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New model for ranking based on Sum Weights Disparity Index in data envelopment analysis in fuzzy condition

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A B S T R A C T

In this research, a new model for ranking is presented based on sum weights disparity index in data envelopment analysis in fuzzy condition. Using disparity index, the input and output of data envelopment analysis is considered according to similarity in one category and, units with the efficiency one can be ranked with this method. The new approach of this research is the evaluation of this model in uncertainty conditions and in fuzzy state. In fuzzy conditions, a new model can be provided and used when there are no definitive data and the application of this model can get closer to the actual situation. In this study, to prove the adequacy of the model, the numerical example is assessed and the results of the proposed model is compared with the results of the fuzzy BCC model; the obtained results are indicative of the superiority of the proposed model.

1. Introduction

The need to investigate the efficiency of decision making units is an essential instrument for determining the status of these units. A decision making unit can be any kind of system, which transforms inputs to outputs. Among them, banks, manufacturing companies, agencies and other systems that have inputs and outputs can be cited. One of the best tools for performance evaluation is the data envelopment analysis method (Charnes et al.,1978). Data Envelopment Analysis is a nonparametric method, which is often used for measuring the efficiency of

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decision making units. In this method, a special and appropriate weight is allocated to inputs and outputs that can best be used when evaluating(Charnes et al.,1978). The first model of this most commonly used method was Charnes, Cooper and Rhodes (CCR), this model presented in the study(Charnes et al.,1978). One important advantage of this method is the assessment of Decision Making Units. Efficient models in data envelopment analysis have the performance equal to one, while in the inefficient models, the performance is less than one. Another model of DEA is Banker, Charnes and Cooper (BCC) model that presented in (Banker et al.,1984). Efficient models can also be discussed on the basis of the super performance. One of the most significant substances discussed in the Data Envelopment Analysis is the number of units with the performance of one and their rankings such that an accurate ranking of efficient units can be done. In the research reported here, literature review is evaluated in section two, the primary principles of the subject are examined in section three, a new model in fuzzy conditions is presented and a numerical example is solved in section four, and the results of the research and suggestions for future studies will be discussed in section five.

2. Literature Review

Finding the most effective decision-making unit has always been an essential concern of investigators. In this regard, Wang and Jiang (2012) considered a mixed integer linear programming model is provided for detecting the performance of Decision Making Units in Data Envelopment Analysis. Kao and Liu (2014) used multi period efficiency measurment in data envelopment analysis on Taiwanese commercial banks, they used measuring the overall efficiency of a set of decision making units. Also Chen et al., (2013) considered integer efficiencies in super performance discussions are evaluated based on distance function. Lau (2013) investigated the feasibility of measuring distribution efficiency of a retail network through data envelopment analysis. Azizi and Wang (2013) have been improved data envelopment analysis models and interval efficiencies are utilized for Decision Making Units. Lee et al, (2014) used bootstrap data envelopment analysis truncated regression for technical efficiency of mainstream airlines and low cost carries. Bal et al., (2008) offered a data envelopment analysis model for ranking units when efficient units are more than one. In this study, first, the CCR data envelopment analysis model is solved and weights optimal values are obtained using this model. Afterwards, the values $\bar{u}$ and $\bar{v}$ are defined, $\bar{u}$ being the outputs mean optimal weights, and $\bar{v}$, the inputs mean optimal weights; using these weights and the definition of the disparity index, efficient units are ranked. Another model, based on the research of Bal et al., (2008), is presented by Jahanshahloo and Shahmirzadi (2013) in which, taking the disparity index into consideration, efficient units have been ranked in a different way. This model can be applied for the evaluation of decision making units when the efficient units are more than one.

3. Basic Principles of the Subject

In this section, the basic principles of the subject are examined.

3.1. Data Envelopment Analysis

Data envelopment analysis is a nonparametric method which can evaluate the decision making unit with several inputs and outputs. This method has different models, of which two primary models are mentioned here.

