Shortest Path Problem in Network with Type-2 Triangular Fuzzy Arc Length

Ranjan Kumar[*], Sripati Jha, Ramayan Singh

Department of Mathematics, National Institute of Technology Jamshedpur, Jharkhand 831014, India.

P A P E R I N F O	A B S T R A C T
	In traditional Shortest Path Problem (SPP) it is always determined that the parameters (Time, Cost and Distance etc.) are fixed between different nodes. But in real life situations where uncertain parameters exist, parameters are considered as fuzzy numbers. In this paper, we explained the application scope of the given fuzzy ranking function. Using this method we can determine both the fuzzy shortest path and fuzzy shortest distance from origin to destination.

1. Introduction

There are many situations in real life where we need to find the shortest path from origin to destination. In traditional Shortest Path Problem (SPP) it is always determined that the parameter are fixed a problem which can be easily solved by fundamental graph theory. For example in Dijkstras algorithm which the weighted graph is used to find the shortest path from origin to destination. Bellman–Ford algorithm solves the single-source problem if edge weights are negative, Floyd–Warshall algorithm solves all pair shortest paths, but there are many situations in real life where we have to face with uncertain parameters between the origin to destination connected via different destinations. In these cases Zadeh [1] introduces the fuzzy numbers and then used to study a variety of problems; see ([2-14] and references their in). We have used the type-2 triangular fuzzy numbers which is an extended concept of ordinary fuzzy set. Type-2 triangular fuzzy numbers is introduced by Zadeh [1]. Type-2 fuzzy sets can be defined as a membership function in which the membership value for each element of this set is fuzzy set in [0,1], contrary to an ordinary fuzzy set where the membership value is a crisp number in [0,1].

Kaufmann et.al. [15] introduced the fuzzy arithmetic theory and application for the first time. Further, Dinagar et.al. [16] also studied arithmetic operations on type-2 triangular fuzzy numbers by using extension principal and proposed ranking function in generalized type-2 triangular fuzzy numbers.

The fuzzy shortest path problem is an extension of fuzzy numbers and it has many real life applications in the field of communication, robotics, scheduling and transportation. Dubois [17] introduced the fuzzy shortest path problem for the first time. Klein [18] introduced a new model to solve the fuzzy shortest path problem for sub-modular functions. Lin and Chern [19] introduced a new design to find the fuzzy shortest path problem on single most vital arc length in a network by using dynamics programming approach. Li et.al. [20] solved the fuzzy shortest path problems by using neural network approach. Chuang et.al. [21] used two steps to find the shortest path from origin to destination. In the first step, they proposed a heuristic procedure to find the fuzzy shortest length among all possible paths in a network; in the second step, they proposed a way to measure the similarity degrees between the fuzzy shortest length and each fuzzy path lengths. The path with the highest similarity degree is the shortest path. Yu and Wei [22] solved the fuzzy shortest path problem by using a linear multiple objective programming. Mahdavi et.al. [23] proposed a dynamic programming approach for the fuzzy shortest chain problem using a fuzzy ranking method to avoid generating the set of non-dominated paths. Zhang et.al. [24] have used a biologically inspired algorithm called Fuzzy Physarum Algorithm (FPA) for fuzzy shortest path problems.

Jain [25] was first introduced the fuzzy ranking index. Researchers and scientists use this theory in many applications. Shortest path problem in fuzzy chain is one of the applications in which we used ranking index to find the shortest path between origin to destination node. Abbasbandy [26] proposed a new approach for ranking the trapezoidal fuzzy numbers. Recently, Anusuya and Sathya [27] have solved the type-2 trapezoidal fuzzy numbers by using linear ranking function. Malini and Ananthanarayanan [28] solved the fuzzy transportation problem using ranking of the trapezoidal fuzzy numbers. But to the best of our knowledge, fuzzy shortest path problems for the type-2 triangular fuzzy numbers by linear ranking function has not yet been solved by any researcher.

