	34,544,928	8,022,420	153	38,214,506
20	15	61,744,956	2,278,347	5,641,748	12,313,951	59,142,584	100,279,001	56,807,993	1,021,561	152,888,295	138,206,446	141,973,103	15,005	157,086,993

As summarized in *Table 3*, applying the proposed non-radial Model to the above data provides efficiency results at the leader level. The results are from adopting *Models (8)* and *(9)*.

Table 3. The Model's results and findings.

Bank Name	Leader Level	Tetan9	Tetan8	Tetan7	Tetan6	Tetan5	Tetan4	Tetan3	Tetan2	Teta1	Follower Level
1	0.9739	0.0268	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
2	0.5669	0.1577	0.3583	0.2918	1.2283	0.6670	0.1129	0.0000	0.4174	1.1664	0.9999
3	0.0914	0.8456	0.3533	0.3469	0.9813	1.6157	0.7031	1.0199	0.3080	4.5038	1.0000
4	0.4597	0.2553	0.1255	0.1269	0.6631	1.0168	0.5584	0.2308	0.6611	1.6642	1.0000
5	0.6786	0.0881	0.0402	0.0000	0.0000	0.7323	0.4676	0.0000	1.1336	1.5812	1.0000
6	0.2713	0.4197	0.3719	0.3084	0.7678	1.8388	0.5579	0.5694	0.3939	6.8188	0.9999
7	0.4022	0.1607	0.3354	0.5109	1.7686	1.2288	0.5533	0.7690	0.9945	1.5356	0.8753
8	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
9	0.5984	0.6712	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
10	0.1770	0.1927	0.1195	0.2309	3.2868	8.7522	4.4177	3.2906	3.8342	22.6283	1.0000
11	0.7223	0.3844	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
12	0.8574	0.1663	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
13	0.4608	0.1356	0.0987	0.1621	2.7620	1.2118	0.4988	0.6309	1.1564	1.4463	1.0000
14	0.4485	0.0671	0.3152	0.3526	1.8825	1.6064	0.5881	1.6172	0.9111	2.0710	1.0000
15	0.7069	0.4145	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
16	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
17	0.7511	0.3313	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
18	0.4465	0.3946	0.0286	0.0000	0.5254	0.8443	0.0711	0.4510	0.9293	0.9055	1.0000
19	0.8741	0.1440	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
20	0.5152	0.1619	0.1587	0.2223	0.7570	1.2705	0.2589	0.2755	0.3585	3.3254	0.8107
21	0.3916	0.6009	0.1869	0.2076	0.0000	0.3454	1.3496	0.3923	0.1134	0.4710	0.6575
22	0.0520	0.4755	0.5651	0.6403	3.6022	3.2309	1.2215	1.9201	2.2246	4.2811	0.6081
23	0.8645	0.1568	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6209
24	0.3820	0.4025	0.1721	0.1335	2.0401	0.9058	0.1834	0.0735	0.7567	1.3975	0.5654
25	0.3872	0.1958	0.0000	0.0678	9.1642	1.9467	0.7601	0.1565	2.1861	2.4909	0.5385
26	0.2513	0.4654	0.1128	0.0675	6.9545	1.5020	0.4373	0.2847	1.7757	1.8073	0.5170
27	0.6423	0.5568	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5031
28	0.7316	0.0161	0.0000	0.0509	1.7457	0.4804	0.0197	0.9202	0.1701	0.5825	0.4851
29	0.4513	0.1702	0.2892	0.2538	0.6737	1.4333	0.2538	0.7236	1.0926	1.9696	0.4765
30	0.0886	0.5221	0.2016	0.2916	2.7188	2.9290	2.1985	1.3025	1.2108	7.0953	0.4803
31	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
32	0.6725	0.4871	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
33	0.4351	0.3794	0.1033	0.1148	0.2957	0.7739	0.4887	0.3587	0.5373	1.0671	0.8382

In general, performance measurement models are divided into two groups: radial models and non-radial models. In radial models, inputs and outputs change in a proportional relationship. In non-radial models, unlike radial models, inputs and outputs do not change proportionally with each other. Therefore, under real-world conditions, non-radial models have a higher discriminative power [21]. As observed, using *Model (8)*, the leader level is examined, and the result shows that branches 8, 16, and 31 are efficient. Then, by applying *Model (9)* at the follower level, it is observed that branches 1, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 31, and 32 have an efficiency score of 1, so they are considered efficient. Thus, since the decision-making unit should be efficient at both leader and follower levels, only branches 8, 16, and 31, which have an efficiency score of 1, are introduced as efficient branches.

4.2 | Discussion and Conclusion

The paper first considered a consecutive bi-level structure for banks and then developed a radial network DEA model. Afterward, the non-radial version of the Model was presented based on the Russell measure. The non-radial Model is superior because it follows the pattern of a strongly efficient unit. The proposed non-radial Model was employed to evaluate 33 bank branches in 2021. The research findings can be analyzed from various perspectives. The conventional models with a black-box view fail to determine the performance details in each stage and only provide the given bank with an overall efficiency score, which is neither an actual evaluation nor a representation of the source of inefficiency for inefficient banks. Future research can be improved like what was done in [22], [22].