3.1.1. CCR Model

The purpose of this model is finding a virtual decision making unit which can produce $Y_o$output with minimal input. Model 1 is indicative of CCR model in input identity (Charnes et al., 1978).

min (1)

s.t:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \Theta x_{io}\ , \qquad i = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}\ , \qquad r = 1, \dots, s$$

$$\lambda_j \geq 0\ , \qquad j = 1, \dots, n$$

3.1.2. BCC Model

This model was presented based on CCR model. Sum possibility production of this model is obtained by eliminating the endless radiation principle from sum principles of Data Envelopment Analysis. Model 2 is indicative of BCC model in input identity(Banker et al., 1984).

min (2)

s.t:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \Theta x_{io}\ , \qquad i = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}\ , \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0\ , \qquad j = 1, \dots, n$$

3.2. Fuzzy Data Envelopment Analysis

The studies about fuzzy data envelopment analysis were initiated by Sengupta's research (1992). That research suggested two approaches for the solution of data envelopment analysis problems, into which uncertain data was incorporated. The first one is a possible approach for the solution of problems and the second an approach based on fuzzy systems. The second approach is based on Zadeh's studies (1978). In Sengupta's research (1992), two membership functions are discussed for fuzzy numbers, the linear and nonlinear membership functions. One of the advantages of using the linear membership function is the ability to use the benefits of the linear programming in the solution of data envelopment problems in uncertain conditions. In this paper also, an approach based on linear programming in the said mixed model (the combination of Data Envelopment Analysis and diagnostic analysis in fuzzy condition) has been utilized.

3.3. Defuzzification Method

One method of defuzzification is the method presented by (Jimenez , 1996) and (Jimenez et al., 2007). According to that method, both sides of the equation, i.e. $\tilde{a}$ as coefficients of restriction and $\breve{b}$ as the number on the right and E_1^a is mean of center and left for fuzzy number a ,and E_2^a is mean of right and center for fuzzy number a. as the mean , which are in fuzzy condition, can be converted to non-fuzzy numbers. This method uses the equations of (3) for this:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & if\ E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & if 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & IF\ E_1^a - E_2^b > 0 \end{cases} \quad (3)$$

When $\mu_M(\tilde{a}, \tilde{b}) > \alpha$, it can be thus said that $\tilde{a} \geq \tilde{b}$ and, considering the α degree, it is written as $\breve{a} \geq_{\propto} \tilde{b}$. Based on the fuzzy equation definition by Parra et al., (2005), for each pair of fuzzy numbers $\tilde{a}, \tilde{b}$, it can be said that $\breve{a}$ is equal to $\tilde{b}$ in the α degree, if the above equations simultaneously exist as $\breve{a} \leq_{\frac{\propto}{2}} \tilde{b}$, $\breve{a} \geq_{\frac{\propto}{2}} \tilde{b}$. These equations are written as equation (4).

$$\frac{\alpha}{2} \leq \mu_M(\tilde{a}, \tilde{b}) \leq 1 - \frac{\alpha}{2} \quad (4)$$

Now if the sample fuzzy model exists as model (5):

$$\min z \quad \tilde{c}^t x \quad (5)$$

s.t:

$$\tilde{a}_i x \geq \tilde{b}_i \ , \ i = 1, \dots, l$$
$$\tilde{a}_i x = \tilde{b}_i \ , \ i = l + 1, \dots, m$$

$x \geq 0$

Ultimately, the defuzzified model based on model (5) is presented with regard to triangular fuzzy condition and as defuzzified in the form of model (6).

$$\min \ EV(\tilde{c})\ x \quad (6)$$

s.t:

$$[(1-\alpha)E_2^{a_i x} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \quad i = 1, \dots, l$$

$$\left[\left(1 - \frac{\alpha}{2}\right) E_2^{a_i x} + \frac{\alpha}{2} E_1^{a_i}\right] x \geq \frac{\alpha}{2} E_2^{b_i} + (1 - \frac{\alpha}{2}) E_1^{b_i}, \quad i = l + 1, \dots, m$$

$$\left[\frac{\alpha}{2} E_2^{a_i x} + (1 - \frac{\alpha}{2}) E_1^{a_i}\right] x \geq \left(1 - \frac{\alpha}{2}\right) E_2^{b_i} + \frac{\alpha}{2} E_1^{b_i}, i = l + 1, \dots, m$$

$x \geq 0$

The values of E_1 represent the middle ground between the triangular fuzzy middle number and left number and E_2 values represent the middle ground between the triangular fuzzy middle number and right number. By changing the value of α, the model can be evaluated in various defuzzified conditions.

