J. Appl. Res. Ind. Eng. Vol. 4, No. 2 (2017) 89–96

JARIE

Journal of Applied Research on Industrial Engineering

www.journal-aprie.com

A new model for solving fuzzy linear fractional programming problem with ranking function

Sapan Kumar Das*, Tarni Mandal

Department of Mathematics, National Institute of Technology Jamshedpur, India

PAPER INFO	ABSTRACT
Chronicle: Received: 10 May 2017 Revised: 02 July 2017 Accepted: 10 August 2017 Available : 10 August 2017 **Keywords** : Triangular fuzzy number. Fuzzy linear fractional programming. Ranking function. Multi Objective Programming.	In this paper, we studied fuzzy linear fractional programming (FLFP) problems with trapezoidal fuzzy numbers where the objective functions are fuzzy numbers and the constraints are real numbers. In this study, in order to obtain the fuzzy optimal solution with unrestricted variables and parameters, a new efficient method for FLFP problem has been proposed. These proposed methods are based on crisp linear fractional programming and newly transformation technique is also used. A computational procedure has been presented to obtain an optimal solution. To show the efficiency of our proposed method a real life example has been illustrated.

1. Introduction

Linear fractional programming (LFP) problem is one of the most important techniques in operation research. Many real world problems can be transformed to linear fractional programming model; hence this model is an indispensable tool for today's applications such as financial sector, hospitality, industrial sector etc. It is a mathematical technique for optimal allocation to several activities on the basis of given decision of optimality. This type of problem is evidently an uncertain optimization problem due to its decision-based system. So, it leads to proposition of a new concept in fuzzy optimization by Bellman and Zadeh [5]. Lotfi et al. [3] introduced a method to obtain the approximate solution of fully fuzzy linear programming problems. Amit Kumar et al. [2] proposed a method for solving fully fuzzy linear programming problems using idea of crisp linear programming and ranking function.

Recently, Veeramani and Sumathi [4] established a new method for solving fuzzy linear fractional Programming problem and they have transformed the problem into a multi objective linear programming problem. Ganesan and Veeramani [6] introduced the fuzzy linear programming problem

* Corresponding author
E-mail address: cool.sapankumar@gmail.com
DOI: 10.22105/jarie.2017.48543

with trapezoidal fuzzy numbers without converting them to crisp linear programming problem. Ebrahimnejad and Tavana [1] proposed a new concept in which the coefficients of objective function and the values of the right hand side are represented by trapezoidal fuzzy numbers and other parts are represented by real numbers. They converted the fuzzy linear programming problem into an equivalent crisp linear programming problem and solved by simplex method. Das et al. [7, 8] have proposed so many methods for solving fuzzy linear fractional programming problem by using various methods. Das et al. [7] proposed a general form of fuzzy linear fractional programming problem with trapezoidal fuzzy numbers. Hatami and Kazemipoor [9] have solved fuzzy linear fractional programming problem by using Big-M method. Saberi Najafi and Edalatpanah [10] have proposed method for solving linear programming problems by using Homotopy perturbation method. Saberi Najafi et al. [11] have proposed method a nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. Hosseinzadeh and Edalatpanah [12] have proposed a method for solving fully fuzzy linear programming by using the lexicography method; see also [13-21].

In this paper, a new method is proposed for finding the fuzzy optimal solution of FLFP problems with inequality constraints. The coefficients of the objective function are represented by trapezoidal fuzzy numbers and the constraints are represented by real numbers. We introduce a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and propose a method for solving fuzzy linear fractional programming problem with converting them to crisp linear fractional programming problems. After that, we used a new transformation technique to solve these crisp linear fractional programming problems. To illustrate the proposed method, numerical examples are solved.

This study is organized as follows: In Section 2, some basic definitions of fuzzy symmetric trapezoidal fuzzy number and some arithmetic results are presented. In Section 3, formulation of FLFP problems and application of ranking function for solving FLFP problems are established. A new method is proposed for solving FLFP problems in section 4. In Section 5, we give a numerical example including symmetrical trapezoidal fuzzy numbers to illustrate the theory developed in this paper. Finally, in Section 6 we present the conclusion part.

2. Preliminaries

In this section some notations and results of fuzzy set theory are presented and discussed:

Definition 2.1 [1]. A convex fuzzy set $\tilde{A}$ on R is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous.

(b) There exist three intervals [a, b], [b, c] and [c, d] such that $\mu_{\tilde{A}}$ is increasing on [a, b], equal to 1 on [b, c], decreasing on [c, d] and equal to 0 elsewhere.

