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Addendum to "A generalized Nash equilibrium for a bioeconomic problem of fishing"

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P A P E R I N F O	A B S T R A C T
Chronicle: Received: 12 September 2017 Revised: 16 November 2017 Accepted: 18 November 2017 Available: 20 November 2017 **Keywords** : Steady state. Stability study. Variational matrix. Routh Hurwitz stability criterion.	In [1] the authors have set up a bio-economic equilibrium model to understand the interactions between three fish populations and seek to maximize the profits of fishermen. The authors then studied the existence of stationary states and their stability using eigenvalue analysis. This is not correct in a dimension 3.In this paper, we correct the proof which appeared in page 190 in [1] by using Routh Hurwitz stability Criterion.

In our earlier paper [1] page 190, we have used the trace and the determinant of the variational matrix to proof that the last steady state $P(B_1, B_2, B_3)$ is locally asymptotically stable (see [1] for notations). Whereas this method is not correct in the case of three species. Actually, the correct proof is given by using Routh Hurwitz stability Criterion as follows:

Theorem. The steady state $P(B_1, B_2, B_3)$ is locally asymptotically stable.

Proof: The characteristic polynomial of the variational matrix of system (1) in [1] at the steady state $P(B_1, B_2, B_3)$ is:

$$P(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3. \quad (1)$$

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Where,

$$a_1 = \frac{r_1}{K_1}B_1 + \frac{r_2}{K_2}B_2 + \frac{r_3}{K_3}B_3. \quad (2)$$

$$a_2 = \left(\frac{r_1}{K_1}\frac{r_2}{K_2} - c_{12}c_{21}\right)B_1B_2 + \left(\frac{r_1}{K_1}\frac{r_3}{K_3} - c_{13}c_{31}\right)B_1B_3 + \left(\frac{r_2}{K_2}\frac{r_3}{K_3} - c_{23}c_{32}\right)B_2B_3. \quad (3)$$

And,

$$a_3 = \left(\frac{r_1}{K_1}\frac{r_2}{K_2}\frac{r_3}{K_3} + c_{12}c_{23}c_{31} + c_{13}c_{32}c_{21} - \frac{r_1}{K_1}c_{23}c_{32} - \frac{r_2}{K_2}c_{13}c_{31} - \frac{r_3}{K_3}c_{12}c_{21}\right)B_1B_2B_3. \quad (4)$$

It is easy to see that:

$$a_1 > 0. \quad (5)$$

Using that (see [1])

$$r_ir_j - c_{ij}c_{ji}K_iK_j > 0; i,j = 1,2,3. \quad (6)$$

Then,

$$a_2>0,\ a_3>0,\ and\ a_1a_2 > a_3. \quad (7)$$

In fact:

$$\begin{aligned} a_1a_2 - a_3 = {} & r_1\frac{B_1}{K_1}r_2\frac{B_2}{K_2}r_3\frac{B_3}{K_3} - c_{12}B_1c_{23}B_2c_{31}B_3 + r_1\frac{B_1}{K_1}r_2\frac{B_2}{K_2}r_3\frac{B_3}{K_3} - c_{21}B_1c_{32}B_2c_{31}B_3 \\ & + r_1\frac{B_1}{K_1}\left(r_1\frac{B_1}{K_1}r_2\frac{B_2}{K_2} - c_{12}B_1c_{21}B_2\right) + r_2\frac{B_2}{K_2}\left(r_2\frac{B_2}{K_2}r_3\frac{B_3}{K_3} - c_{23}B_2c_{32}B_3\right) \\ & + r_3\frac{B_3}{K_3}\left(r_1\frac{B_1}{K_1}r_3\frac{B_3}{K_3} - c_{13}B_1c_{31}B_3\right) + r_1\frac{B_1}{K_1}\left(r_1\frac{B_1}{K_1}r_3\frac{B_3}{K_3} - c_{13}B_1c_{31}B_3\right) \\ & + r_2\frac{B_2}{K_2}\left(r_2\frac{B_2}{K_2}r_1\frac{B_1}{K_1} - c_{12}B_1c_{21}B_2\right) + r_3\frac{B_3}{K_3}\left(r_2\frac{B_2}{K_2}r_3\frac{B_3}{K_3} - c_{23}B_2c_{32}B_3\right) \end{aligned} \quad (8)$$

From [1], we deduce that:

$$a_1a_2 - a_3 > 0. \quad (9)$$

Then, using the Routh Hurwitz stability criterion we conclude that the steady state point

$P(B_1, B_2, B_3)$ is locally asymptotically stable.

References

[1] Elfoutayeni, Y., Khaladi, M., & Zegzouti, A. (2012). A generalized Nash equilibrium for a bioeconomic problem of fishing. *Stud. Inform. Univ.*, *10*(1), 186-204.