3.4. Data Envelopment Analysis Using Sum Weights Disparity Index

If the number of efficient units is hypothesized as k and the total number of decision making units n, in the first step, model (7) presented by Charnes et al., 1978, should be solved to determine the amount of weight coefficients for inputs v_i and the weight coefficients for outputs v_r.

$$Max \sum_{r=1}^{s} u_r Y_{r\,o} \tag{7}$$

s.t:

$$\sum_{i=1}^{m} v_i X_{i\,o} = 1$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \text{ , j} = 1, \dots, \text{n}$$

$$u_r \ge 0 \text{ , } r = 1, \dots, s$$

$$v_i \ge 0 \text{ , } i = 1, \dots, m$$

Subsequently, using the equations 8 and 9, the values $\bar{v}_i$ and $\bar{u}_r$ should be determined.

$$\bar{v}_i = \frac{1}{n} \sum_{j=1}^{n} v_{i\,j} \text{ , } i = 1, \dots, m \tag{8}$$

$$\bar{u}_r = \frac{1}{n} \sum_{j=1}^{n} v_{r\,j} \text{ , } r = 1, \dots, s \tag{9}$$

After solving model 7 and obtaining the values $\bar{v}_i$ and $\bar{u}_r$ from the equations 8 and 9, the proposed model in research (Jahanshahloo and Shahmirzadi, 2013), shown as model 10:

$$Min \sum_{r=1}^{s} |u_{rk} - \bar{u}_r| + \sum_{i=1}^{m} |v_{ik} - \bar{v}_i| \tag{10}$$

s.t:

$$\sum_{i=1}^{m} v_{i\,k} X_{ik} = 1$$

$$\sum_{r=1}^{s} u_{r\,k} Y_{rj} - \sum_{i=1}^{m} v_{i\,k} X_{ij} \le 0 \text{ , j} = 1, \dots, \text{n}$$

$$u_{r\,k} \ge 0 \text{ , } r = 1, \dots, s$$

$$v_{i\,k} \ge 0 \text{ , } i = 1, \dots, m$$

Using this model and obtaining the results achieved from solving this model, the efficient units are ranked.

4. Discussion

According to the innovation of the present study in applying models in fuzzy conditions, model (7) is mixed with the proposed method in researches (Jimenez , 1996) and (Jimenez et al., 2007) in triangular fuzzy numbers, and this model is presented as model 11 in defuzzified condition.

$$Max \sum_{r=1}^{s} u_r\left[(1-\alpha)E_2^{Y_{ro}} + \alpha E_1^{Y_{ro}}\right] \quad (11)$$

s.t:

$$\sum_{i=1}^{m} v_i \left[(1-\alpha)E_2^{X_{io}} + \alpha E_1^{X_{io}}\right] = 1$$

$$\sum_{r=1}^{s} u_r \left[(1-\alpha)E_2^{Y_{rj}} + \alpha E_1^{Y_{rj}}\right] - \sum_{i=1}^{m} v_i \left[(1-\alpha)E_2^{X_{ij}} + \alpha E_1^{X_{ij}}\right] \le 0 \text{ , j} = 1, \dots, \text{n}$$

$$u_r \ge 0 \ , r = 1, \dots, s$$

$$v_i \ge 0 \ , i = 1, \dots, m$$

In this model, n is detected as the decision making unit j=1,…,n with m as input i=1,…,m and s as output r=1,…,s, in which the inputs are displayed as x_{ij} and the outputs are demonstrated as y_{rj}. Moreover, E_1 values represent the middle ground between the triangular fuzzy middle number and left number and E_2 values represent the middle ground between the triangular fuzzy middle number and right number. By changing the value of α, the model can be evaluated in different defuzzified conditions. New models have been presented which are demonstrated in this research as model 12.

$$\mu_k = Min \sum_{r=1}^{s} |u_{rk} - \bar{u}_r| + \sum_{i=1}^{m} |v_{ik} - \bar{v}_i| \quad (12)$$

s.t:

$$\sum_{i=1}^{m} v_{i\,k} \tilde{X}_{ik} = 1$$

$$\sum_{r=1}^{s} u_{r\,k} \tilde{Y}_{rj} - \sum_{i=1}^{m} v_{i\,k} \tilde{X}_{ik} \le 0 \text{ , j} = 1, \dots, \text{n}$$

$$u_{r\,k} \ge 0 \ , r = 1, \dots, s$$

$$v_{i\,k} \ge 0 \ , i = 1, \dots, m$$

Using the equations in research 6, model 12 can be revised as the defuzzified model 13.