Definition 2.2 [1]. The arithmetic operations on two symmetric trapezoidal fuzzy numbers

$\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ are given by:

$$\tilde{A}+\tilde{B}=(a^L+b^L,a^U+b^U,\alpha+\beta,\alpha+\beta),$$

$$\tilde{A}-\tilde{B}=(a^L-b^L,a^U-b^U,\alpha-\beta,\alpha-\beta),$$

$$\tilde{A}\tilde{B}=((\frac{a^L+a^U}{2})\,(\frac{b^L+b^u}{2})-t,(\frac{a^L+a^U}{2})\,(\frac{b^L+b^U}{2})+t,\ \left|a^U\beta+b^U\alpha\right|,\ \left|a^U\beta+b^U\alpha\right|),$$

Where,

$$t=\frac{t_2-t_1}{2},\ \ t_1=\min\{a^Lb^L,a^Ub^U,a^Ub^L,a^Ub^U\},t_2=\max\{a^Lb^L,a^Ub^U,a^Ub^L,a^Ub^U\}.$$

$$k\tilde{A}=\begin{cases}(ka^L,ka^U,k\alpha,k\alpha) & k\geq 0.\\ (ka^U,ka^L,-k\alpha,-k\alpha) & k<0.\end{cases}$$

Definition 2.3 [1]. Let $\tilde{A}=(\mathrm{a}^L,\mathrm{a}^U,\alpha,\alpha)$ and $\tilde{\mathrm{B}}=(\mathrm{b}^L,\mathrm{b}^U,\beta,\beta)$ be two symmetric trapezoidal fuzzy numbers. The relations $\tilde{\leq}$ and $\approx$ are defined as follows:

(i) $\frac{(a^L-\alpha)+(a^U-\alpha)}{2}<\frac{(b^L-\beta)+(b^U-\beta)}{2}$, that is $\frac{a^L+a^U}{2}<\frac{b^L+b^U}{2}$ (in this case, we may write $\tilde{A}\tilde{<}\tilde{B}$), or

(ii) $\frac{a^L+a^U}{2}=\frac{b^L+b^U}{2}, b^L<a^L, a^U<b^U$ (in this case we say $\tilde{A}\approx\tilde{B}$), or

(iii) $\frac{a^L+a^U}{2}=\frac{b^L+b^U}{2}, b^L=a^L, a^U=b^U, \alpha\leq\beta$ (in this case we say $\tilde{A}\approx\tilde{B}$).

Definition 2.4 [1]. A fuzzy set $\tilde{A}$ on R is called a symmetric trapezoidal fuzzy number if its membership function is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha\leq x\leq a^L\\ 1 & a^L\leq x\leq a^U\\ \frac{(a^u+\alpha)-x}{\alpha} & a^U\leq x\leq a^U+\alpha\\ 0 & else\end{cases}$$

3. Proposed method to convert LFP into LP

In this section we refer to [18] a transformation method to convert linear fractional to equivalent linear programming as follows:

Transformation of Objective:

$$\text{Max (Z)}=\frac{c^t x+\alpha}{d^t x+\beta}$$

$$\text{Max (Z)}=\frac{c^t x\beta+\alpha\beta}{\beta(d^t x+\beta)}$$

$$=(c^t-d^t*\frac{\alpha}{\beta})*\frac{x}{d^t x+\beta}+\frac{\alpha}{\beta}$$

$$\text{Max (Z)} = p^t y+g$$

Where, $p^t=(c^t-d^t*\frac{\alpha}{\beta}),\ y=\frac{x}{d^t x+\beta},\ g=\frac{\alpha}{\beta}$

Transformation of constraints:

$$Ax-b\le 0$$

$$=(A+d^t*\frac{b}{\beta})*\frac{x}{d^t x+\beta}\le 0$$

$$=Gy\le h$$

Where, $G=(A+d^t*\frac{b}{\beta}),\ h=\frac{b}{\beta}$

We get the linear programming problem as

Max (Z)= $p^t y+g$

Subject to $Gy\le h$

$y\ge 0.$

4. Proposed method to find the fuzzy optimal solution of FLFP problems

In this section, a new method is proposed to find the fuzzy optimal solution of the following type of fuzzy linear fractional programming (FLFP) problems:

$$\text{Maximize (or Minimize)}=\frac{\tilde{c}^t x+\tilde{\alpha}}{\tilde{d}^t x+\tilde{\beta}} \quad (1)$$

Subject to $A\otimes x\le b$,

$x\ge 0$.