Acknowledgment

The authors would like to make special thanks to the reviewers for their careful reading of our manuscript and their relevant and useful comments that helped us improve the paper.

Conflict of Interest

The authors declare that they have no conflict of interest.

Funding

No funding was received.

References

[1] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, *2*(6), 429–444.

[2] Färe, R., Grosskopf, S., & Lovell, C. A. K. (1985). *The Measurement of Efficiency of Production*. , 6 The Measurement of Efficiency of Production (Vol. 6). Springer Science \& Business Media.

[3] Sherman, H. D., & Gold, F. (1985). Bank branch operating efficiency. Evaluation with Data Envelopment Analysis. *Journal of banking and finance*, *9*(2), 297–315. DOI:10.1016/0378-4266(85)90025-1

[4] Vassiloglou, M., & Giokas, D. (1990). A study of the relative efficiency of bank branches: An application of data envelopment analysis. *Journal of the operational research society*, *41*(7), 591–597. DOI:10.1057/jors.1990.83

[5] Oral, M., & Yolalan, R. (1990). An empirical study on measuring operating efficiency and profitability of bank branches. *European journal of operational research*, *46*(3), 282–294. DOI:10.1016/0377-2217(90)90002-S

[6] Sherman, H. D., & Ladino, G. (1995). Managing Bank Productivity Using Data Envelopment Analysis (DEA). *Interfaces*, *25*(2), 60–73. DOI:10.1287/inte.25.2.60

[7] Wang, L. (1997). The soil seed bank and understorey regeneration in Eucalyptus regnans forest, Victoria. *Austral ecology*, *22*(4), 404–411. DOI:10.1111/j.1442-9993.1997.tb00690.x

[8] Seiford, L. M., & Zhu, J. (1999). Profitability and marketability of the top 55 U.S. commercial banks. *Management science*, *45*(9), 1270–1288. DOI:10.1287/mnsc.45.9.1270

[9] Luo, X. (2003). Evaluating the profitability and marketability efficiency of large banks: An application of data envelopment analysis. *Journal of business research*, *56*(8), 627–635. DOI:10.1016/S0148-2963(01)00293-4

[10] Yu, C. S. (2012). Factors affecting individuals to adopt mobile banking: Empirical evidence from the utaut model. *Journal of electronic commerce research*, *13*(2), 105–121.

[11] Lin, T. Y., & Chiu, S. H. (2013). Using independent component analysis and network DEA to improve bank performance evaluation. *Economic modelling*, *32*(1), 608–616. DOI:10.1016/j.econmod.2013.03.003

[12] Hakim, B., & Uddin, M. A. (2016). *Does Islamic bank financing lead to economic growth: An empirical analysis for Malaysia*.

[13] Shafiee, M., Lotfi, F. H., Saleh, H., & Ghaderi, M. (2016). A mixed integer bi-level DEA model for bank branch performance evaluation by Stackelberg approach. *Journal of industrial engineering international*, *12*(1), 81–91. DOI:10.1007/s40092-015-0131-9

[14] Zha, Y., Liang, N., Wu, M., & Bian, Y. (2016). Efficiency evaluation of banks in China: A dynamic two-stage slacks-based measure approach. *Omega (united kingdom)*, *60*, 60–72. DOI:10.1016/j.omega.2014.12.008

[15] Kong, W. H., Fu, T. T., & Yu, M. M. (2017). Evaluating Taiwanese Bank Efficiency Using the Two-Stage Range DEA Model. *International journal of information technology and decision making*, *16*(4), 1043–1068. DOI:10.1142/S0219622017500031

[16] Färe, R., & Lovell, C. A. K. (1978). Measuring the technical efficiency of production. *Journal of economic theory*, *19*(1), 150–162.

[17] Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). Enhanced DEA Russell graph efficiency measure. *European journal of operational research*, *115*(3), 596–607. DOI:10.1016/S0377-2217(98)00098-8

[18] Von Stackelberg, H. (1952). *The theory of the market economy*. William Hodge.

[19] Sakawa, M., & Nishizaki, I. (2012). Interactive fuzzy programming for multi-level programming problems: A review. *International journal of multicriteria decision making*, *2*(3), 241–266. DOI:10.1504/IJMCDM.2012.047846

[20] Wu, D. D. (2010). BiLevel programming data envelopment analysis with constrained resource. *European journal of operational research*, *207*(2), 856–864. DOI:10.1016/j.ejor.2010.05.008

[21] Zhou, P., Ang, B. W., & Poh, K. L. (2006). Slacks-based efficiency measures for modeling environmental performance. *Ecological economics*, *60*(1), 111–118.

[22] Monzeli, A., Daneshian, B., Tohidi, G., Sanei, M., & Razavian, S. (2020). Efficiency study with undesirable inputs and outputs in DEA. *Journal of fuzzy extension and applications*, *1*(1), 73–80. http://www.journal-fea.com/article_114143.html%0Ahttp://www.journal-fea.com/article_114143_14d2503c8af0eb814f633f168ed3772e.pdf