$$\mu_k = Min \sum_{r=1}^{s} |u_{rk} - \bar{u}_r| + \sum_{i=1}^{m} |v_{ik} - \bar{v}_i| \quad (13)$$

s.t:

$$\sum_{i=1}^{m} v_{i\,k} \left[(1-\alpha)E_2^{X_{ik}} + \alpha E_1^{X_{ik}}\right] = 1$$

$$\sum_{r=1}^{s} u_{r\,k} \left[(1-\alpha)E_2^{Y_{rj}} + \alpha E_1^{Y_{rj}}\right] - \sum_{i=1}^{m} v_{i\,k} \left[(1-\alpha)E_2^{X_{ij}} + \alpha E_1^{X_{ij}}\right] \le 0 \text{ , j} = 1, \dots, \text{n}$$

$$u_{r\,k} \ge 0 \ , r = 1, \dots, s$$

$$v_{i\,k} \ge 0 \ , i = 1, \dots, m$$

In model (13), each of the decision making units, previously announced efficient by the model (11), is investigated and each, that has less value of the target function, is placed in a higher rank compared with other efficient units.

4.1. Numerical Example

In this part, the numerical example is provided in order to explain the subject more. This example includes five decision making units which have two inputs and two outputs. Furthermore, all the numbers are triangular fuzzy numbers. The fuzzy number tables have been presented in research (Guo and Tanaka,2001). Table (1) is indicative of the numbers applied in this example.

Using the input and output numbers, the five data envelopment analysis models are solved by applying model (11) in Lingo software, the results of which are demonstrated in table (2). According to Table (2), the decision making units 5, 4 and 2 have the efficiency of one and the other inefficient decision making units are announced. For the accurate ranking of efficient units, model (13) is utilized and the results of ranking are displayed in Table (3). Moreover, the example of interest was assessed using BCC fuzzy model, the results of which are demonstrated in table (4). According to the obtained results in table (4), it can be concluded that the proposed model performs a comprehensive ranking for efficient units, whereas the BCC fuzzy model does not conduct a comprehensive ranking in regard with efficient units.

Table 1. Fuzzy Inputs and Outputs

	DMU 1	DMU 2	DMU3	DMU 4	DMU 5
I1	(3.5,4,4.5)	(2.9,2.9,2.9)	(4.4,4.9,5.4)	(3.4,4.1,5.8)	(5.9,6.5,7.1)
I2	(1.9,2.1,2.3)	(1.4,1.5,1.6)	(2.2,2.6,3)	(2.2,2.3,2.4)	(3.6,4.1,4.6)
O1	(2.4,2.6,2.8)	(2.2,2.2,2.2)	(2.7,3.2,3.7)	(2.5,2.9,3.3)	(4.4,5.1,5.8)
O2	(3.8,4.1,4.4)	(3.3,3.5,3.7)	(4.3,5.1,5.9)	(5.5,5.7,5.9)	(6.5,7.4,8.3)

Table 2. results of solving the example using model (11) with Lingo

	DMU1	DMU 2	DMU3	DMU4	DMU 5
efficiency	0.8548	1	0.8607	1	1
u_1	0.3287	0.4545	0.1834	0.2189	0.196
u_2	0	0	0.0536	0.064	0
v_1	0.2104	0	0.1989	0.2374	0.1538
v_2	0.0754	0.6666	0.0096	0.0115	0

Table 3. results of solving efficient units using model (13) with Lingo

	DMU2	DMU4	DMU5
μ_k	0.1057	0.0018	0.2225
u_{1k}	0.2763	0.2763	0.1799
u_{2k}	0.0235	0.0235	0
v_{1k}	0.2658	0.1620	0.0575
v_{2k}	0.1526	0.1526	0.1526
Ranking	2	1	3