This paper organized as follows: Some basic knowledge concepts on fuzzy set theory used throughout this paper are introduced in Section 2. Section 3 includes the existing method for finding shortest path and shortest path length from source node to destination node. In Section 4, some numerical examples have been given to reveal the effectiveness of existing model. Finally some conclusions have been drawn in the last section.

2. Preliminaries

In this section some knowledge concepts on fuzzy set theory and ranking function theory used throughout this paper are reviewed.

Definition 2.1. ([1]). A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0, 1].

Definition 2.2. ([1]). Let $\tilde{\eta}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{\eta}$ is normal,

2. $\tilde{\eta}$ is a convex set,

3. The support of $\tilde{\eta}$ is closed and bounded, then $\tilde{\eta}$ is called a type-2 fuzzy number.

Definition 2.3. (Type-2 triangular fuzzy number) A type-2 triangular fuzzy number ω on R is given by $\tilde{\tilde{\omega}} = \left\{ x, \left(\mu_\omega^1(x), \mu_\omega^2(x), \mu_\omega^3(x), \right); x \in R \right\}$ and $\mu_\omega^1(x) \le \mu_\omega^2(x) \le \mu_\omega^3(x)$ for all $x \in R$. Denote $\tilde{\tilde{\omega}} = \left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} \right)$, where

$$\tilde{\alpha} = \left(\tilde{\alpha}_1^M, \tilde{\alpha}_1^N, \tilde{\alpha}_1^O \right), \tilde{\beta} = \left(\tilde{\beta}_1^M, \tilde{\beta}_1^N, \tilde{\beta}_1^O \right) \text{ and } \gamma = \left(\gamma_1^M, \gamma_1^N, \gamma_1^O \right)$$

are the same type of fuzzy numbers.

Definition 2.4. Arithmetic operation on type-2 triangular fuzzy numbers:

Addition [29]: Let $\tilde{\tilde{A}} = \left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} \right) = \left\{ \left(\tilde{\alpha}_1^M, \tilde{\alpha}_1^N, \tilde{\alpha}_1^O \right), \left(\tilde{\beta}_1^M, \tilde{\beta}_1^N, \tilde{\beta}_1^O \right), \left(\tilde{\gamma}_1^M, \tilde{\gamma}_1^N, \tilde{\gamma}_1^O \right) \right\}$ and $\tilde{\tilde{B}} = \left(\tilde{\varphi}, \tilde{\delta}, \tilde{\chi} \right) = \left\{ \left(\tilde{\varphi}_1^M, \tilde{\varphi}_1^N, \tilde{\varphi}_1^O \right), \left(\tilde{\delta}_1^M, \tilde{\delta}_1^N, \tilde{\delta}_1^O \right), \left(\tilde{\chi}_1^M, \tilde{\chi}_1^N, \tilde{\chi}_1^O \right) \right\}$, then $\tilde{\tilde{A}} + \tilde{\tilde{B}}$ are Type-2 triangular fuzzy numbers, defined as:

$$\tilde{\tilde{A}} + \tilde{\tilde{B}} = \left\{ \begin{array}{l} \left(\tilde{\alpha}_1^M + \tilde{\phi}_1^M, \tilde{\alpha}_1^N + \tilde{\phi}_1^N, \tilde{\alpha}_1^O + \tilde{\phi}_1^O \right), \left(\tilde{\beta}_1^M + \tilde{\delta}_1^M, \tilde{\beta}_1^N + \tilde{\delta}_1^N, \tilde{\beta}_1^O + \tilde{\delta}_1^O \right), \\ \left(\tilde{\gamma}_1^M + \tilde{\chi}_1^M, \tilde{\gamma}_1^N + \tilde{\chi}_1^N, \tilde{\gamma}_1^O + \tilde{\chi}_1^O \right) \end{array} \right\}$$