Where, $\tilde{c}^t=[\tilde{c}_j]$ is 1 by *n* matrix; $\tilde{d}^t=[\tilde{d}_j]$ is 1 by *n* matrix; $x=[x_j]$ is *n* by 1 matrix; $A=[a_{ij}]$ is *m* by *n* matrix; $b=[b_{ij}]$ is a *m* by 1 matrix; $\tilde{\alpha}=[\tilde{\alpha}_j]$ and $\tilde{\beta}=[\tilde{\beta}_j]$ are the scalars.

Based on definition 2.3, we define a rank for each symmetric trapezoidal fuzzy number for comparison purposes. Assuming that $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is a symmetric trapezoidal fuzzy number, then $R(\tilde{A}) = \dfrac{a^L + a^U}{2}$. This equation allows us to convert the fuzzy linear fractional programming (FLFP) problem in to a crisp linear fractional programming (CLFP) problem. We substitute the rank order of each fuzzy number for the corresponding fuzzy number in the fuzzy problem under consideration. This leads to an equivalent crisp linear fractional programming problem which can be solved by standard method. In the following part, we are going to introduce an algorithm to find an exact optimal solution of FLFP problem. The steps of the proposed algorithm are given as follows:

Step 1. With respect to the general form of our problem, it can be written as follows:

$$\text{Maximize (or Minimize)} = \frac{\tilde{c}^t x + \tilde{\alpha}}{\tilde{d}^t x + \tilde{\beta}} \tag{2}$$
$$\text{Subject to} \quad A \otimes x \leq b,$$
$$x \geq 0.$$

Step 2. Regarding definitions 2.3, the problem (2) is converted to the crisp linear fractional programming problem may be written as follows:

$$\text{Maximize (or Minimize)} = \frac{c^t x + \alpha}{d^t x + \beta} \tag{3}$$
$$\text{Subject to} \quad A \otimes x \leq b,$$
$$x \geq 0.$$

Step 3. In terms of the objective function, the transformation method will be used into an equivalent crisp linear programming problem, so we have:

$$\text{Max (or Min)} = p^t y + g \tag{4}$$
$$\text{Subject to} \quad Gy \leq h$$
$$y \geq 0.$$

Step 4. Solve the problem (4) with the help of Lingo software: we get the optimal solution.

5. Numerical Example:

In this section, we illustrate the proposed algorithm using a real life problem. Linear fractional programming problem is evidently an uncertain optimization problem due to its variations in the maximum daily requirements. So, the amount of each product of ingredient will be uncertain. Hence, we will model the problem as a Fuzzy linear fractional programming (FLFP) problem. We use trapezoidal fuzzy numbers for each uncertain value. Also, the mathematical programming problem will be solved by *Lingo*.

5.1. Example (Production Planning)

A company manufactures two kinds of products A and B with profit around (4,6,3,3) and around (1,5,1,1) dollar per unit, respectively. However the cost for each one unit of the above products is around (3,7,2,2) and around (3,1,1,1) dollars respectively. It is assumed that a fixed cost of around (1,1,2,2) dollars is added to the cost function due to the expected duration through the process of production. Supposing the raw material needed for manufacturing product A and B, about 3 units per pound and about 5 units per pound respectively, the supply for this raw material is restricted to about 15 pounds. Man-hours per unit for the product A is about 5 hours and product B is about 2 hours per unit for manufacturing but total the Man-hour available is about 10 hour daily. Determine how many products A and B should be manufactured in order to maximize the total profit.

This real life problem can be formulated to the following FLFP problem:

$$\begin{aligned} \text{Max } & \frac{(4,6,3,3)x_1 + (1,5,1,1)x_2}{(3,7,2,2)x_1 + (3,1,1,1)x_2 + (1,1,2,2)} \\ \text{s.t. } & 3x_1 + 5x_2 \le 15, \\ & 5x_1 + 2x_2 \le 10, \\ & x_1, x_2 \ge 0. \end{aligned} \tag{5}$$

So, with respect to Step 2 we convert the problem (2) into an equivalent crisp linear programming problem as follows:

$$\begin{aligned} \text{Max } & \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1} \\ \text{s.t. } & 3x_1 + 5x_2 \le 15, \\ & 5x_1 + 2x_2 \le 10, \\ & x_1, x_2 \ge 0. \end{aligned} \tag{6}$$

Now, the crisp linear fractional programming problem (6) is converting into an equivalent crisp linear programming problem by using Step 3.

$$\begin{aligned} \text{Max } & 5y_1 + 3y_2 \\ \text{s. t. } & 78y_1 + 35y_2 \le 15, \\ & 55y_1 + 22y_2 \le 10, \\ & y_1, y_2 \ge 0. \end{aligned} \tag{7}$$

The problem (7) is the crisp linear programming problem. Now solved by simplex method we get the result is:

$y_1=0$, $y_2=0.42$, and the objective function value is Z=1.28.