Table 4. results of solving the model using BCC fuzzy with Lingo software

	DMU1	DMU2	DMU3	DMU4	DMU5
Fuzzy BCC	0.842	1	0.838	1	1

5. Conclusions

In this research, a new model for ranking was provided based on sum weights disparity index in data envelopment analysis in fuzzy conditions. One of the problems of data envelopment analysis is announcing a great number of decision making units as efficient units. Using data envelopment analysis with sum weights disparity index, units with the efficiency of one can be ranked. According to the nonexistence of this model in an uncertain condition, the available model is applied in fuzzy conditions such that the extent of using the model is increased and the model is closer to the actual conditions of decision making. To prove the efficiency of the model, the numerical example was evaluated and the results of this example was compared with the BCC fuzzy condition and the results indicate comprehensive ranking in the proposed model while, in BCC fuzzy, comprehensive ranking does not occur for efficient units. In future studies, this model can be considered in probable condition or interval numbers and or interval numbers of the second type so that the model is applied in other conditions of uncertainty and the decision making units can be evaluated using new models.

6. References

Azizi, H., Wang, Y.M., (2013). "Improved DEA models for measuring interval efficiencies of decision-making units", *Measurement*, Vol. 46, No. 3, pp. 1325-1332.

Bal, H., orkcu, H.H., & Celebioglu, S.(2008). "A new method based on the dispersion of weights in data envelopment analysis". *Journal of Computers Industrial Engineering*, Vol. 54, No. 3, pp. 502–512.

Banker, R.D., Charnes, A., and Cooper, W.W. (1984). "Some models for estimating technical and scale efficiency in data envelopment analysis". *Management Science*, Vol. 30, No. 9, pp. 1078–1092.

Charnes, A., Cooper, W.W., and Rhodes,E.(1978). "Measuring the efficiency of decision making units". *European Journal of Operational Research*, Vol. 2, No. 6, pp. 429–444.

Chen, Y., Djamasbi, S., Juan, D., and Lim, S., (2013). "Integer-valued DEA super-efficiency based on directional distance function with an application of evaluating mood and its impact on performance", *International Journal of Production Economics*, Vol. 146, No. 2, pp. 550-556.

Guo, P., and Tanaka, H. (2001). "Fuzzy DEA: A perceptual evaluation method". *Fuzzy Sets and Systems*, Vol. 119, No. 1, pp. 149–160.

Kao, C., and Liu, T.S., (2014). "Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks". *Omega*, Vol. 47, No. 1, pp. 90-98.

Lau, K.H. (2013). "Measuring distribution efficiency of retail network through data envelopment analysis". *International journal of production economics*, Vol. 146, No. 3, pp. 598-611.

Lee, B.L., and Worthington, C.A., (2014). "Technical efficiency of mainstream airlines and low-cost carriers: New evidence using bootstrap data envelopment analysis truncated regression". *Journal of Air Transport Management*, Vol. 38, No. 1, pp. 15-20.

Parra, M.A., Terol, A.B., Gladish, B.P., and Rodriguez Uria, M.V. (2005). "Solving a multi objective possibilistic problem through compromise programming", *European Journal of Operational Research* , Vol. 164, No. 3, pp. 748–759.

Sengupta, J. K. (1992). “A fuzzy systems approach in data envelopment analysis”. *Computers and Mathematics with Applications.* Vol. 24, No. 8-9, pp. 259-266.

Jahanshahloo, G. R., Shahmirzadi, P., (2013). “New methods for ranking decision making units based on the dispersion of weights and Norm 1 in Data Envelopment Analysis”. *Computers & Industrial Engineering*, Vol. 65, No. 2, pp. 187-193.

Jimenez, M. (1996). “Ranking fuzzy numbers through the comparison of its expected intervals”, *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, Vol. 4 , No. 4, pp. 379–388.

Jimenez, M., Arenas, A., & Bilbao, A., Rodriguez, M.V. (2007). “Linear programming with fuzzy parameters: an interactive method resolution”, *European Journal of Operational Research*, Vol. 177, No. 3, pp. 1599–1609.

Wang, Y. M., Jiang, P., (2012). “Alternative mixed integer linear programming models for identifying the most efficient decision making unit in data envelopment analysis”, *Computers and Industrial Engineering*, Vol. 62, No. 2, pp. 546–553.

Zadeh, L.A. (1978). “Fuzzy sets as a basis for a theory of possibility”. *Fuzzy sets and systems*, Vol. 1, No. 1, pp. 3-28.