Subtraction [29]: If $\tilde{\tilde{A}} > \tilde{\tilde{B}}$ then :

$$\tilde{\tilde{A}} - \tilde{\tilde{B}} = \left\{ \begin{array}{l} \left(\tilde{\alpha}_1^M - \tilde{\phi}_1^M, \tilde{\alpha}_1^N - \tilde{\phi}_1^N, \tilde{\alpha}_1^O - \tilde{\phi}_1^O \right), \left(\tilde{\beta}_1^M - \tilde{\delta}_1^M, \tilde{\beta}_1^N - \tilde{\delta}_1^N, \tilde{\beta}_1^O - \tilde{\delta}_1^O \right), \\ \left(\tilde{\gamma}_1^M - \tilde{\chi}_1^M, \tilde{\gamma}_1^N - \tilde{\chi}_1^N, \tilde{\gamma}_1^O - \tilde{\chi}_1^O \right) \end{array} \right\}.$$

Scalar multiplication [29]: If $K \ge 0$ and $K \in R$ then:

$$K\tilde{\tilde{A}} = K \left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} \right) = \left(K \tilde{\alpha}_1^M, K \tilde{\alpha}_1^N, K \tilde{\alpha}_1^O \right), \left(K \tilde{\beta}_1^M, K \tilde{\beta}_1^N, K \tilde{\beta}_1^O \right), \left(K \tilde{\gamma}_1^M, K \tilde{\gamma}_1^N, K \tilde{\gamma}_1^O \right)$$

If K<0 and $K \in R$ then:

$$K\tilde{\tilde{A}} = K \left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} \right) = \left(\left(K \tilde{\alpha}_1^M, K \tilde{\alpha}_1^N, K \tilde{\alpha}_1^O \right), \left(K \tilde{\beta}_1^M, K \tilde{\beta}_1^N, K \tilde{\beta}_1^O \right), \left(K \tilde{\gamma}_1^M, K \tilde{\gamma}_1^N, K \tilde{\gamma}_1^O \right) \right)$$

Definition 2.5. [29]. The existing ranking function: Let F(R) be the set of all type-2 normal triangular fuzzy numbers. Using ranking function we can compare two fuzzy Type-2 triangular numbers for solving numerical value problem, define a linear ranking function

$\tilde{R} : F(R) \rightarrow R$ for mapping each fuzzy number into R. for ordering the elements of F(R). Consider $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ as follows:

$$\tilde{\tilde{A}} = (\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = \left\{ \left(\tilde{\alpha}_1^M, \tilde{\alpha}_1^N, \tilde{\alpha}_1^O \right), \left(\tilde{\beta}_1^M, \tilde{\beta}_1^N, \tilde{\beta}_1^O \right), \left(\tilde{\gamma}_1^M, \tilde{\gamma}_1^N, \tilde{\gamma}_1^O \right) \right\},$$

$$\tilde{\tilde{B}} = (\tilde{\varphi}, \tilde{\delta}, \tilde{\chi}) = \left\{ \left(\tilde{\varphi}_1^M, \tilde{\varphi}_1^N, \tilde{\varphi}_1^O \right), \left(\tilde{\delta}_1^M, \tilde{\delta}_1^N, \tilde{\delta}_1^O \right), \left(\tilde{\chi}_1^M, \tilde{\chi}_1^N, \tilde{\chi}_1^O \right) \right\}.$$

Then linear rank of $\tilde{\tilde{A}}$ is defined as $\tilde{R}(\tilde{\tilde{A}})$ and linear rank of $\tilde{\tilde{B}}$ is defined as $\tilde{R}(\tilde{\tilde{B}})$. If the order of F(R) are as follows then :

Case I: If rank of $\tilde{\tilde{A}} \geq$ rank of $\tilde{\tilde{B}}$ then $R(\tilde{\tilde{A}}) \geq R(\tilde{\tilde{B}})$ (i.e. A is greater than B);

Case II: If rank of $\tilde{\tilde{A}} \leq$ rank of $\tilde{\tilde{B}}$ then $R(\tilde{\tilde{A}}) \leq R(\tilde{\tilde{B}})$ (i.e. B is greater than A);

Case III: And If rank of $\tilde{\tilde{A}} =$ rank of $\tilde{\tilde{B}}$ then $R(\tilde{\tilde{A}}) = R(\tilde{\tilde{B}})$ (i.e. A is equal to B).