By comparing the result of proposed method with Ebrahimnejad method et al. we conclude that our result is more effective, because:

$1.25=(Z)_{\text{Ebrahimnejad method etal.}}<(Z)_{\text{proposed method}}=1.28.$

6. Conclusion:

In the past few years, a growing interest has been shown in Fuzzy linear fractional programming and several methods for solving FLFP problem have been suggested. In this paper, a new efficient method has been proposed, in order to obtain the fuzzy optimal solution of fuzzy linear fractional programming problems with inequality constraints occurring in daily real life problem. We showed that the method proposed in this paper is highly reliable and applicable. To illustrate the proposed method numerical examples are solved.

References

[1] Ebrahimnejad, A., & Tavana, M. (2014). A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. *Applied Mathematical Modelling*, *38*(17), 4388-4395.

[2] Kumar, A., Kaur, J., & Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. *Applied Mathematical Modelling*, *35*(2), 817-823.

[3] Lotfi, F. H., Allahviranloo, T., Jondabeh, M. A., & Alizadeh, L. (2009). Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied Mathematical Modelling*, *33*(7), 3151-3156.

[4] Veeramani, C., & Sumathi, M. (2014). Fuzzy mathematical programming approach for solving fuzzy linear fractional programming problem. *RAIRO-Operations Research*, *48*(1), 109-122.

[5] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141.

[6] Ganesan, K., & P. Veeramani, V., (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. Ann. Oper. Res. 143, 305-315.

[7] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO-Operations Research*, *51*(1), 285-297.

[8] Das, S. K., & S. A. Edalatpanah (2016). A General Form of Fuzzy Linear Fractional Programs with Trapezoidal Fuzzy Numbers. *International Journal of Data Envelopment Analysis and *Operations Research*, 2* (1), 16-19.

[9] Hatami, A., & Kazemipoor, H. (2013). Fuzzy big-M method for solving fuzzy linear programs with trapezoidal fuzzy numbers. *International Journal of Research in Industrial Engineering*, *2*(3), 1-9.

[10] Najafi, H. S., & Edalatpanah, S. A. (2014). Homotopy perturbation method for linear programming problems. *Applied Mathematical Modelling*, *38*(5), 1607-1611.

[11] Saberi Najafi, H., Edalatpanah, S. A., and Dutta, H. (2016). A nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. Alexandria Engineering Journal, 55(3), 2589-2595.

[12] Hosseinzadeh, A., & Edalatpanah, S. A. (2016). A New Approach for Solving Fully Fuzzy Linear Programming by Using the Lexicography Method. Adv. Fuzzy Syst., http://dx.doi.org/10.1155/2016/1538496, 6 pages 1538496.

[13]Maleki, H. R, Tata, M., Mashinchi, M., (1996). Fuzzy number linear programming, ProcInternat Conf. on intelligent and Cognitive Systems FSS Sponsored by IEE and ISRF, Tehran, Iran ,145-148.

[14] Tanaka, H., Okuda, T., & Asai, K. (1974). On fuzzy mathematical programming, *J. Cybern*, 3(4), 37-46.

[15] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, *1*(1), 45-55.

[16] Zimmermann, H. J. (1982). Trends and new approaches in European operational research. *Journal of the Operational Research Society*, 597-603.

[17] Das, S., & Mandal, T. (2015). A single stage single constraints linear fractional programming problem. *Oper Res and Appl: An Intern J*, *2*, 1-5.

[18] Sapan, K. D., Mandal, T., & Edalatpanah, S. A. (2015). A note on "A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers".

[19] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied Intelligence*, *46*(3), 509-519.

[20] Torabi, N.,& Najafi, S. E. (2015). New model for ranking based on Sum Weights Disparity Index in data envelopment analysis in fuzzy condition. *Journal of Applied Research on Industrial Engineering*, 2(2), 111-119.

[21] Sheikhi, H. (2012). A Novel Approach for Solving Fuzzy Multi-Objective Zero-One Linear Programming Problems. *International Journal of Research in Industrial Engineering*, 1(1), 42-63.