## 3. Proposed algorithm to find the SPP on type-2 triangular fuzzy numbers

***Step I*:** To find maximum numbers of possible path from source node to destination node and we define path lengths as $S_i, i = 1, 2, 3, \ldots\ldots, m$ for m-possible numbers of paths.

***Step II*:** To calculate the ranking function for all possible path lengths this is defined as $R(S_i)$, where $i = 1, 2, 3, \ldots\ldots, m$

***Step III*:** Now find out the shortest path length and shortest path with the minimum rank.

4. Numerical Example

To show the effectiveness of the existing model on type-2 triangular fuzzy numbers, we consider an example with eight vertices and each of which are connected at least to one of the vertexes on the given graph below.
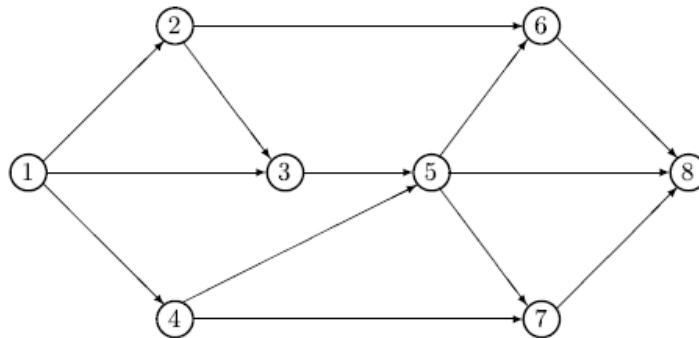
Fig 1. Network with eight vertices.

Table 1. List of Arc lengths

Arc(i, j)	type-2 triangular fuzzy Arc length
(1,2)	{(1.3, 1.8, 2.0), (1.2, 1.5, 2.0), (1.1 , 1.3, 1.5)}
(1,4)	{(1.7, 1.8, 1.8), (1.5, 1.7, 2.0), (1.3 , 1.5, 2.0)}
(1,3)	{(1.8, 1.9, 2.2), (1.6, 1.9, 2.2), (1.5 , 1.7, 2.1)}
(2,3)	{(1.3, 1.5, 1.9), (1.3, 1.4, 2.3), (1.2 , 1.6, 2.3)}
(2,6)	{(1.3, 1.8, 2.0), (1.2, 1.5, 2.0), (1.1 , 1.3, 1.5)}
(3,5)	{(1.5, 1.7, 2.0), (1.3, 1.6, 2.2), (1.2 , 1.6, 2.3)}
(4,5)	{(1.5, 1.7, 2.0), (1.8, 1.9, 2.3), (1.3 , 1.5, 1.9)}
(4,7)	{(1.6, 1.9, 2.2), (1.2, 1.6, 2.3), (1.3 , 1.5, 2.0)}
(5,6)	{(1.5, 1.8, 2.0), (1.7, 1.8, 1.8), (1.8 , 1.9, 2.2)}
(5,7)	{(1.3, 1.5, 1.9), (1.8, 1.9, 2.3), (1.5 , 1.7, 2.0)}
(5,8)	{(1.2, 1.5, 2.0), (1.5, 1.7, 2.0), (1.6 , 1.9, 2.2)}
(6,8)	{(1.3, 1.4, 2.3), (1.6, 1.8, 2.3), (1.3 , 1.6, 2.2)}
(7,8)	{(1.1, 1.3, 1.5), (1.3, 1.5, 2.0), (1.5 , 1.7, 2.1)}

Solution:

Step 1 , i.e. 'maximum numbers of possible paths from source to destination' are as follows:

Table 2. Possible Shortest path From Origin to Destination

Path name	Possible shortest path
S_1	1-2-6-8
S_2	1-2-3-5-6-8
S_3	1-2-3-5-8
S_4	1-2-3-5-7-8
S_5	1-3-5-6-8
S_6	1-3-5-7-8
S_7	1-3-5-8
S_8	1-4-5-7-8
S_9	1-4-7-8
S_{10}	1-4-5-6-8
S_{11}	1-4-5-8

Now the rank of $\tilde{\tilde{s}}_1, \tilde{\tilde{s}}_2, \tilde{\tilde{s}}_3, \tilde{\tilde{s}}_4, \tilde{\tilde{s}}_5, \tilde{\tilde{s}}_6, \tilde{\tilde{s}}_7, \tilde{\tilde{s}}_8, \tilde{\tilde{s}}_9, \tilde{\tilde{s}}_{10}, \tilde{\tilde{s}}_{11}$ are as follows: $R\left(\tilde{\tilde{s}}_1\right) = 5.133$, $R\left(\tilde{\tilde{s}}_2\right) = 8.467$, $R\left(\tilde{\tilde{s}}_3\right) = 6.611$, $R\left(\tilde{\tilde{s}}_4\right) = 8.200$, $R\left(\tilde{\tilde{s}}_5\right) = 7.000$, $R\left(\tilde{\tilde{s}}_6\right) = 6.7333$, $R\left(\tilde{\tilde{s}}_7\right) = 5.144$, $R\left(\tilde{\tilde{s}}_8\right) = 6.967$, $R\left(\tilde{\tilde{s}}_9\right) = 5.167$, $R\left(\tilde{\tilde{s}}_{10}\right) = 7.233$, $R\left(\tilde{\tilde{s}}_{11}\right) = 5.378$ Now we can decide the shortest path with

minimum rank i.e. $R\left(\tilde{\tilde{s}}_1\right)=5.133$, so the corresponding shortest path is 1-2-6-8 with the shortest path length as {(4.4, 5.1, 6.6), (4.4, 5.1, 6.6) , (3.9, 4.5, 5.6)}.

5. Conclusion

In this paper, we used the linear ranking function method to find the shortest path and shortest path length on Type-2 triangular fuzzy number from source node to destination node. Thus it can be concluded that the current method is more effective and versatile to solve problems by dealing with uncertainty and ambiguity and consequently making the required decision as it has been derived from Type-2 triangular fuzzy numbers.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.
[2] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141.
[3] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information sciences*, *8*(3), 199-249.
[4] Zimmermann, H. J. (2011). *Fuzzy set theory—and its applications*. Springer Science & Business Media.
[5] Sheikhi, H. (2012). A novel approach for solving fuzzy multi-objective zero-one linear programming problems. *International Journal of Research in Industrial Engineering*, *1*(1), 42-63.
[6] Rajabi, F., Najafi, S. E., Keshteli, M. H., & Zavardehi, S. M. A. (2013). Solving fuzzy step fixed charge transportation problems via metaheuristics. *International Journal of Research in Industrial Engineering*, *2*(3), 24-34.
[7] Bezdek, J. C. (2013). *Pattern recognition with fuzzy objective function algorithms*. Springer Science & Business Media.
[8] Najafi, H. S., & Edalatpanah, S. A. (2013). A note on "A new method for solving fully fuzzy linear programming problems". *Applied Mathematical Modelling*, *37*(14), 7865-7867.
[9] Zamani, S., Farughi, H., & Soolaki, M. (2013). Contractor selection using fuzzy hybrid AHP-VIKOR. *International Journal of Research in Industrial Engineering*, *2*(4), 26-40.
[10] Shamooshaki, M. M., Hosseinzadeh, A., & Edalatpanah, S. A. (2014). A new method for solving fully fuzzy linear programming with LR-type fuzzy numbers. *International Journal of Data Envelopment Analysis and Operations Research*, *1*(3), 53-55.
[11] Torabi, N., Najafi, E. (2015). New model for ranking based on sum weights disparity index in data envelopment analysis in fuzzy condition. *Journal of Applied Research on Industrial Engineering,* 2(2), 111-119.
[12] Shamooshaki, M. M., Hosseinzadeh, A., and Edalatpanah, S. A. (2015). A new method for solving fully fuzzy linear programming problems by using the lexicography method. *Applied and Computational Mathematics,* 1, 53-55.
[13] Najafi, H. S., Edalatpanah, S. A., & Dutta, H. (2016). A nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. *Alexandria Engineering Journal*, *55*(3), 2589-2595.
[14] Oztaysi, B., Onar, S. C., Kahraman, C., & Yavuz, M. (2017). Multi-criteria alternative-fuel technology selection using interval-valued intuitionistic fuzzy sets. *Transportation Research Part D: Transport and Environment*, *53*, 128-148.

[15]   Kauffman, A., & Gupta, M. M. (1985). *Introduction to fuzzy arithmetic, theory and application.* New York, Van Nostrand Reinhold.

[16]   Dinagar, D. S., & Latha, K. (2013). Some types of type-2 triangular fuzzy matrices. *International Journal of Pure and Applied Mathematics*, 82(1), 21-32.

[17]   Dubois, D. J. (1980). *Fuzzy sets and systems: theory and applications* (Vol. 144). Academic press.

[18]   Klein C.M. (1991). Fuzzy shortest paths. *Fuzzy Sets and Systems,* 39, 27-41.

[19]   Lin, K. C., & Chern, M. S. (1993). The fuzzy shortest path problem and its most vital arcs. *Fuzzy Sets and Systems*, 58(3), 343-353.

[20]   Li, Y., Gen, M., & Ida, K. (1996). Solving fuzzy shortest path problems by neural networks. *Computers & industrial engineering*, 31(3), 861-865.

[21]   Chuang, T. N., & Kung, J. Y. (2005). A new approach for the fuzzy shortest path problem. *Computational Intelligence for Modelling and Prediction*, 2, 89-100.

[22]   Yu, J. R., & Wei, T. H. (2007). Solving the fuzzy shortest path problem by using a linear multiple objective programming. *Journal of the Chinese Institute of Industrial Engineers*, 24(5), 360-365.

[23]   Mahdavi, I., Nourifar, R., Heidarzade, A., & Amiri, N. M. (2009). A dynamic programming approach for finding shortest chains in a fuzzy network. *Applied Soft Computing*, 9(2), 503-511.

[24]   Zhang, Y., Zhang, Z., Deng, Y., & Mahadevan, S. (2013). A biologically inspired solution for fuzzy shortest path problems. *Applied Soft Computing*, 13(5), 2356-2363.

[25]   Jain R. (1976). Decision-making in the presence of fuzzy variables. *IIIE Transactions on Systems, Man and Cybernetics*, 6, 698-703.

[26]   Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & Mathematics with Applications*, 57(3), 413-419.

[27]   Anusuya, V., & Sathya, R. (2013). Type-2 fuzzy shortest path. *International Journal of fuzzy mathematical Archive*, 2, 36-42.

[28]   Malini, P., & Ananthanarayanan, M. (2016). Solving fuzzy transportation problem using ranking of octagonal fuzzy numbers. *International Journal of Pure and Applied Mathematics*, 110(2), 275-282.

[29]   Dinagar, D. S., & Latha, K. (2013). Some types of type-2 triangular fuzzy matrices. *International Journal of Pure and Applied Mathematics*, 82(1), 21-32.